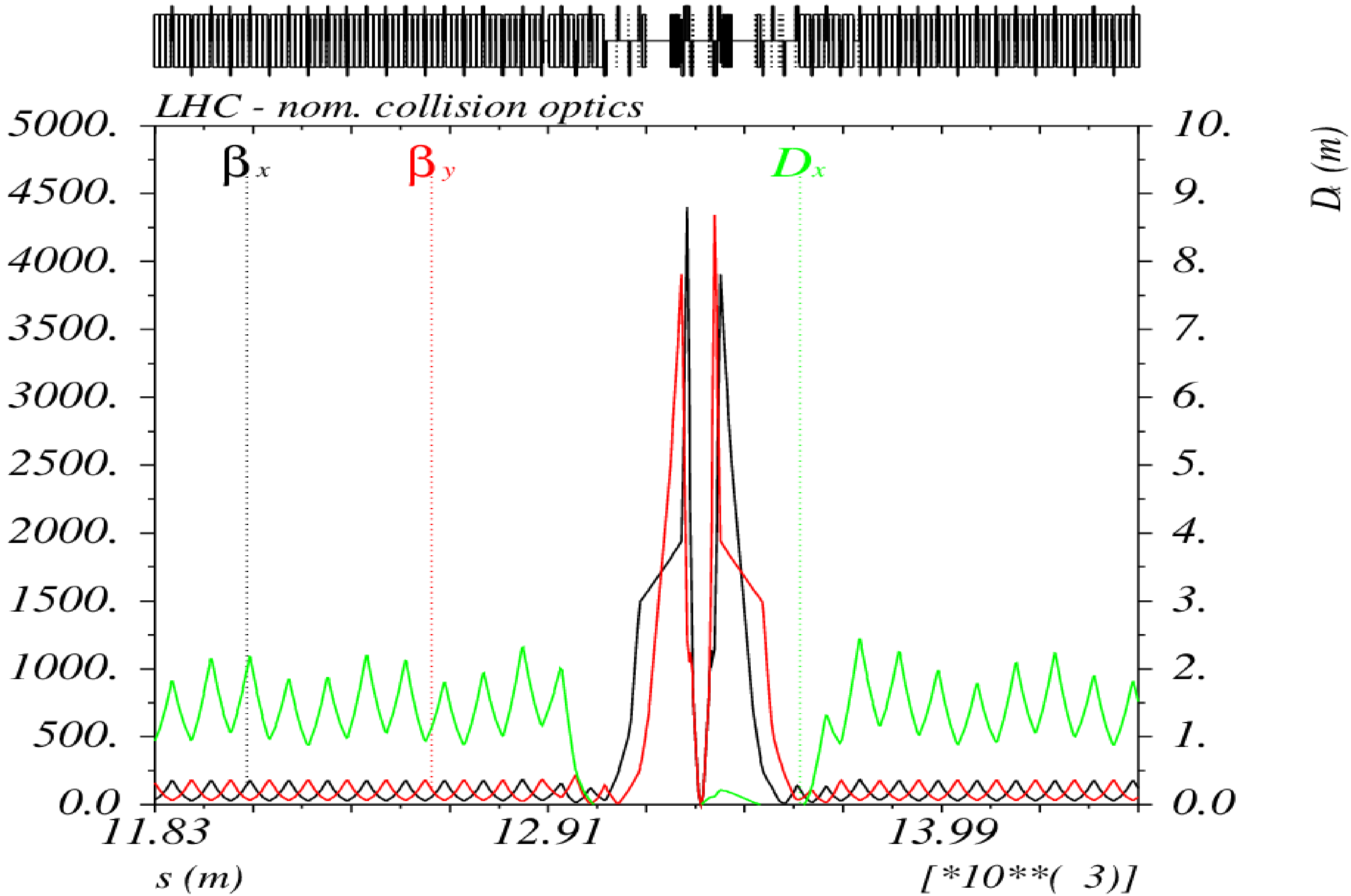
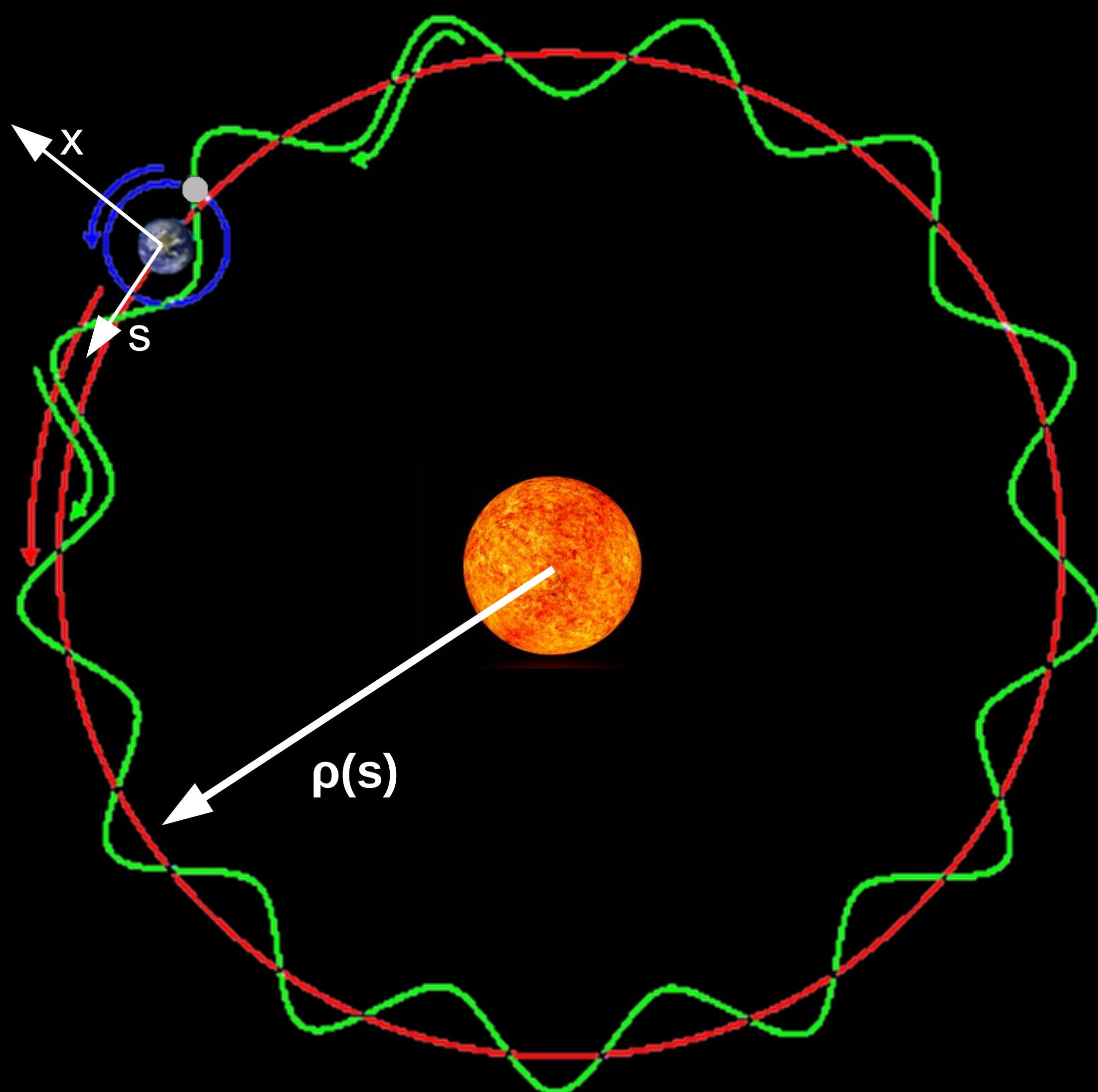
Accelerator Lattice Design – Part II Insertions

Ralph J. Steinhagen, CERN



- Acknowledgements and credits to: W. Herr, B. Holzer, A. Streun, A. Wolski

Moon's Trajectory around Sun

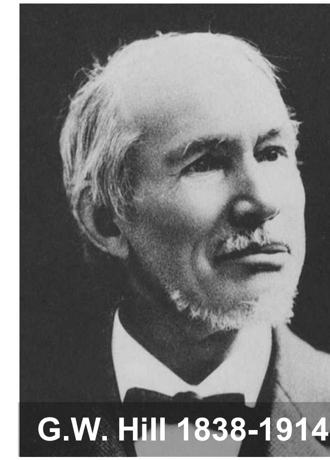


Recap: Transverse Beam Dynamics

- Hill's equation[1,2]:

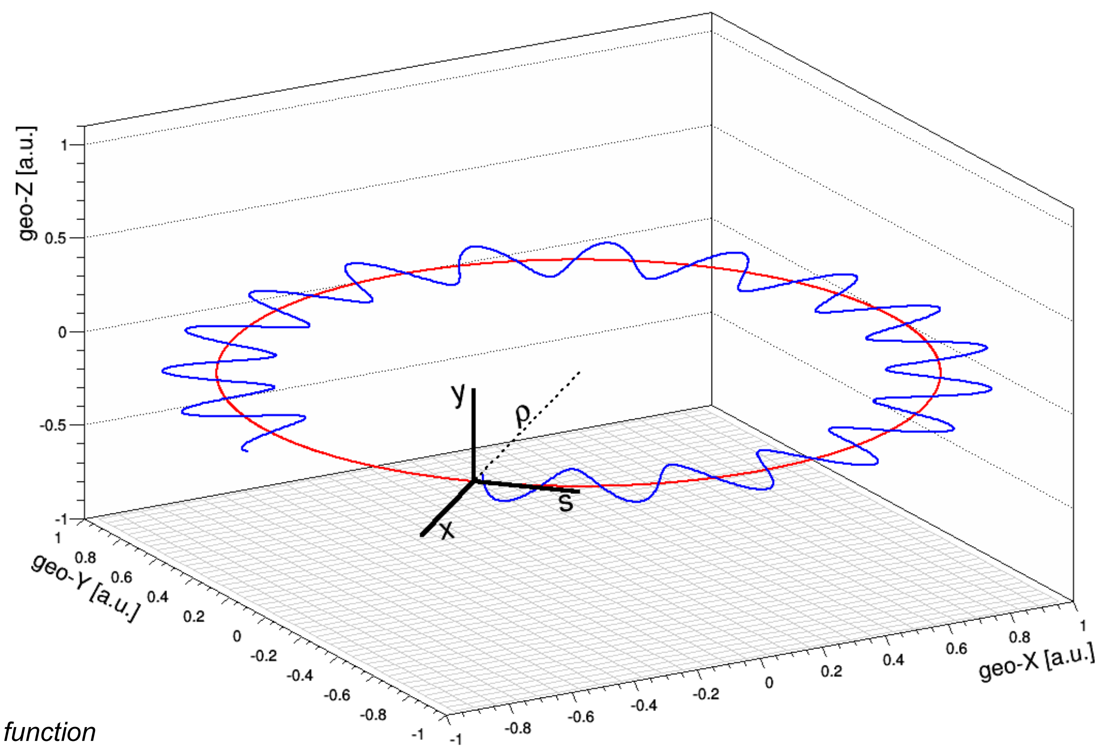
$$z'' + K(s)\cdot z = f(s,t)$$

$$K(s) = \underbrace{\frac{q}{p} B_{dipole}}_{weak\ focusing:\ \frac{1}{\rho}} - \underbrace{\frac{q}{p}\frac{\partial B_y}{\partial x}}_{strong\ focusing:\ k(s)}$$


