Introduction to Beam Diagnostics – Part II

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- Acknowledgements: F. Caspers, M. Betz, E. Jensen et al.

- Excellent online resource: http://www.microwaves101.com/

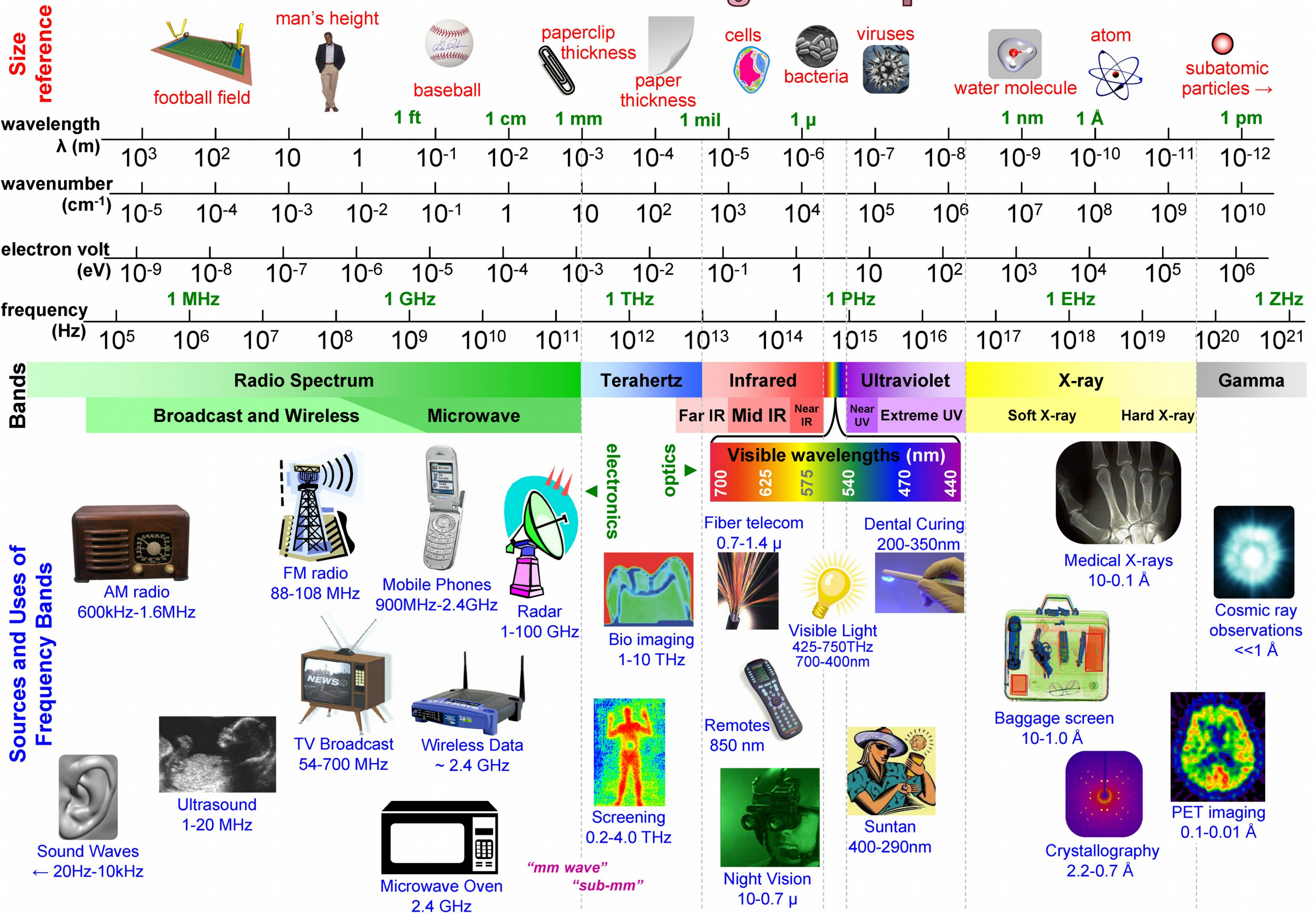
Introduction to RF – Part II

- Aim: learn how high-frequency signals are transmitted

- Part II – RF Transmission, S-Parameter & Noise
  - Signal transmission and reflection → S-parameters
  - Low-, band- and high-pass filters
  - Amplification and noise figure

- Laboratory:
  - In || lab measurements with VNA and various RF components (hair-pin filter, strip-lines, diplexer, attenuator, low-pass, etc.)
  - Measure cable response (terminated, open,short) – see defects
  - Cantenna + VNA → building their own radar
  - demonstrate equivalence between 'scope+pulse generator' and VNA
  - RF mixer (DIY radar or FM modulation generation, check with SA and RFD)

Chart of the Electromagnetic Spectrum



$\lambda = 3 \times 10^{8}/\text{freq} = 1/(\text{wn}*100) = 1.24 \times 10^{-6}/\text{eV}$

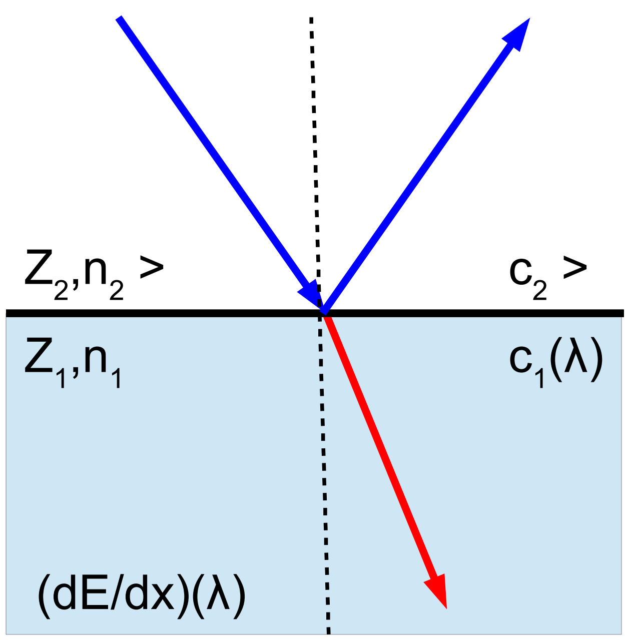
SURA Southeastern Universities Research Association ®