G.W. Hill 1838-1914

  – k(s): focusing strength, defines:
    - betatron function β(s) → envelope of the oscillation
    - dispersion function D(s) → trajectory for off-momentum $\Delta p/p_0$ particles

  – f(s,t): external driving force



[1]George William Hill, *"On the part of the motion of the lunar perigee which is a function of the mean motions of the sun and moon"*, Acta Mathematica, 8:1–36, 1886
[2]coordinate 'z' being place holder for either x,y

Recap: Transverse Beam Dynamics

- Hill's equation for non-zero momentum spread $\Delta p/p_0$

$$z'' + K(s)\cdot z = \frac{\Delta p}{p_0}\cdot\frac{1}{\rho}$$

$$K(s) = \underbrace{\left(\frac{q}{p}B_{dipole}\right)^2}_{weak\ focusing:\frac{1}{\rho^2}} - \underbrace{\frac{q}{p}\frac{\partial B_y}{\partial x}}_{strong\ focusing:k(s)}$$

- General Solution

$$z(s) = \underbrace{z_{co}(s)}_{closed\ orbit} + \underbrace{D(s)\cdot\frac{\Delta p}{p_0}}_{dispersion\ orbit} + \underbrace{z_\beta(s)}_{betatron\ oscillations}$$

- Propagation through the ring:

$$D(s) = \underbrace{m_{12}(s)}_{\sim\cos(\mu)}\cdot\oint\frac{1}{\rho(\tilde{s})}m_{11}(\tilde{s})\,d\tilde{s} - \underbrace{m_{21}(s)}_{\sim\sin(\mu)}\cdot\oint\frac{1}{\rho(\tilde{s})}m_{12}(\tilde{s})\,d\tilde{s}$$

- Two important take-away messages:
  - weak dipoles → large bending radius → small dispersion
  - Drifts or quadrupoles: $1/\rho = 0$

Recap: Transverse Beam Dynamics

- Defines add. 'Twiss' functions[1]: betatron phase advance μ(s), α(s) & γ(s)

$$\Delta\mu(s) := \int_0^s \frac{1}{\beta(s')}\,ds' \qquad \alpha(s) := -\frac{\beta'(s)}{2} \qquad \gamma(s) := \frac{1+\alpha^2(s)}{\beta(s)}$$

- First-order solution to Hill's equation:

$$z(s) = \underbrace{z_{co}(s)}_{\text{closed orbit}} + \underbrace{D(s)\cdot\frac{\Delta p}{p_0}}_{\text{dispersion orbit}} + \underbrace{z_\beta(s)}_{\text{betatron oscillations}}$$

→ sinusoidal particle motion in accelerators:

$$z_\beta(s) = \sqrt{\epsilon_i\beta(s)}\cdot\sin(\mu(s)+\phi_i)$$

ε_i, Φ_i : initial particle state

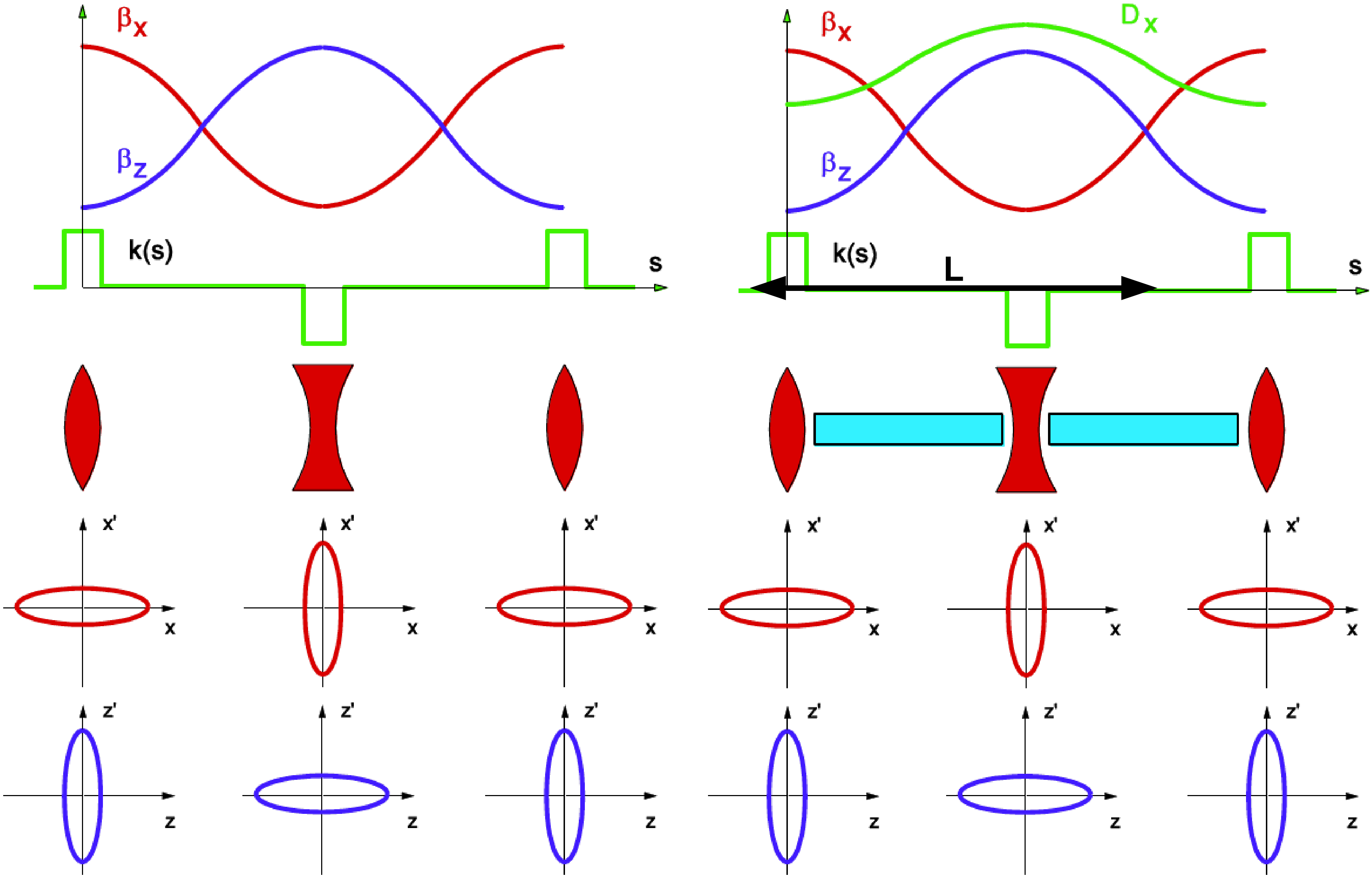
- Courant-Synder invariant of motion (↔ energy conservation)

$$\epsilon = \beta(s)\cdot x'^2 + 2\alpha(s)\cdot xx' + \gamma(s)\cdot x^2$$

[1]Richard Q. Twiss and N. H. Frank, "Orbital stability in a proton synchrotron", Rev. Sci. Instr., 20(1):1–17, January 1949.