Reflection and Transmission

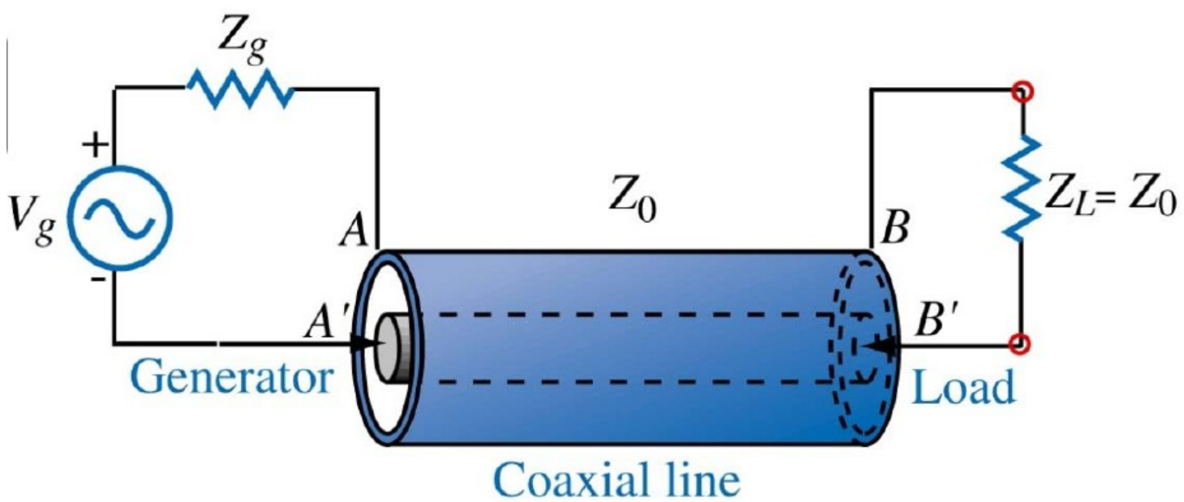
$$c = \lambda f$$

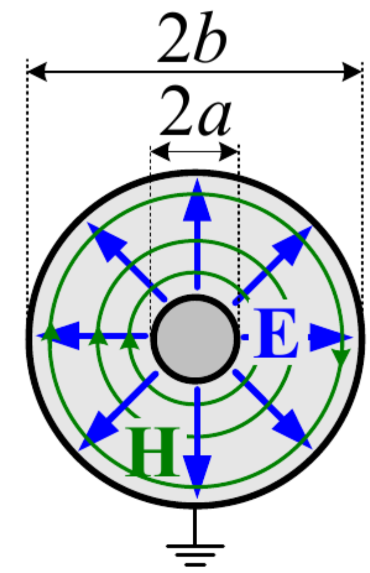
- Until now we assumed that the component sizes are small compared to the wavelength λ → can use circuit network theory
  - doesn't apply for distributed-parameter networks
  - still: component size not much larger than the wavelength – not yet fully optical

- From a physics point of view there aren't many differences between RF, microwave or optical electro-magnetic radiation
  - reflection & transmission
    - optical domain: refractive index 'n'
    - RF & MW: characteristic impedance Z_0
      - free-space: $Z_0 := E/H = \mu_0 \varepsilon_0 = (\varepsilon_0 c)^{-1} = 377\ \Omega$
    - If 'n1 ≠ n2' or '$Z_1 \neq Z_2$' → signal reflection
  - Absorption: dE/dx != 0
  - Dispersion: '$c=c(\lambda)$' or '$\gamma=\gamma(f)$'



Transmission Lines you know I/II

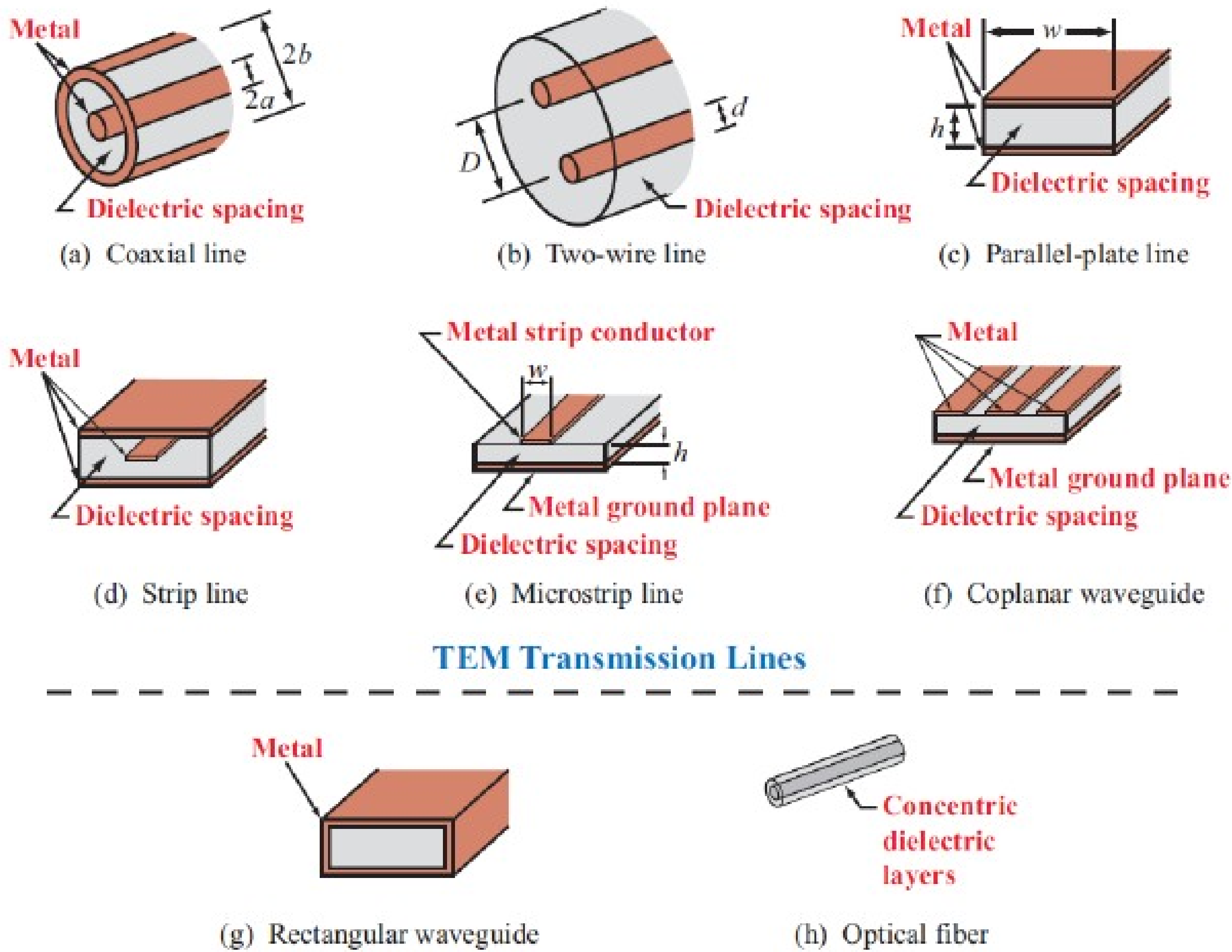




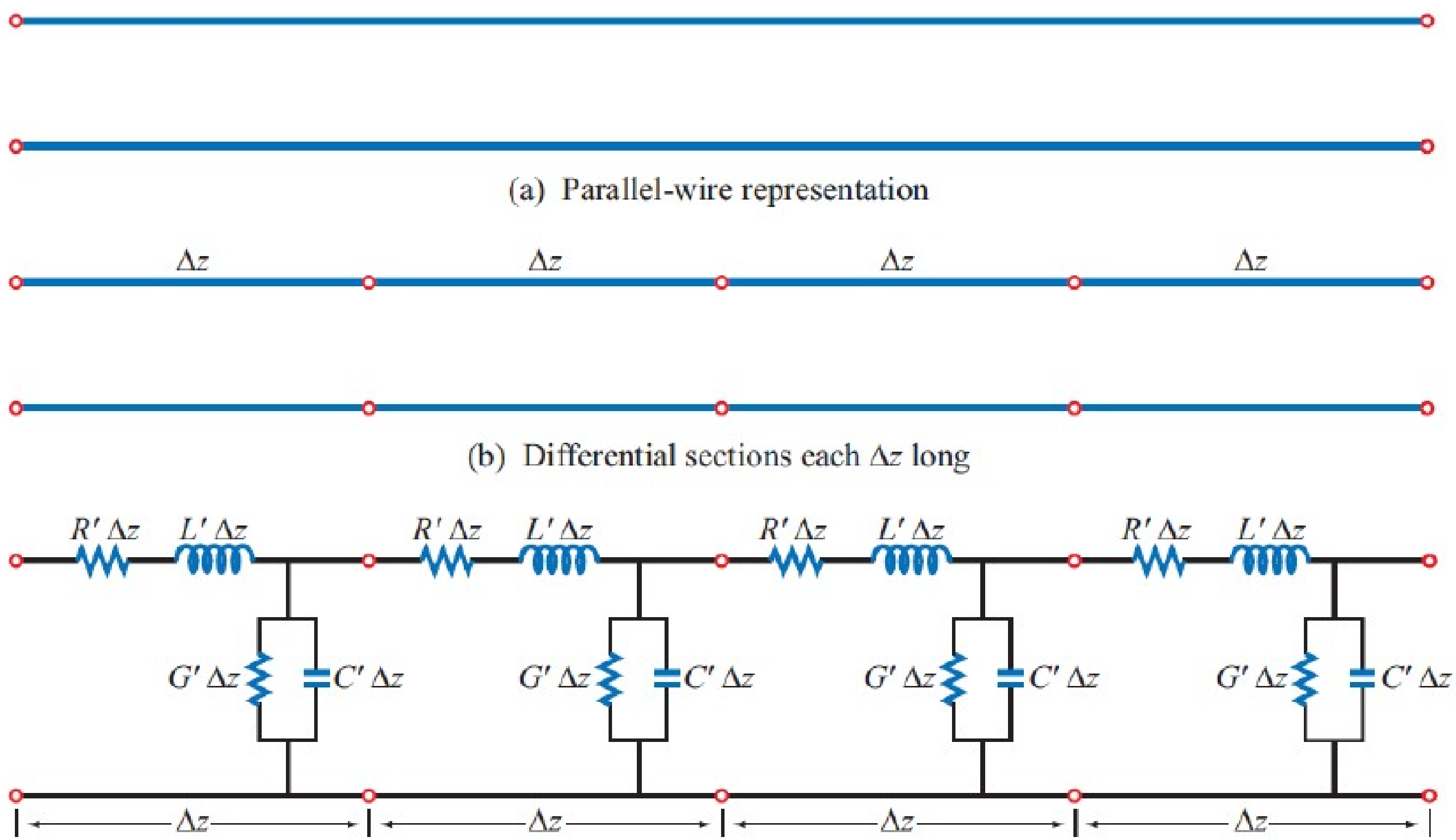
- Impedance
  - free-space: Z_0 = 377 Ω
  - Cable come typically in Z_0 = 50 Ω or 75 Ω
    - 75 Ohm: least signal losses
    - 50 Ohm: best power transmission (break-down condition), compromise between 30 (power) and 77 (low-loss for air-dielectric)

$$Z_0 \approx \frac{138}{\sqrt{\epsilon_r}} \log\left(\frac{b}{d}\right) \quad [\Omega]$$

Transmission Lines you know II/II



Transmission Line Model I/III



(c) Each section is represented by an equivalent circuit

- R': The combined *resistance* of both conductors per unit length, in Ω/m,
- L': The combined *inductance* of both conductors per unit length, in H/m,
- G': The *conductance* of the insulation medium between the two conductors per unit length, in S/m, and
- C': The *capacitance* of the two conductors per unit length, in F/m.

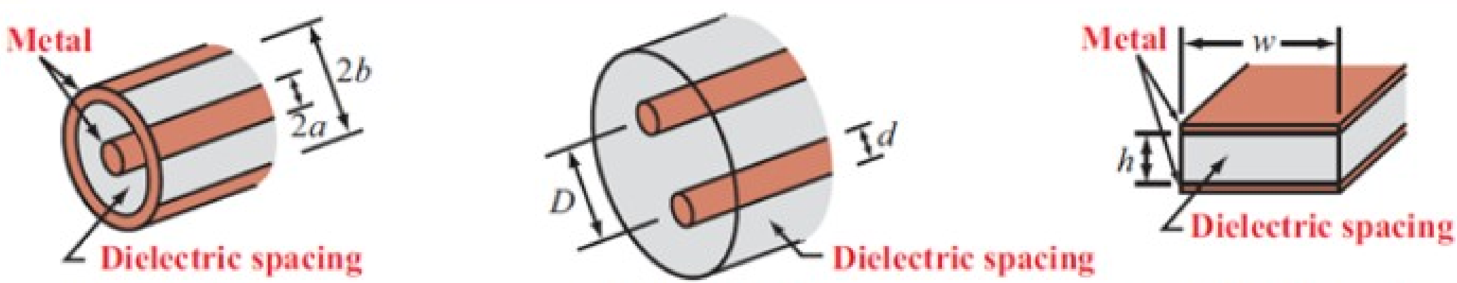
courtesy Farid Farahmand

Transmission Line Model II/III

Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

Notes: . (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c / \sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \simeq \ln(2D/d)$.

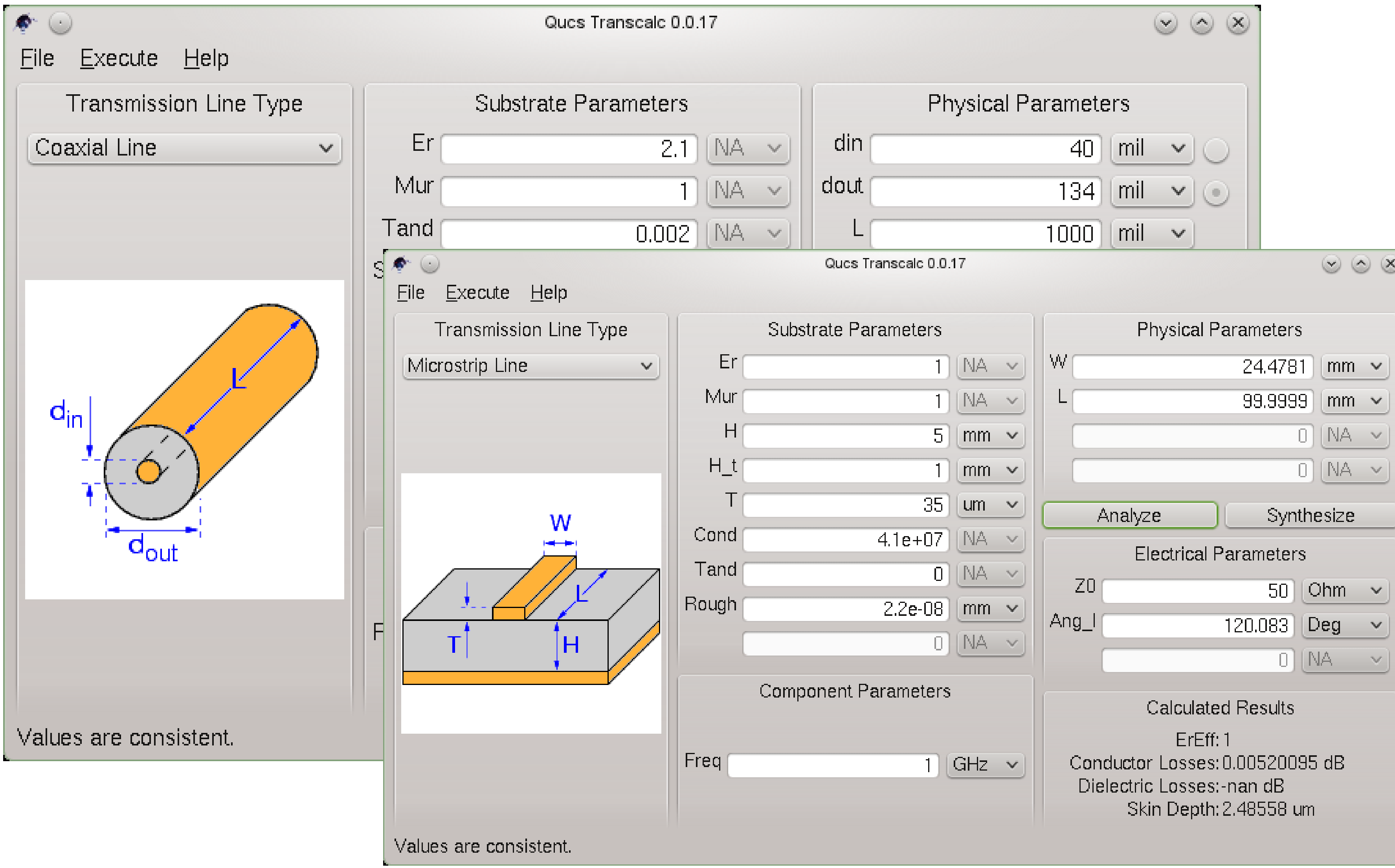


(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

courtesy Farid Farahmand

Transmission Lines in Qucs

- Tools → Line Calculation & Components → Transmission Lines



Transmission Line Model III/III

- Wave equation:

$$\frac{d^2\tilde{V}(z)}{dz^2}-\gamma\tilde{V}(z)=0 \quad \text{with: } \gamma=\alpha+j\beta=\sqrt{(R'+j\omega L')(G'+j\omega C')}$$

propagation "constant"

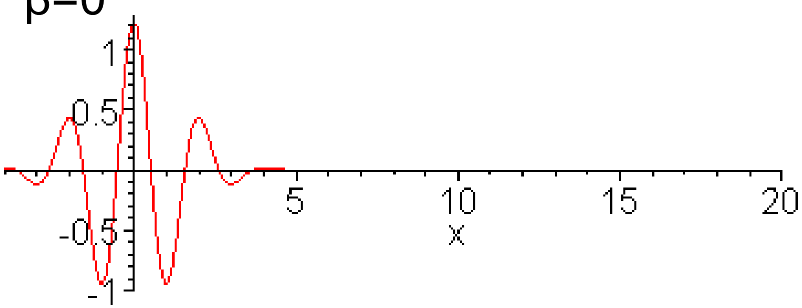
attenuation "constant" [Neper/m]

phase "constant"

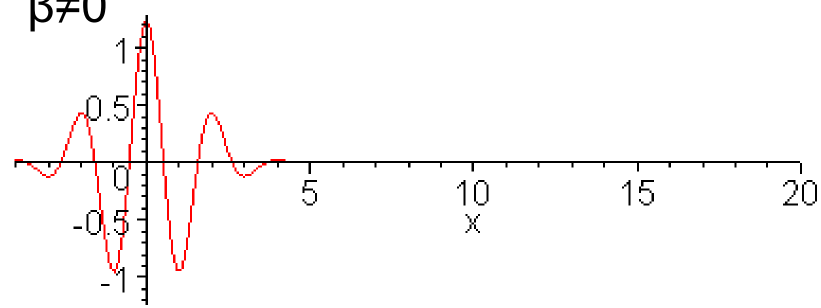
$$Z_0=\sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}\overset{\text{if loss-less}}{=}\sqrt{\frac{L}{C}}$$

$$\tilde{V}(z)=V_0\cdot e^{j\omega t\pm\gamma z}=V_0\cdot e^{j\omega t\pm\beta z}e^{\pm\alpha z}$$