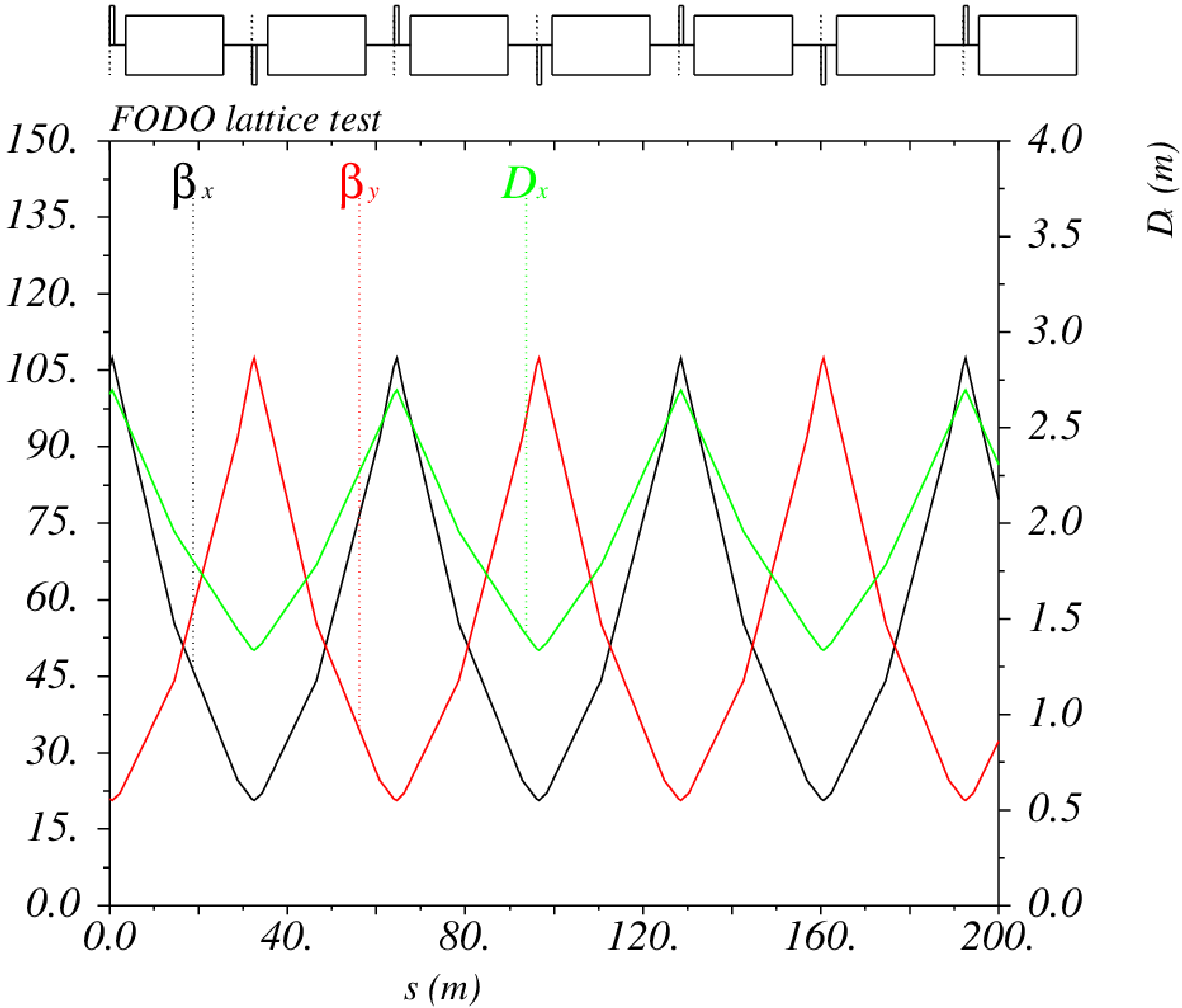
shorthand: $x' = \frac{dx}{ds}$

Recap: FoDo Lattice



Courtesy L. Rivkin, EPFL & PSI

Recap: FoDo Lattice



Recap: FoDo Lattice – Summary

- FoDo cell transfer matrix (→ tutorial)

$$M_{FoDo} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & L\left(1 + \frac{L}{2f}\right) \\ \left(\frac{L^2}{2f^3} - \frac{L}{f^2}\right) & 1 - \frac{L^2}{2f^2} \end{pmatrix}$$

- Phase advance per cell (→ tutorial)
  - N.B. also correct for non-FoDo cells

$$\cos\mu_{cell} = \frac{1}{2}\, trace(M)$$

$$f = \pm\frac{L}{4\sin\frac{\mu}{2}} = (k\, l_q)^{-1}$$

$$\beta^{\pm} = \frac{L\left(1 \pm \sin\frac{\mu}{2}\right)}{\sin\frac{\mu}{2}}$$

$$\alpha^{\pm} = \frac{\mp 1 - \sin\frac{\mu}{2}}{\cos\frac{\mu}{2}}$$

$$D^{\pm} = \frac{L\varphi\left(1 \pm \frac{1}{2}\sin\frac{\mu}{2}\right)}{4\sin^2\frac{\mu}{2}}$$

$$\xi_{FODO} = -\frac{1}{\pi}\tan\frac{\mu}{2}$$

- Equations guide 1st-order cell design
→ input for non-linear numerical optimisations (tutorial)

Overview

- Part I – we learned …
  - … how to design simple FoDo lattice,
  - … how to compute optical functions,
  - … to match simple global parameters (tune, chromaticity),
  - … and did some simple particle tracking.
- Part II
  - Machines with imperfections and corrections (dispersion)
  - Design of insertions
    - Dispersion suppressor
    - Low-β insertion

Matching Local Optical Parameters

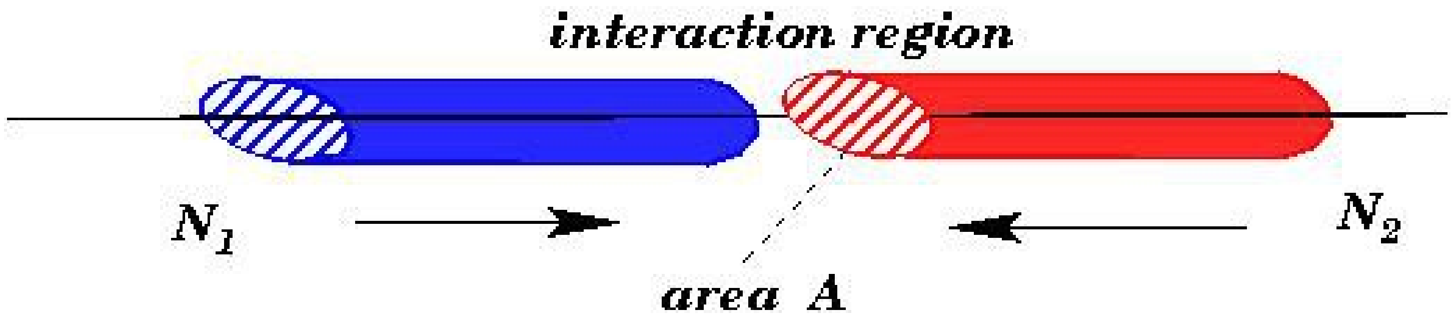
- To get the optical configuration you want matching

- Main applications:
  - Setting global optical parameters
    - Tune and chromaticity
  - Setting local optical parameters
    - Low (or high) β insertions
    - Dispersion suppressors
  - Correction of imperfections

→ Adjust strength of individual machine elements

Luminosity: $dN_{event}/dt = L\,\sigma_{process}$

- Collider design:



$$L_{peak} \approx \frac{f_{rev} k_b \cdot N_b^2}{4\pi\sigma_x\sigma_y} \cdot F = \frac{f_{rev}\,\gamma\,k_b \cdot N_b^2}{4\pi\beta^*\epsilon_n} \cdot F$$

$$\sigma_{x,y} = \sqrt{\frac{\epsilon^*\beta(s)}{\gamma}} + \ldots$$

- N_b: number of particles per bunch,
- k_b: total number of bunches,
- σ_x, σ_y: hor./vert. r.m.s. beam size in IR,
- f_{rev}: revolution or repetition frequency,
- $F_{corr.}$: numerical correction factors (hour-glass, crossing angle, ...),
- ε: emittance (invariant of motion, ~"temperature of bunch")

Most Simple Example – Drift Space I/II

$$M_{drift} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

- Particle coordinates:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$x(L) = x_0 + L \cdot x'_0$$
$$x'(L) = x'_0$$

- Transformation of Twiss parameters

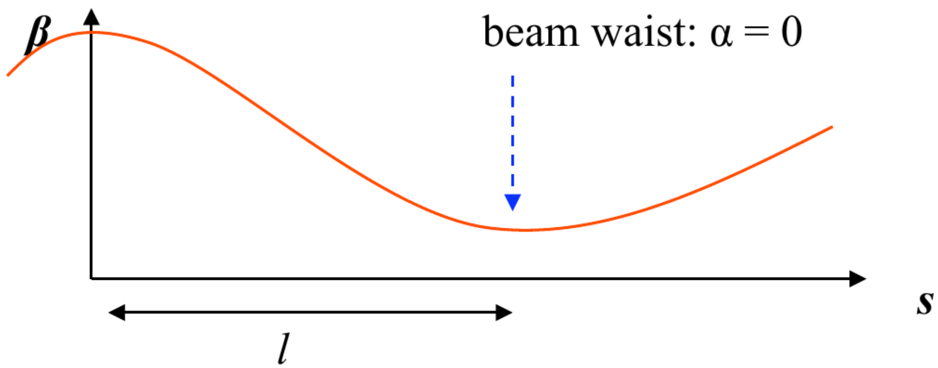
$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 & -2\mathrm{L} & L^2 \\ 0 & 1 & -L \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

$$\beta(s) = \beta_0 - 2\alpha_0 \cdot L + \gamma_0 \cdot L^2$$

$$\alpha(s) = \alpha_0 - L \cdot \gamma_0$$

→ equation being important for low-beta insertions

Most Simple Example – Drift Space II/II



(I) $\beta(s) = \beta_0 - 2\alpha_0 \cdot L + \gamma_0 \cdot L^2$

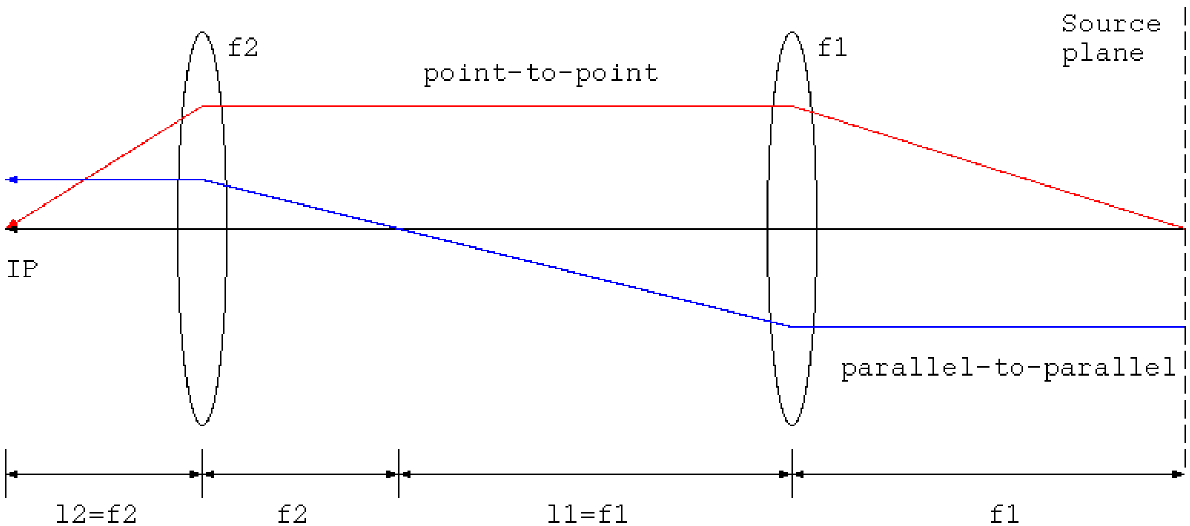
(II) $\alpha(s) = \alpha_0 - L \cdot \gamma_0$

- Beam size is smallest at $\alpha(s) = 0 \rightarrow \alpha_0 = \gamma_0 \cdot s$ $\rightarrow L = \alpha_0/\gamma_0$
- Beam size at that point: $\gamma(L) = \gamma_0$ & $\alpha(L) = 0$ $\rightarrow \beta(L) = 1/\gamma_0$
- Inserting in (I):

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

- Phase advance: $\Delta\mu(s) := \int_{-L}^{+L} \frac{1}{\beta(s')} ds' \approx \pi$

Telescope and low β



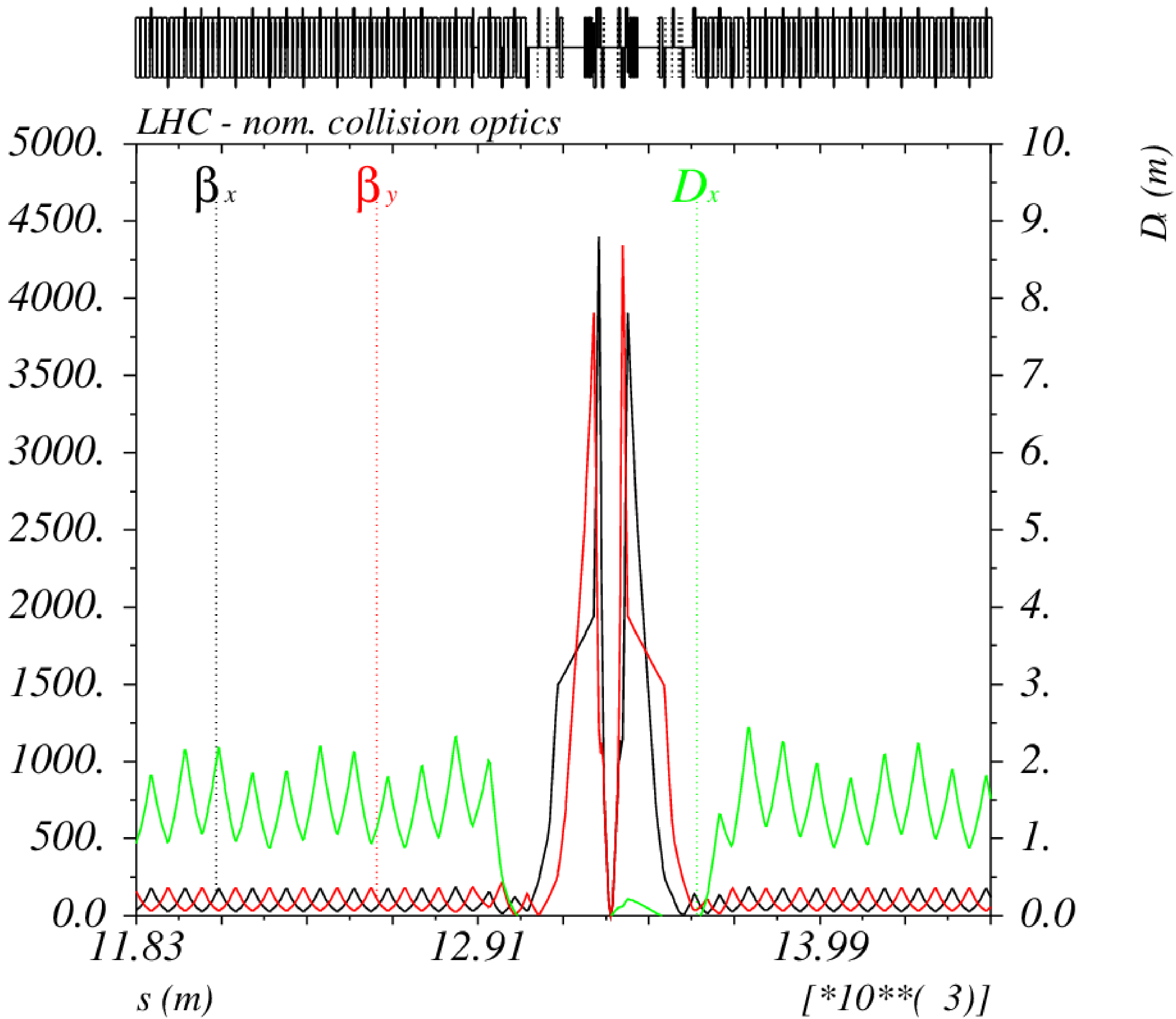
$$\begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - l_1/f_1 & 2l_1 - l_1^2/f_1 \\ -1/f_1 & 1 - l_1/f_1 \end{pmatrix}$$

For one module with l1=f1 $\begin{pmatrix} 0 & f_1 \\ -1/f_1 & 0 \end{pmatrix}$

For both modules: $R = \begin{pmatrix} -f_2/f_1 & 0 \\ 0 & -f_1/f_2 \end{pmatrix}$

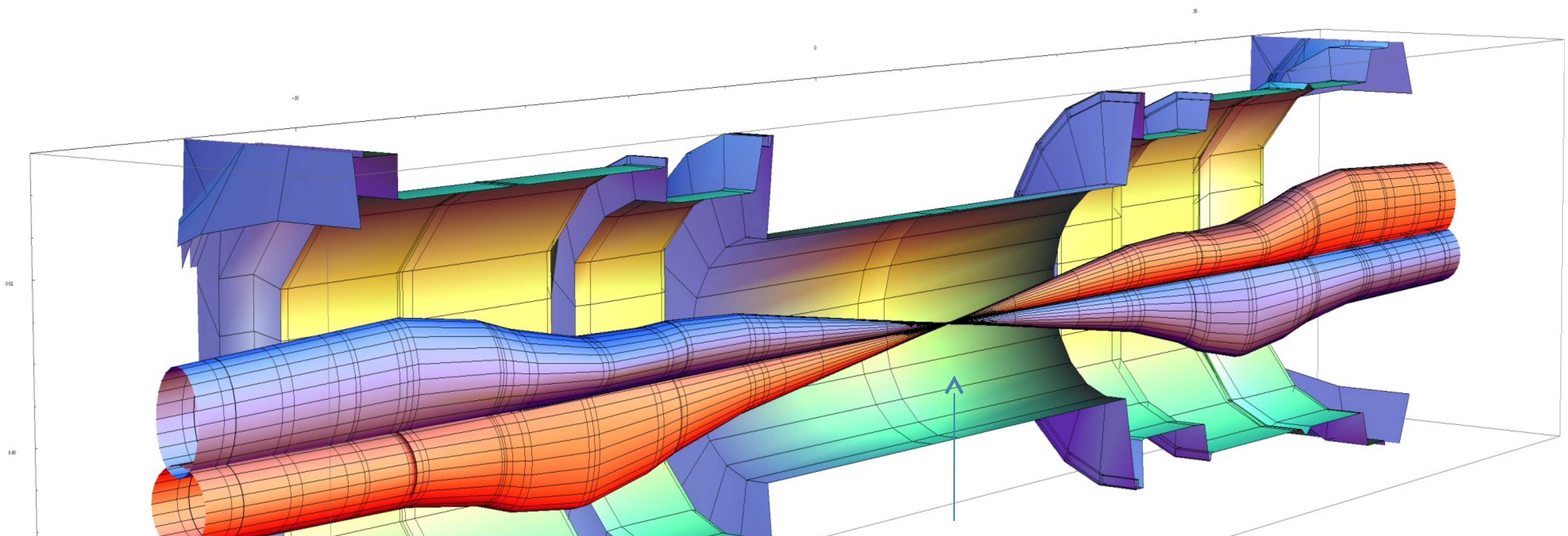
- Doublet or Triplet
- Point-to-point R12=0
- Parallel to parallel R21=0
- R11=demagnification
- Ratio of focal lengths
- Needs to work in both planes with doublets/triplets

Example LHC



LHC - nom. collision optics
β x
β y
Dx
5000.
4500.
4000.
3500.
3000.
2500.
2000.
1500.
1000.
500.
0.0
10.
9.
8.
7.
6.
5.
4.
3.
2.
1.
0.0
Dx (m)
11.83
12.91
13.99
s (m)
[*10**(3)]

LHC: Squeezing in ATLAS – Beam Envelope



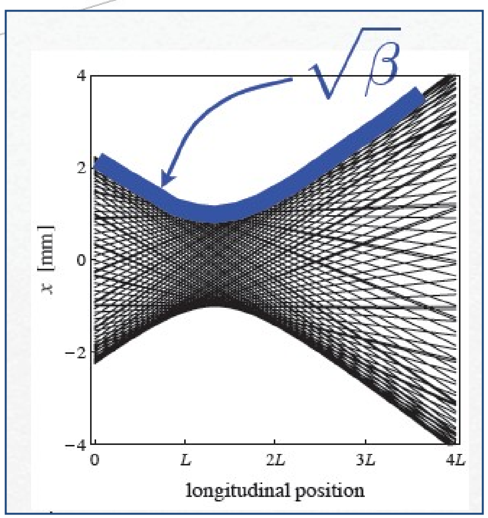
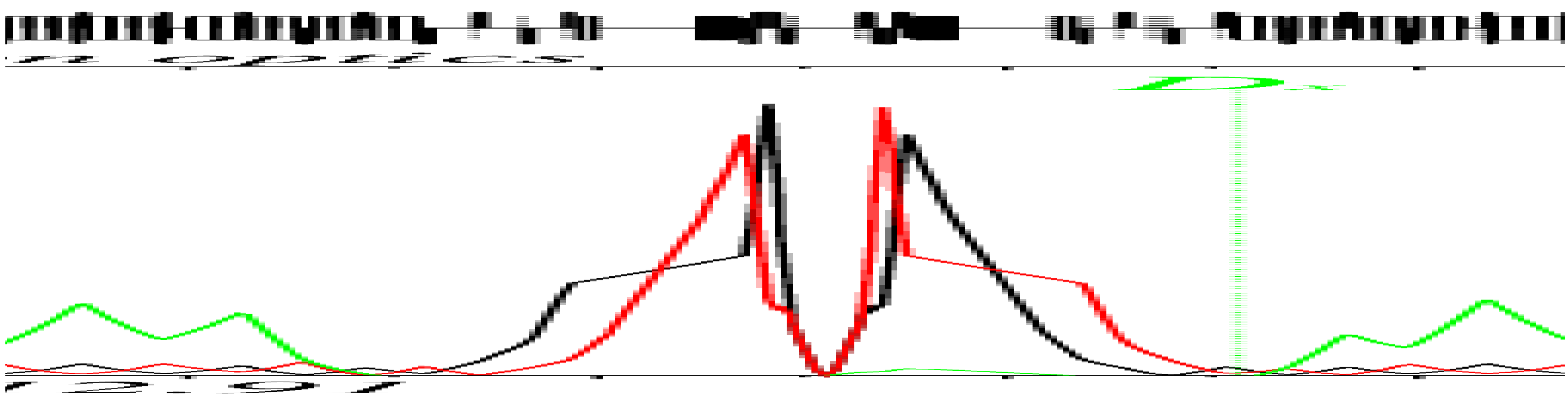