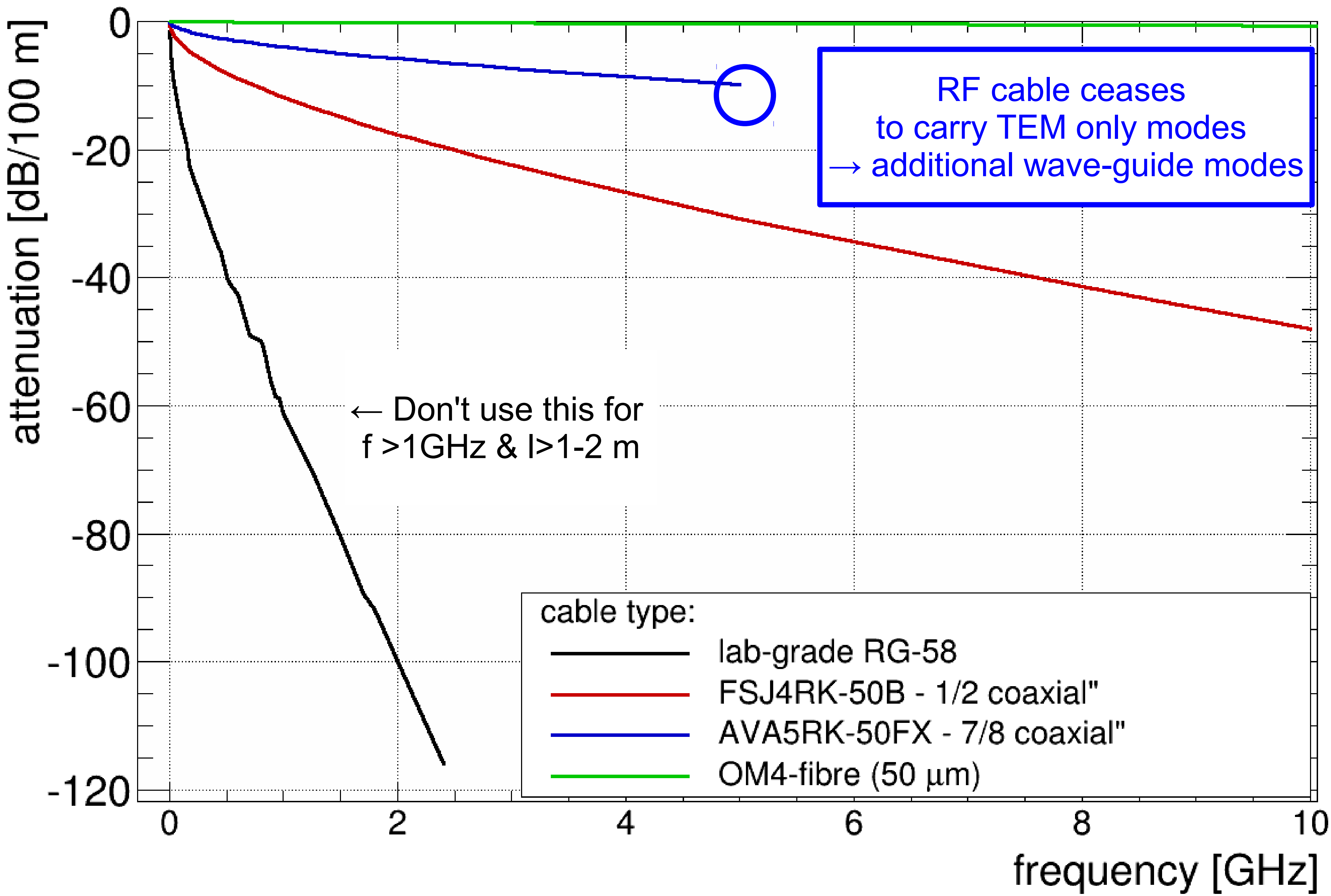
β=0



β≠0

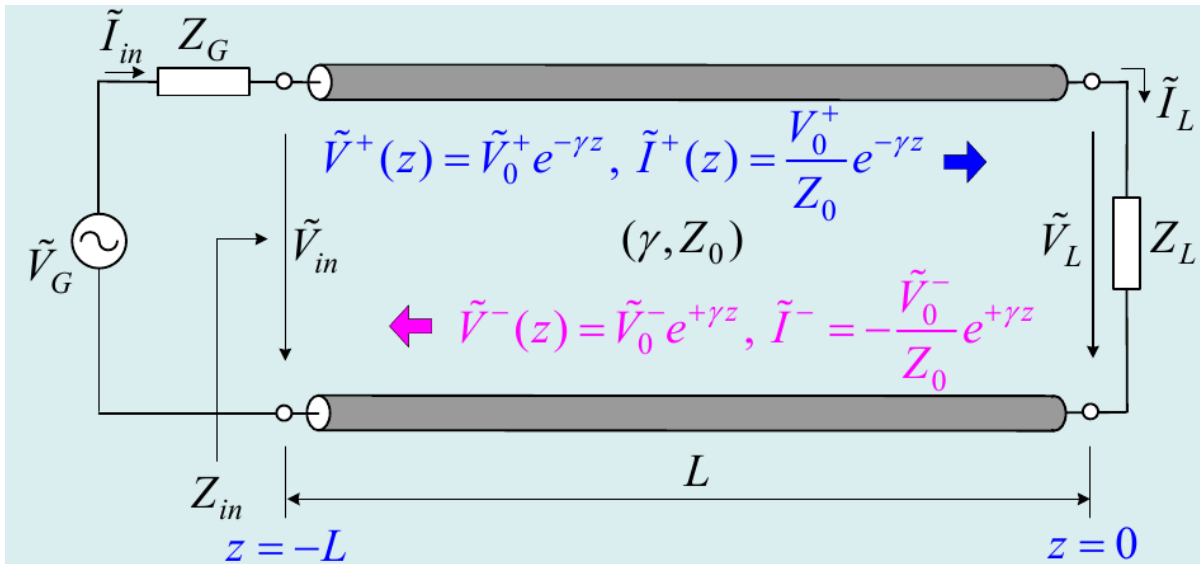


Typical RF Cable Attenuation



Reflection and Transmission

- Terminations of transmission-lines cause reflections analogous to the reflections of plane waves from material interfaces at normal incidence



- The incident and reflected voltage at the load can be expressed in terms of the total voltage and current at the load

$$\begin{cases} \tilde{V}_{(z=0)} = \tilde{V}_L = \tilde{V}_0^+ + \tilde{V}_0^- \\ \tilde{I}_{(z=0)} = \tilde{I}_L = \dfrac{\tilde{V}_0^+}{Z_0} - \dfrac{\tilde{V}_0^-}{Z_0} \end{cases} \Rightarrow \begin{cases} \tilde{V}_0^+ = 0.5(\tilde{V}_L + Z_0 \tilde{I}_L) \\ \tilde{V}_0^- = 0.5(\tilde{V}_L - Z_0 \tilde{I}_L) \end{cases}$$

Reflection Coefficient

- Reflection coefficient Γ and SWR are defined in the same way as with plane-wave reflection at normal incidence
- Γ is the ratio of reflected and incident voltage at the load

$$\Gamma = \frac{\tilde{V}^-_{(z=0)}}{\tilde{V}^+_{(z=0)}} = \frac{\tilde{V}^-_0}{\tilde{V}^+_0} = \frac{\tilde{V}_L - Z_0\tilde{I}_L}{\tilde{V}_L + Z_0\tilde{I}_L} = \frac{(\tilde{V}_L / \tilde{I}_L) - Z_0}{(\tilde{V}_L / \tilde{I}_L) + Z_0} \longleftarrow Z_L = \frac{\tilde{V}_L}{\tilde{I}_L} = \frac{\tilde{V}_{(z=0)}}{\tilde{I}_{(z=0)}}$$

Z_L

$$\Rightarrow \quad \boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}} \quad \boxed{Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right)}$$

- return loss: available (incident) power that is not delivered to the load

$$return \quad loss[dB] = -20\log_{10}\rho$$

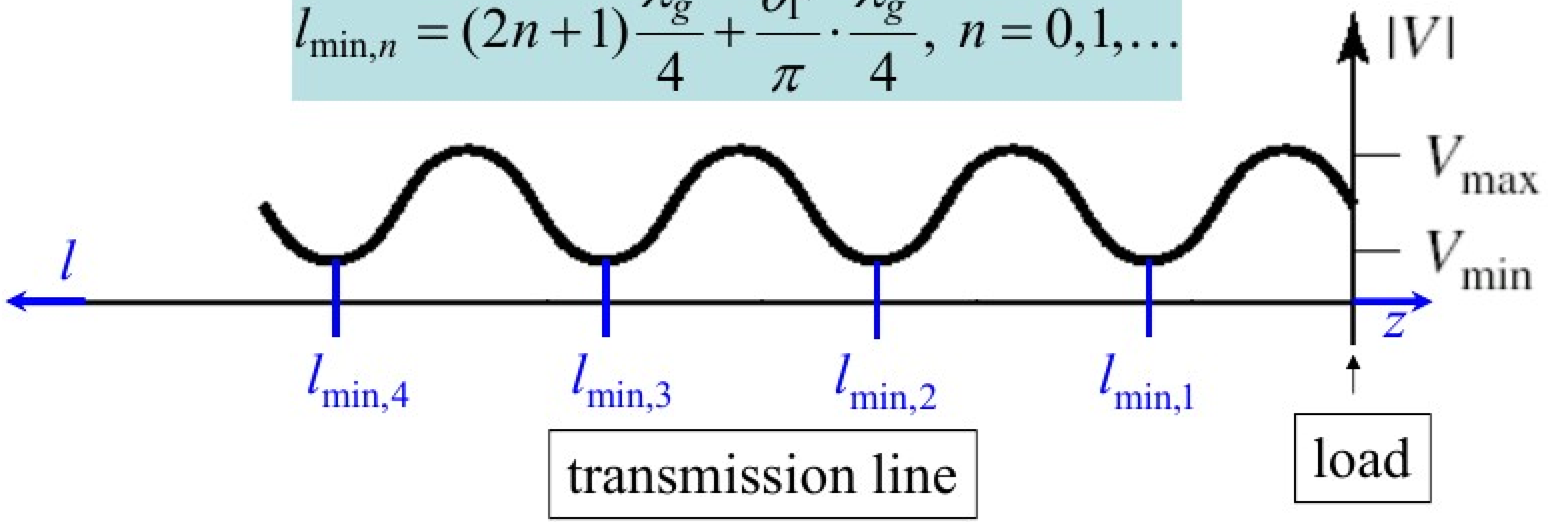
Standing-Wave-Ratio SWR

- the relation between the SWR and Γ is derived in the same manner as for plane waves

$$SWR = \frac{|\tilde{V}(z)|_{max}}{|\tilde{V}(z)|_{min}} \Rightarrow \boxed{SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \geq 1}$$

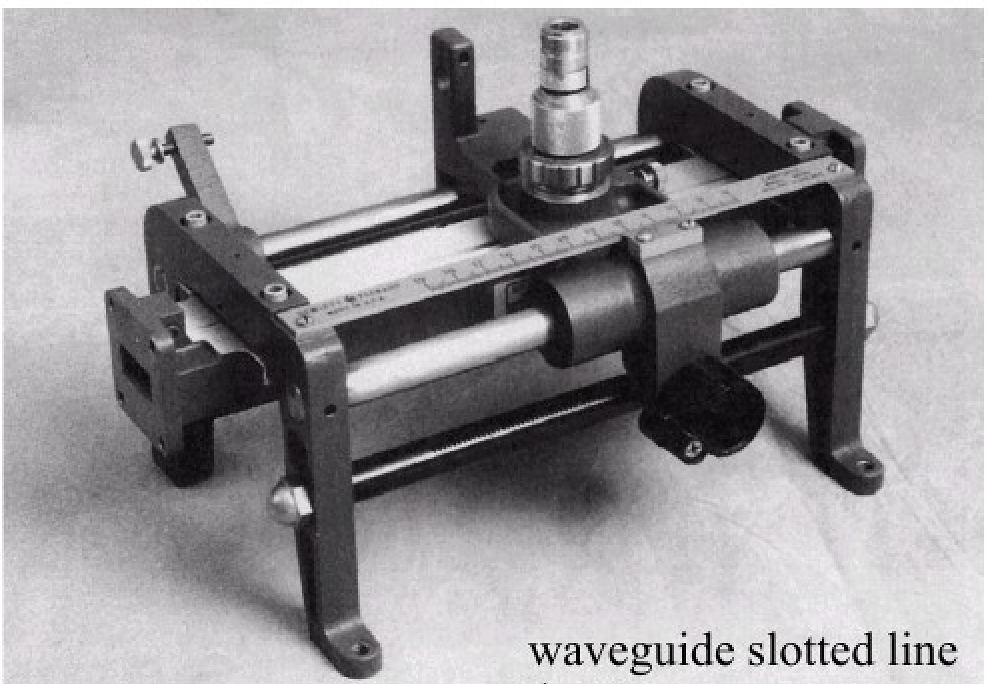
- locations of the voltage minima (current maxima) are found in the same way as for plane waves

$$l_{min,n} = (2n+1)\frac{\lambda_g}{4} + \frac{\theta_\Gamma}{\pi}\cdot\frac{\lambda_g}{4}, \quad n = 0,1,\ldots$$

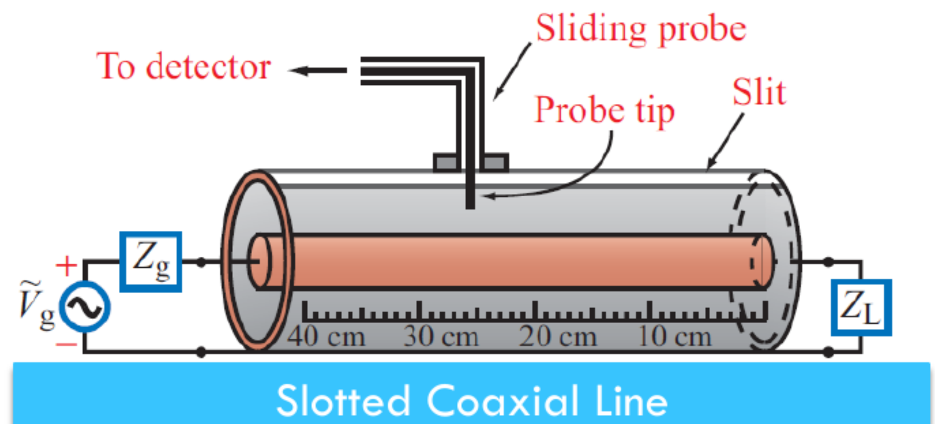


SWR Measurement – Slotted Line

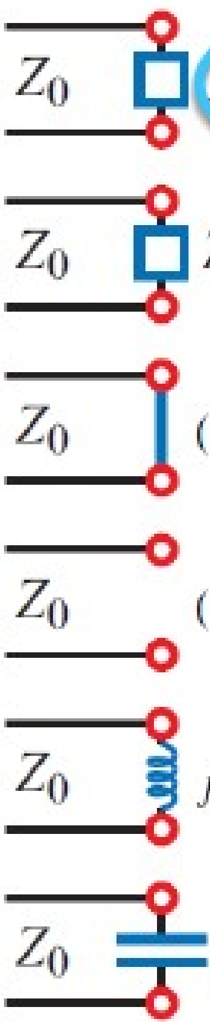
- Allows for sampling of the E field along a terminated TL to determine the load impedance by measuring the SWR



[Pozar, *Microwave Engineering*, 3rd ed.]



Some Common Reflection Coefficients



Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

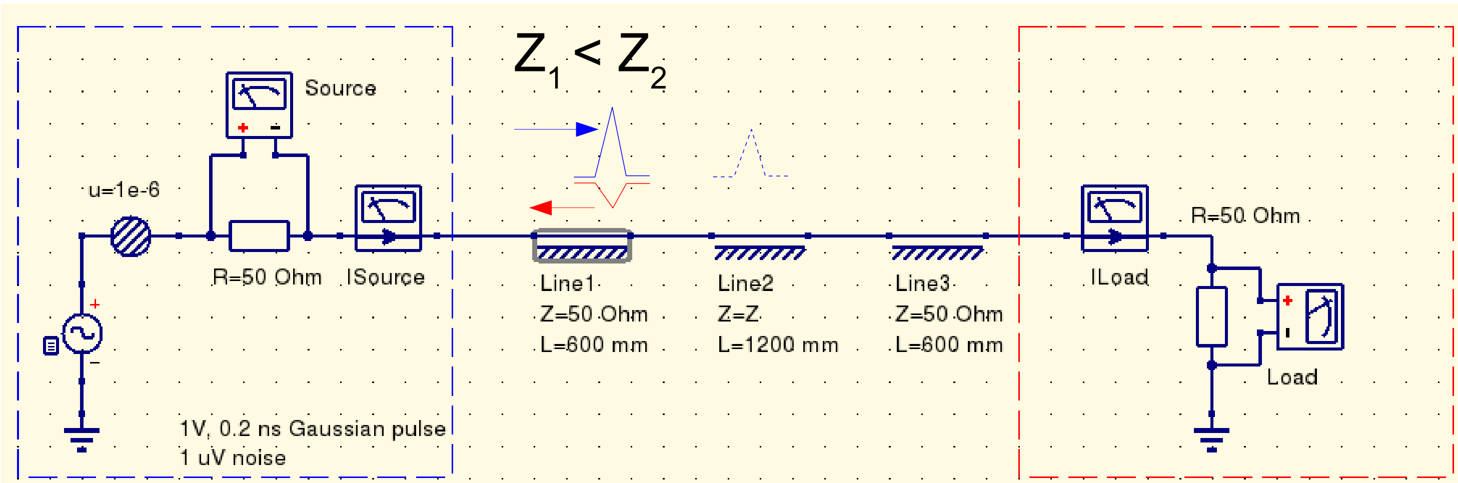
Load	$\lvert\Gamma\rvert$	θ_r
$Z_L = (r + jx)Z_0$	$\left[\dfrac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\dfrac{x}{r-1}\right) - \tan^{-1}\left(\dfrac{x}{r+1}\right)$
Z_0	0 (no reflection)	irrelevant
(short)	1	$\pm 180°$ (phase opposition)
(open)	1	0 (in-phase)
$jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
$jX = \dfrac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

Pay attention!