Image courtesy John Jowett

Low β: Design Guidelines

1. Calculate the periodic solution in the arc (e.g. FODO)
2. Insert the drift space for the insertion devices (e.g. undulator/detector)
3. Put a quadrupole doublet (or: triplet) as close as possible
4. Introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure

- Parameters to be optimised and matched to the periodic solution:
  - β_x, α_x, D_x, β_y, α_y, D_y, Q_x, Q_y → requires (at least) 8 independent quadrupoles



Dispersion Suppressors

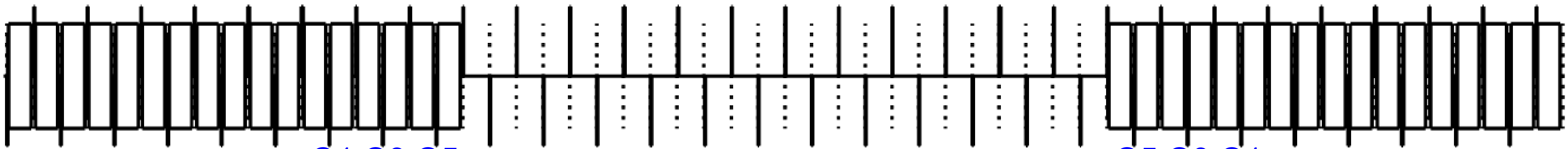
- Beam size:

$$\sigma(s) = \sqrt{\epsilon \cdot \beta(s) + \left(D(s) \cdot \frac{\Delta p}{p_0}\right)^2}$$

  - for small emittances dominated by dispersion
  - need small sizes? → small dispersion!
    - for electron machines this is notably in the vertical plane
    - N.B. off-centre beam in quadrupoles → dipole kick → dipsersion

- Various dispersion Suppressor schemes
  - “Full-Match”
  - “Missing-bend”
  - “Half-bend”

Dispersion Suppressor – Quadrupole Match I/II



- Idea: use 6 additional independent quadrupoles
  - Dispersion suppressed by two quadrupoles
  - β and α restored to the values of the periodic solutions by 4 additional quads

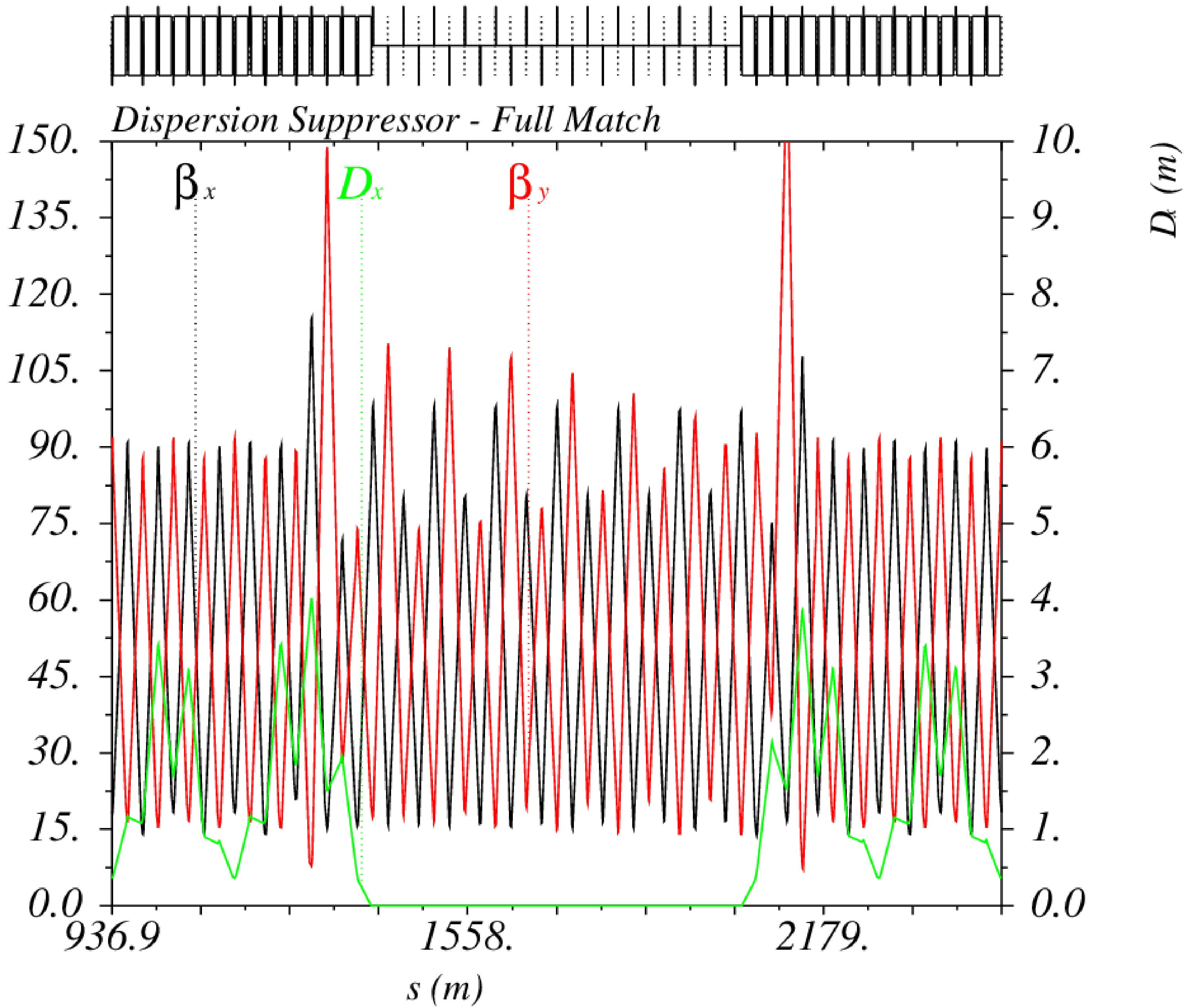
Advantage:

- Easy
- works for any phase-advance per cell
- Does not change the geometry of the storage ring
- Can be used to match between different lattice structures

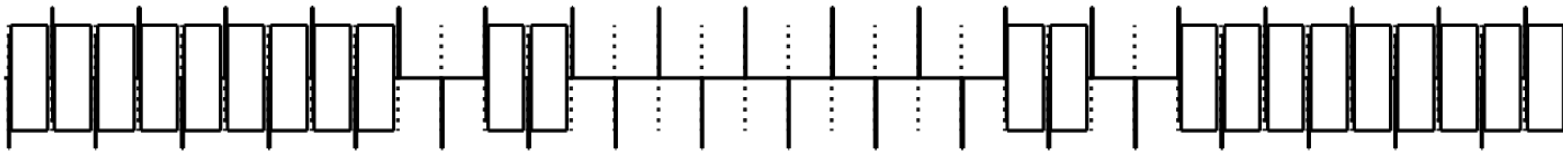
Disadvantage:

- additional power supplies needed (costs!)
- Requires stronger quadrupoles
- Due to higher-β values: more aperture needed

Dispersion Suppressor – Quadrupole Match II/II



Dispersion Suppressor – Missing Bend I/II



- Idea: create dispersion wave matching conditions at the first quadrupole

$$\frac{2m+n}{2}\Phi_C = (2k+1)\frac{\pi}{2}$$

  - m = # of cells without dipoles, followed by n regular arc cells, Φ_c: cell phase-advance

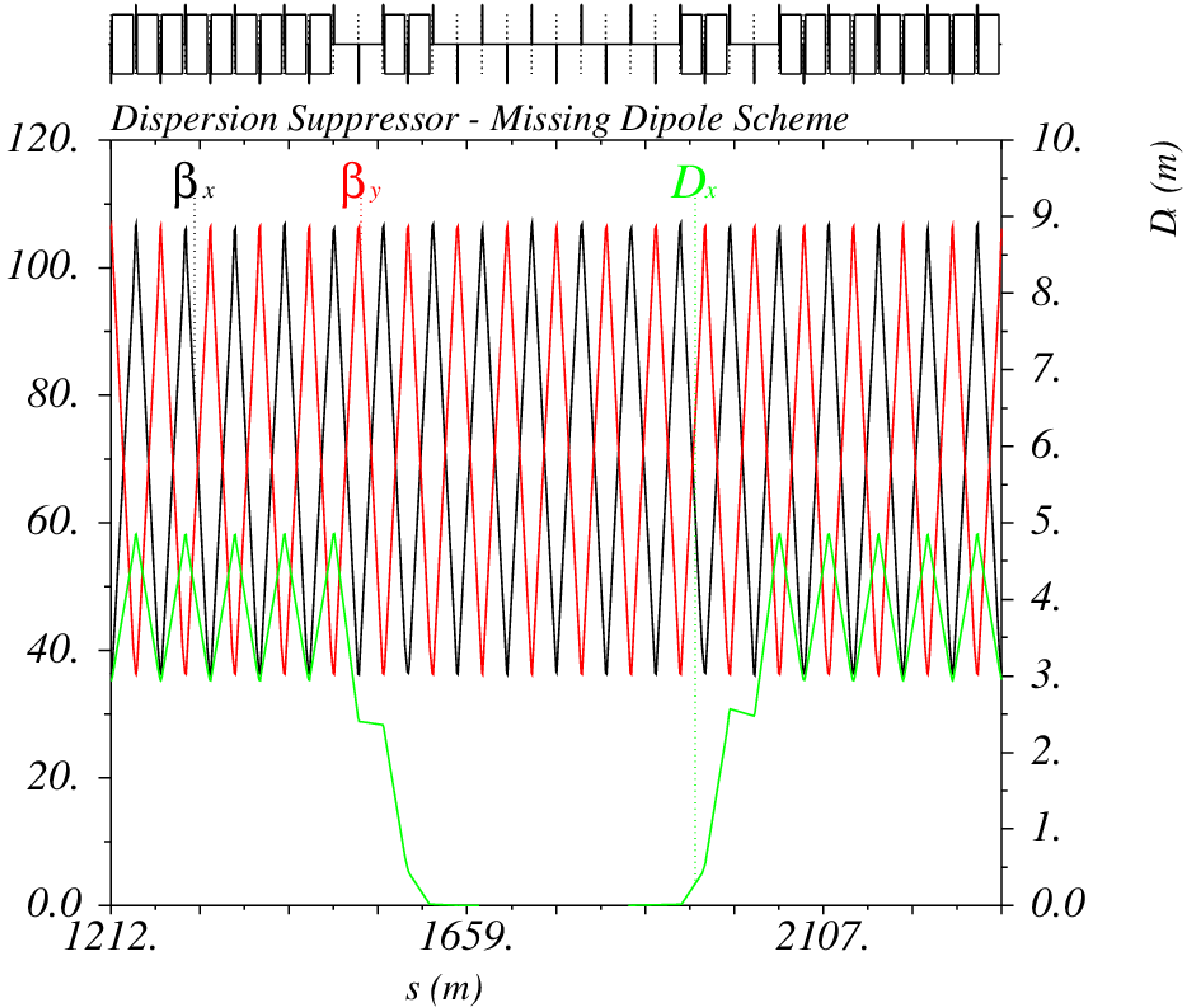
Advantage:

- Cost-effective
- Keeps the geometry of the arc cells

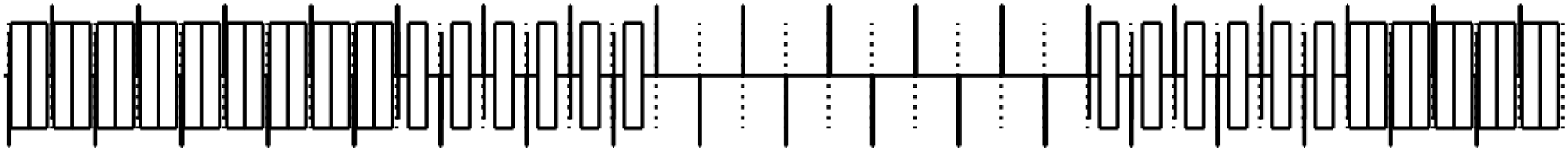
Disadvantage:

- In its simplest form constraints possible cell phase-advances Φ_c

Dispersion Suppressor – Missing Bend II/II



Dispersion Suppressor – Half-Bend I/II



- Idea: condition for vanishing dispersion

$$2 * \delta_{supr} * \sin^2\left(\frac{n\Phi_c}{2}\right) = \delta_{arc}$$

$$\delta_{supr} = \frac{1}{2} * \delta_{arc} \qquad \sin(n\Phi_c) = 0$$

  – n = # of cells with half-strength dipoles, Φ_c: cell phase-advance

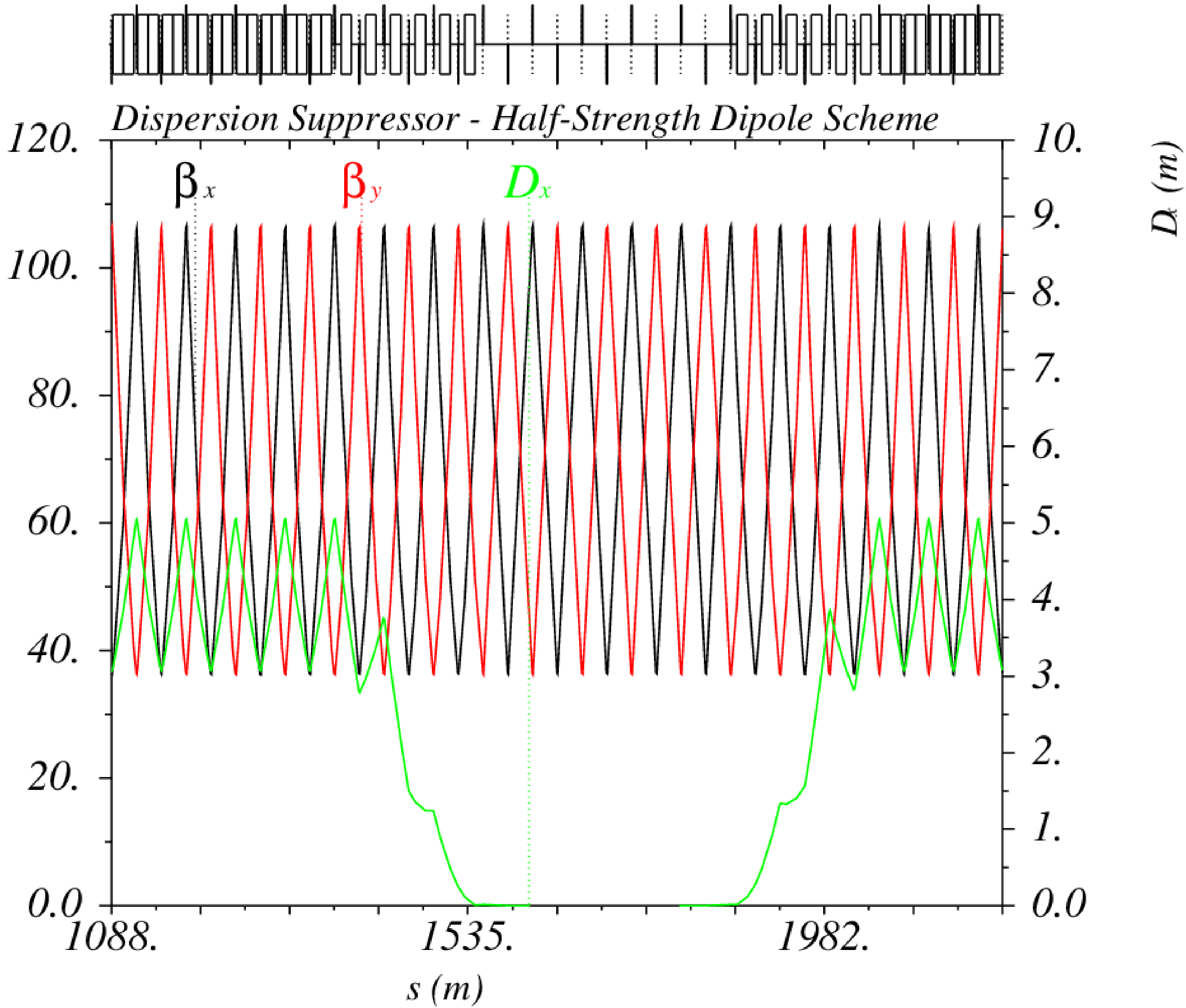
Advantage:

- Cost-effective
- Keeps the geometry of the arc cells

Disadvantage:

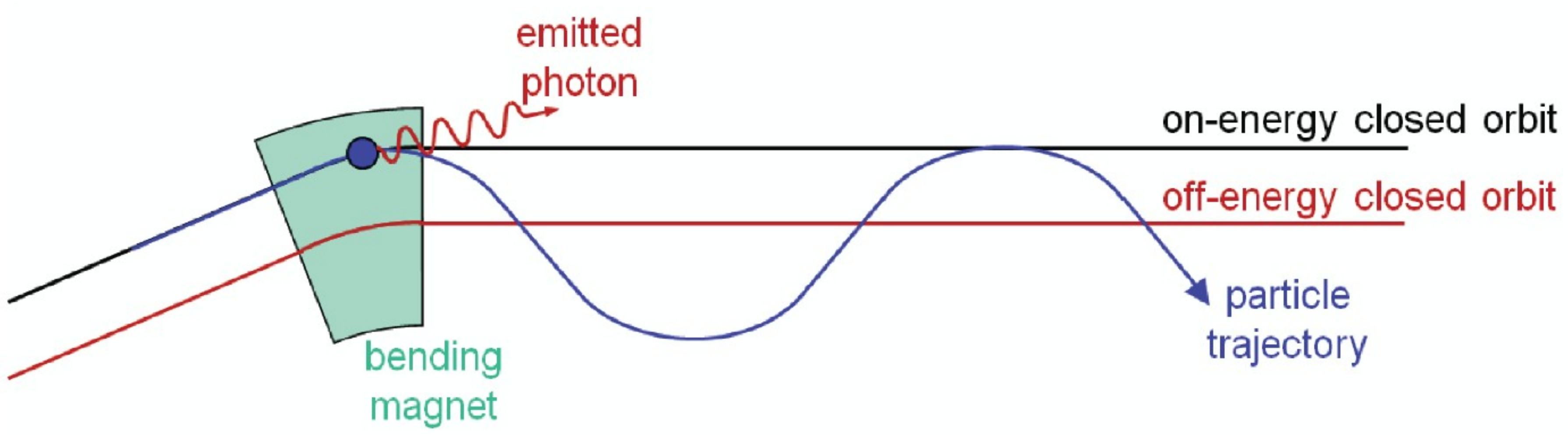
- In its simplest form constraints possible cell phase-advances Φ_c

Dispersion Suppressor – Half-Bend II/II



Natural Emittance

- Quantum nature of synchrotron radiation emission
  → finite electron beam size, emittance and energy spread



- oscillation damping and excitation counterbalance

$$\frac{d\varepsilon}{dt} = \frac{\gamma D^2 + 2\alpha D D' + \beta D'^2}{T} \delta_{rms}^2$$

Radiation Integrals

$$I_1 = \oint \frac{D_x}{\rho} ds, \qquad \alpha_p = \frac{I_1}{C_0}$$

$$I_2 = \oint \frac{1}{\rho^2} ds,$$

$$I_3 = \oint \frac{1}{|\rho^3|} ds,$$

$$I_4 = \oint \frac{D_x}{\rho} \left(\frac{1}{\rho^2} + 2 k_1 \right) ds, \qquad \text{with: } k_1 = \frac{q}{p_0} \frac{\partial B_y}{\partial x}$$

$$I_5 = \oint \frac{H_x}{|\rho^3|} ds, \qquad \text{with: } \boxed{H_x = \gamma_x D_x^2 + 2 \alpha_x D_x D' + \beta D'^2}$$