courtesy Farid Farahmand

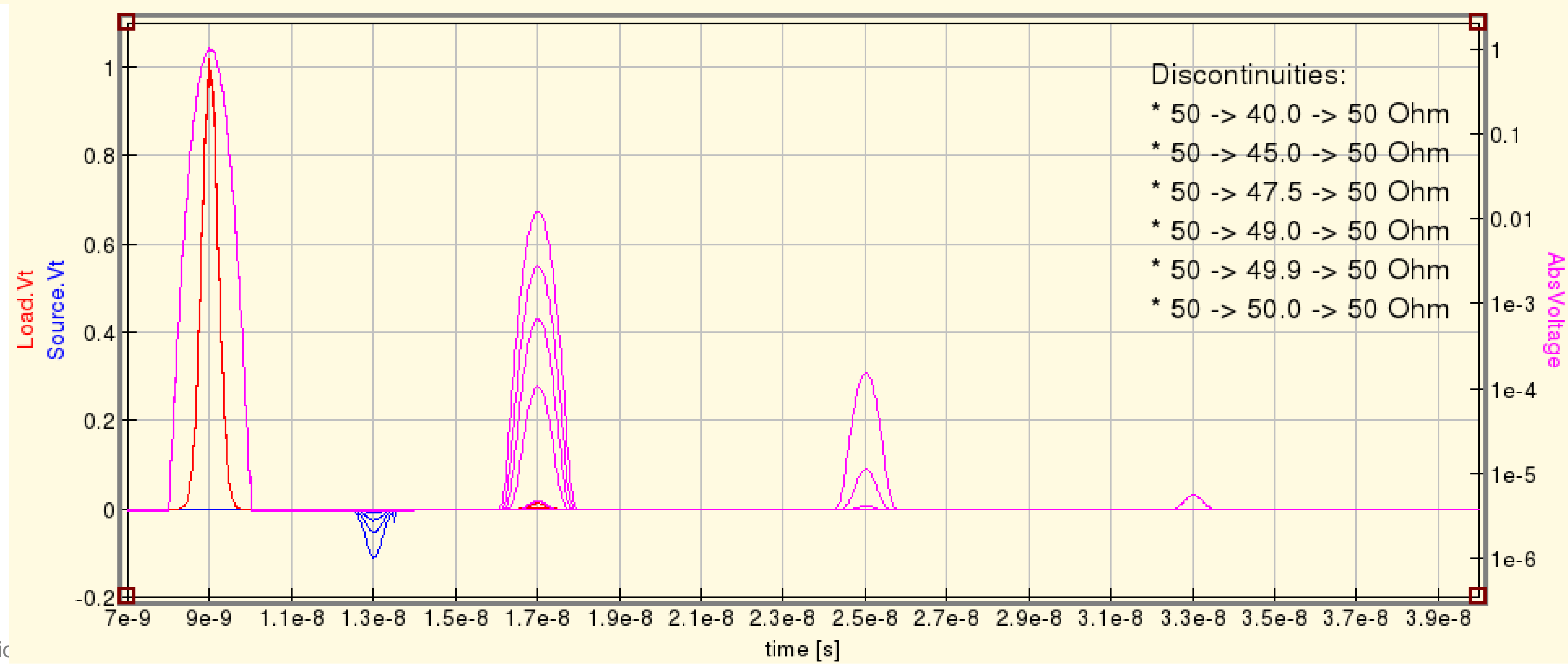
RF Reflections – Definitions

- ...are unavoidable impedance mismatches



$$\rho := \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$VSWR := \frac{1-\rho}{1+\rho}$$



VSWR and Reflection Coefficient

Γ	VSWR	Refl. Power $\|-\Gamma\|^2$
0.0	1.00	1.00
0.1	1.22	0.99
0.2	1.50	0.96
0.3	1.87	0.91
0.4	2.33	0.84
0.5	3.00	0.75
0.6	4.00	0.64
0.7	5.67	0.51
0.8	9.00	0.36
0.9	19	0.19
1.0	∞	0.00

$$V.S.W.R. := \frac{V_{max}}{V_{min}} \qquad \Gamma := \frac{Z_{load} - Z_0}{Z_{load} + Z_0}$$

RF Connector and Cable Geometry

- Selection of common connectors and adapters (H&S):
  - Naively, one would expect these to be inert
  - static and frequency dependent component

- For comparison, a VSWR of
  - 1.02 ↔ r = 1% ↔ 40 dB
  - 1.03 ↔ r = 1.4% ↔ 36.6 dB
  - 1.05 ↔ r = 2.4% ↔ 32.3 dB

- RF transitions are unavoidable in real life
- %-level reflections are common/normal

VSWR ≤ 1.03 + 0.01·f [GHz]

≤ 1.19 + 0.06·f [GHz]

VSWR ≤ 1.03 + 0.004·f [GHz]

VSWR ≤ 1.025 + 0.007·f [GHz]

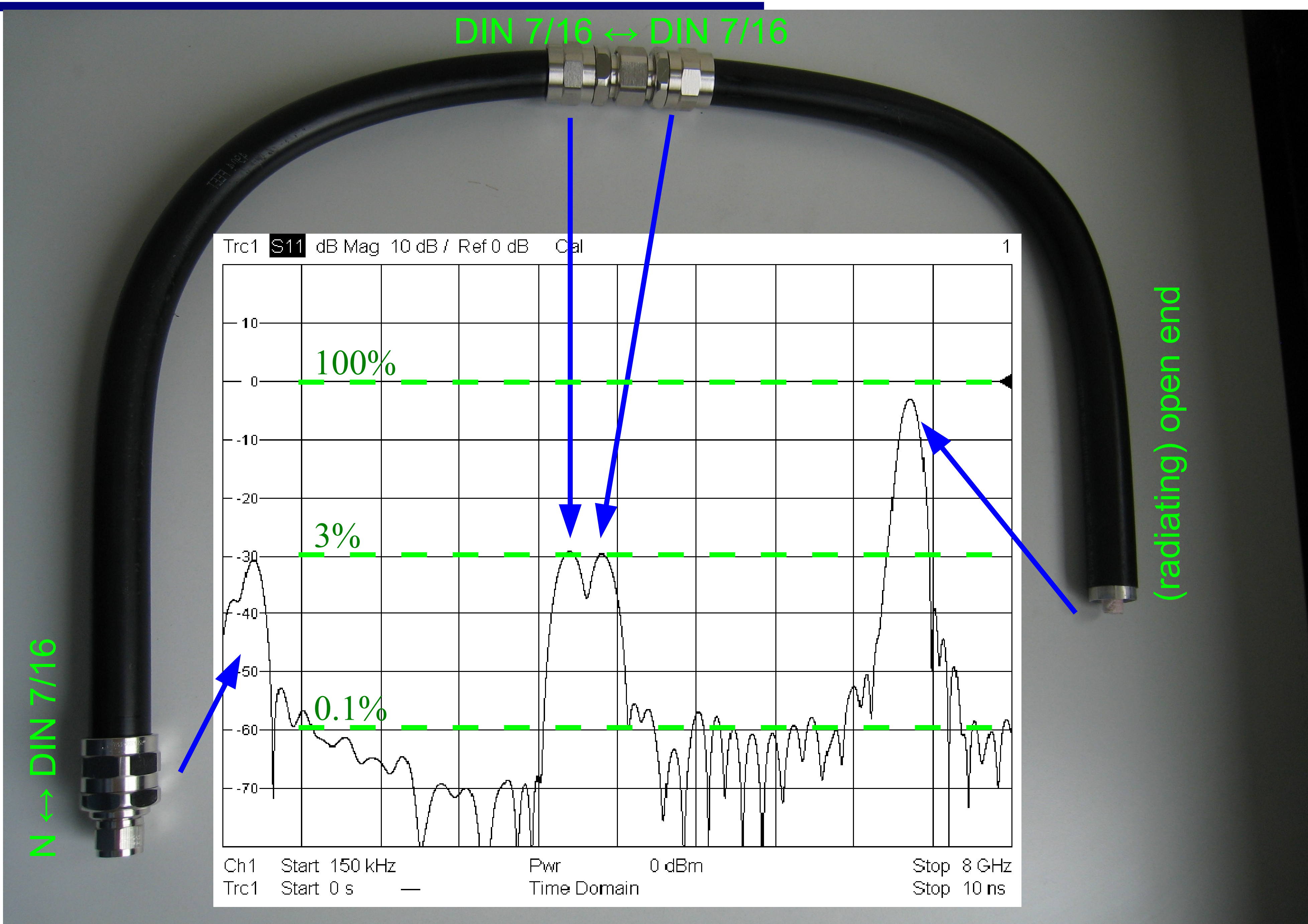
≤ 1.05 + 0.015·f [GHz]

VSWR ≤ 1.06 + ~0.01·f [GHz]