Natural Emittance

- Quantum nature of synchrotron radiation emission
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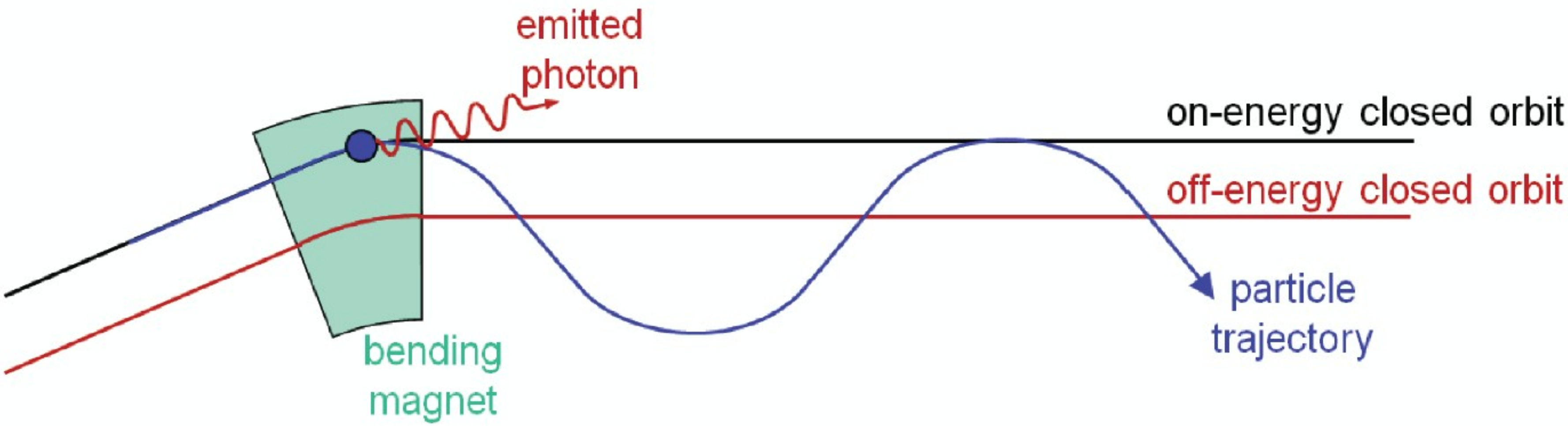


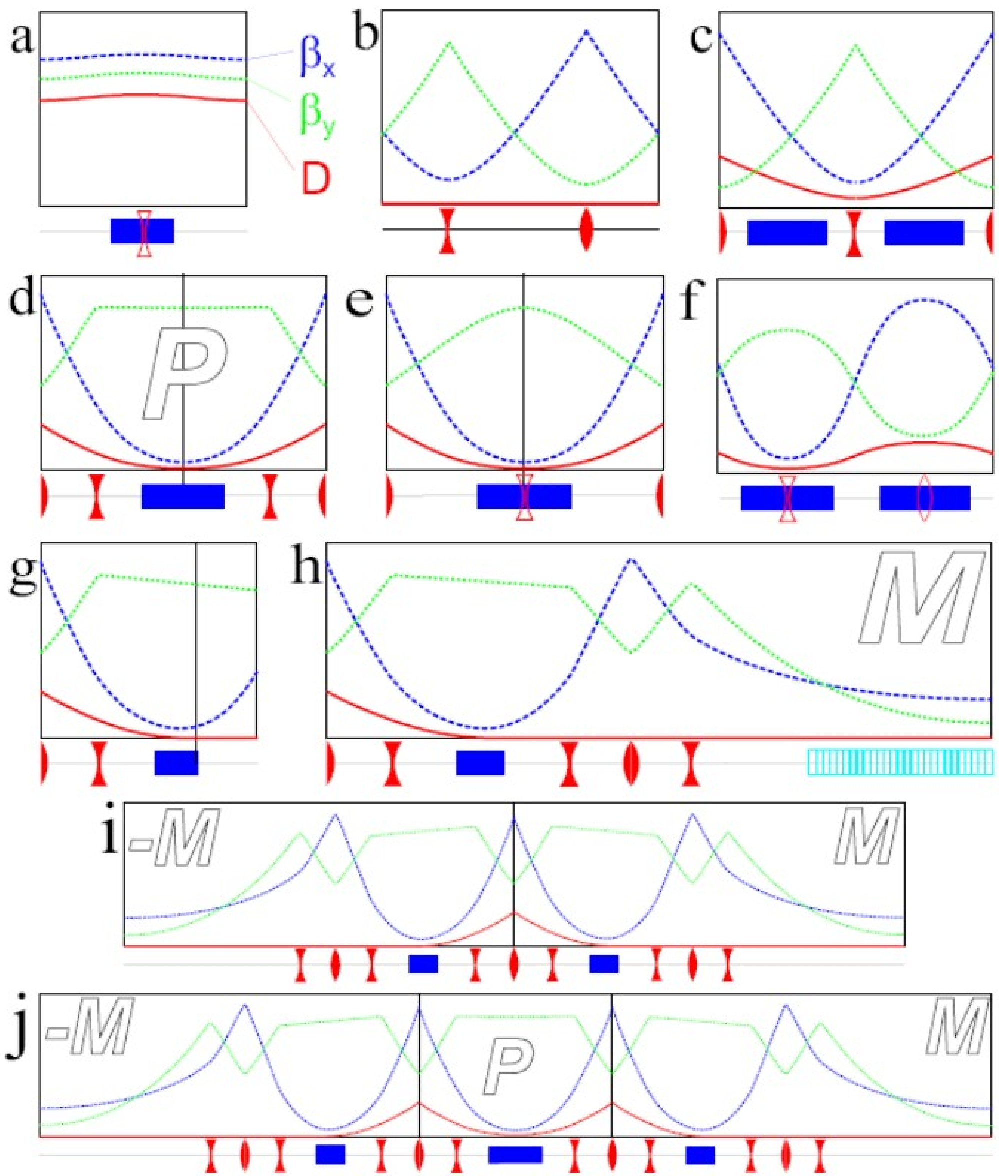
- damping and excitation counterbalance → equilibrium emittance:

$$\epsilon_0 = C_q \gamma^2 \frac{I_5}{j_x I_2}, \qquad j_x = 1 - \underbrace{\frac{I_4}{I_2}}_{\approx 1}, \qquad \text{and} \qquad C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \approx 3.832 \cdot 10^{-13} m$$

$$I_2 = \oint \frac{1}{\rho^2} ds,$$

$$I_5 = \oint \frac{H_x}{|\rho^3|} ds, \qquad \text{with:} \quad H_x = \gamma_x D_x^2 + 2\alpha_x D_x D' + \beta D'^2$$

Many Alternatives – Give it a try!



a) Weak focusing (dipole only)
b) FODO line (w/o dipoles)
c) FODO cell
d) Low-emittance cell
e) CF low-emittance cell
f) Low-emittance FODO
g) Dispersion match
h) Periodic dispersion match
i) Double-bend achromat
j) Triple-bend achromat
k) ...

Very good course on low-emittance lattice design: A.Streun, PSI

Natural Emittance – A bit more Detail

$$\epsilon_0 \approx C_q \gamma^2 \frac{I_5}{j_x I_2}, \qquad I_2 = \underbrace{\oint \frac{1}{\rho^2} ds}_{\sim \langle D \rangle_{ring}}, \qquad I_5 = \oint \frac{H_x}{|\rho^3|} ds, \qquad \text{with: } H_x = \gamma_x D_x^2 + 2\alpha_x D_x D' + \beta D'^2$$

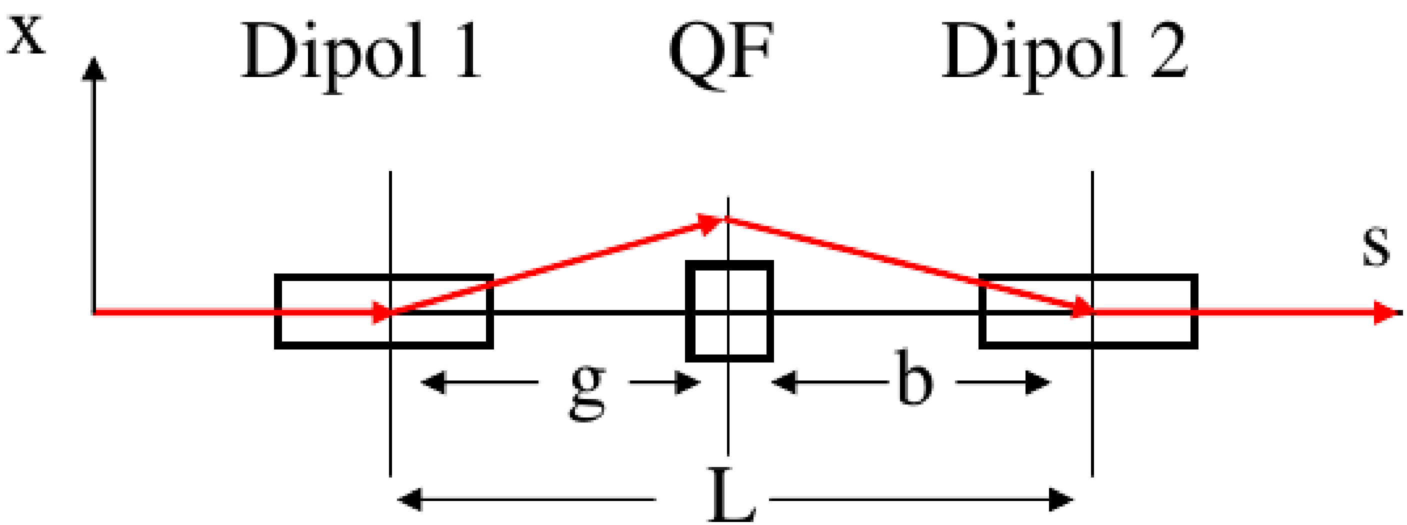
- Important take-aways:
  – minimise dispersion $D_x(s)$
  – minimise deflection angle $|1/\rho^3|$ ~θ^3
  – arrange your dipoles and quadrupoles wisely

- Examples:
  – FoDo $\quad \epsilon_0|_{min} \approx \left(2\sqrt{2} \dots 1.2\right) C_q \gamma^2 \theta^3,$
  – DBA $\quad \epsilon_0|_{min} \approx \frac{1}{4\sqrt{14}} C_q \gamma^2 \theta^3,$
  – TME $\quad \epsilon_0|_{min} \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3,$

Chasman-Green Double-Bend Achromat I/II

- Remember trick from the missing dipole dispersion suppressor..
  - 1st dipole creates dispersion wave,
  - 2nd dipole creates dispersion wave but with opposite sign

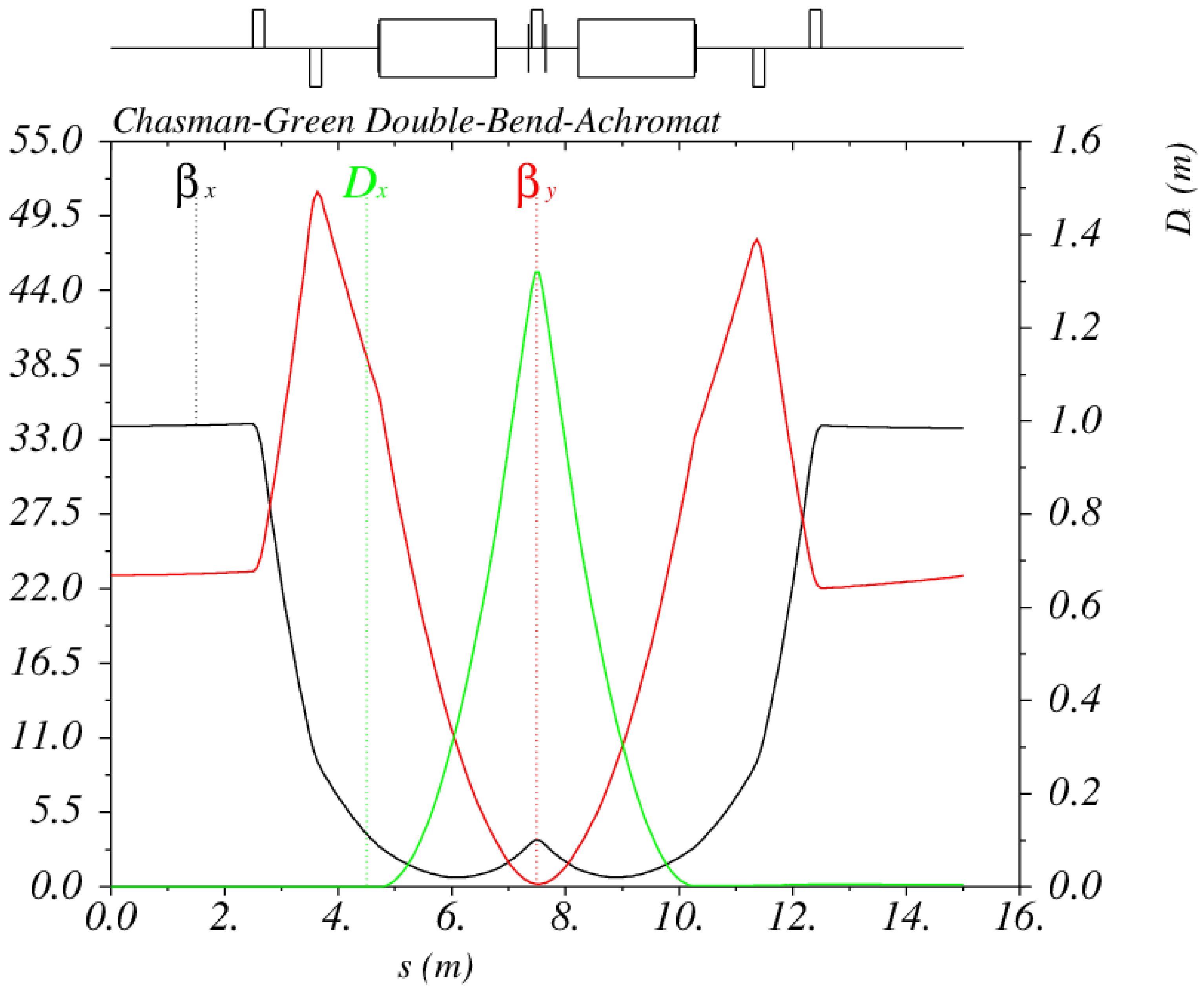
→ design phase advance $\Delta\mu := \pi$



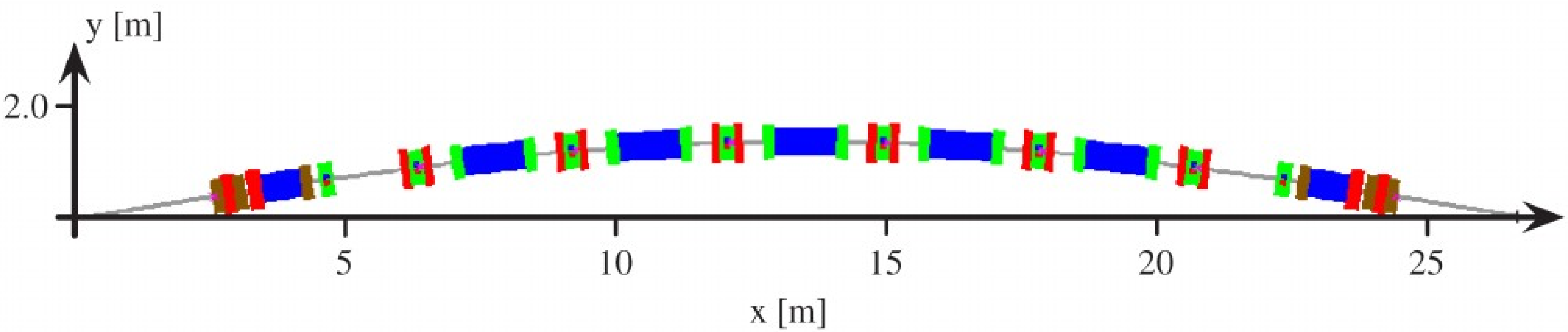
$$\frac{1}{b}+\frac{1}{g}=\frac{1}{f} \qquad f = L/4 \qquad \frac{1}{f} = k \cdot L_{QF} = \frac{4}{L}$$

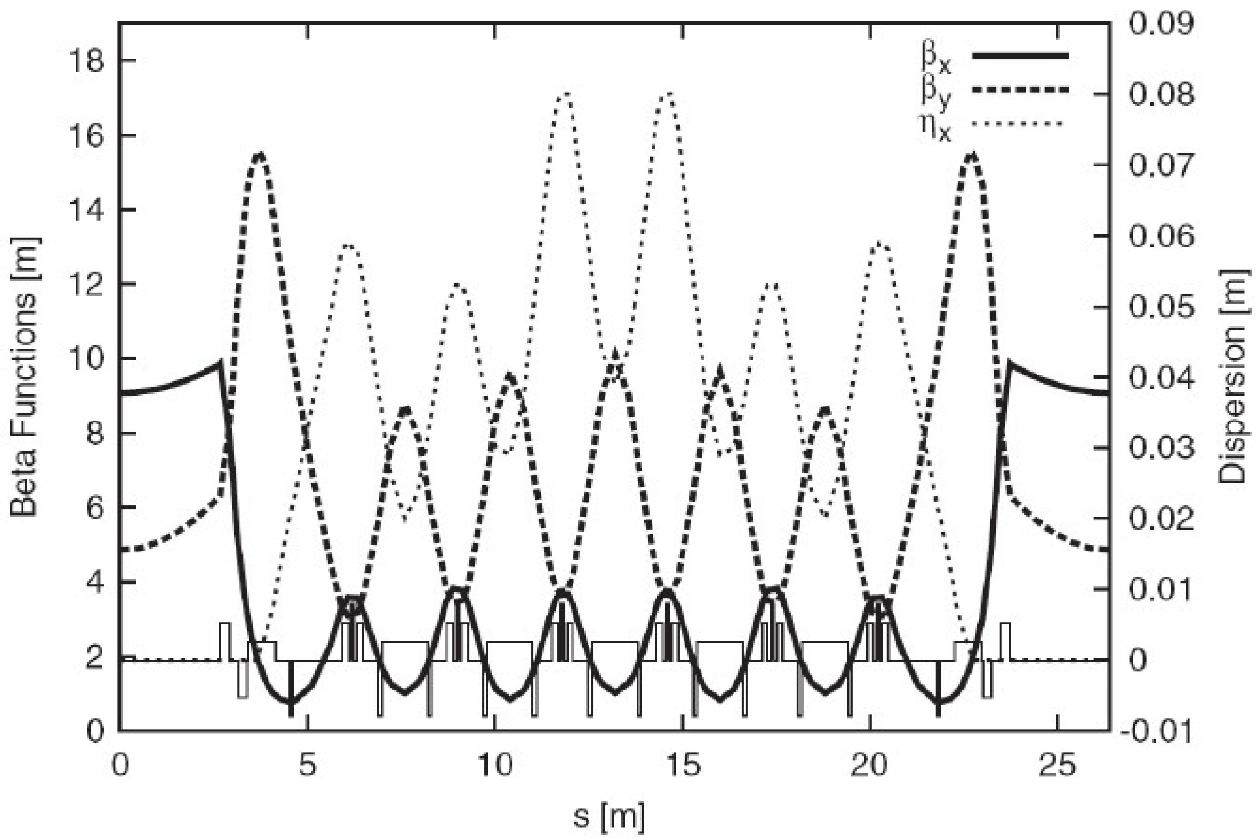
R. Chasman, G.K. Green & E.M. Rowe: "Preliminary Design of a Dedicated Synchrotron Radiation Facility". IEEE Trans. Nucl. Sci. 22 (3), 1975
R. Chasman and G. K. Green: “Design of a National Dedicated Synchrotron Radiation Facility.”, OSTI 7334216, 1976

Chasman-Green Double-Bend Achromat II/II



Seven-Bend Achromat





S.C. Leeman et al, "Beam dynamics and expected performance of Sweden's new storage-ring light source: MAX IV," PRST-AB 12, 120701 (2009).


The King is conducting a survey - how do you feel about ACAS lectures?
Oh, I can't complain.