VSWR ≤ 1.02 + 0.03·f [GHz]

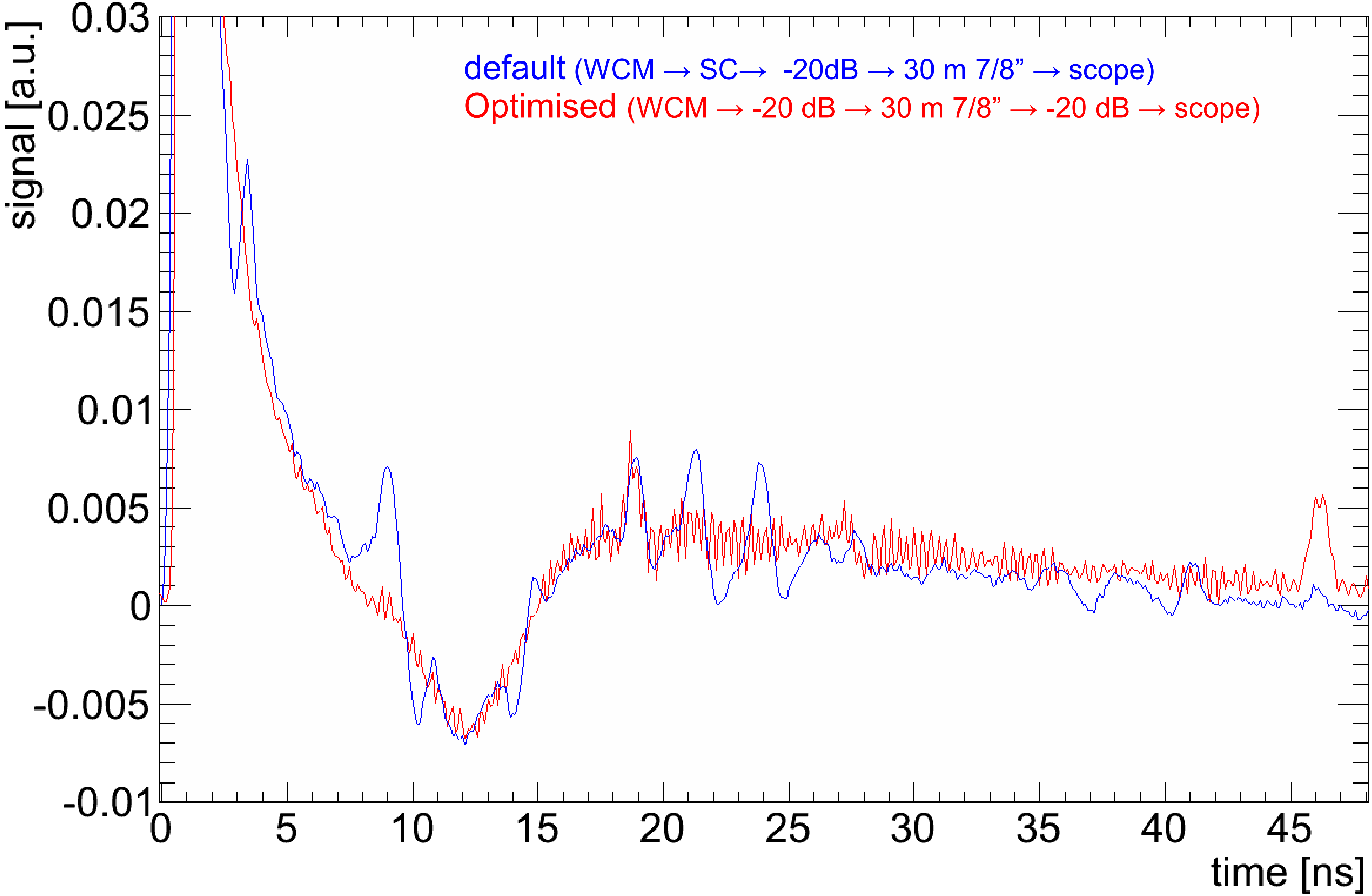
≤ 1.05 @ 6GHz

RF Connector and Cable Geometry Real-Life Example



For comparison: LHC WCM Installation I/II

- Comparison of standard vs. optimised installation:



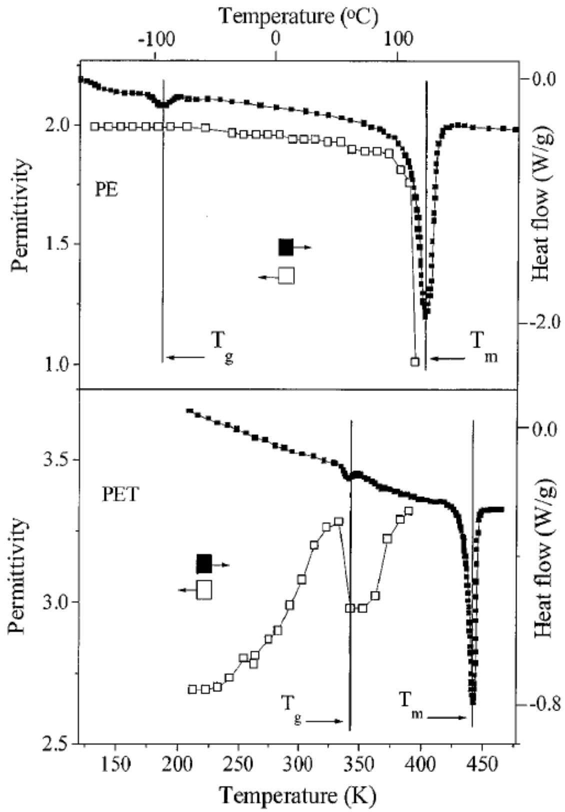
Permittivity and Dependence on Temperature

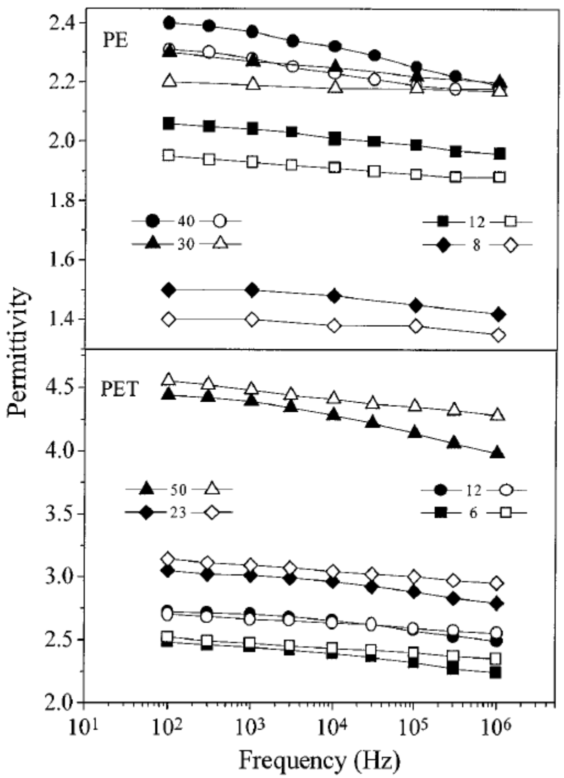
- Permittivity depends on frequency and temperature

N.B. $Z_0 \sim \sqrt{\frac{\mu_r}{\epsilon_r}}$

$$\frac{\partial}{\partial T}\left(\frac{\Delta\epsilon}{\epsilon}\right) \sim \pm 30\ ppm/{}^\circ C \quad (e.g.\ ceramics)$$

$$\frac{\partial}{\partial T}\left(\frac{\Delta\mu}{\mu}\right) \sim 0.1\ldots 1\cdot 10^{-2}/{}^\circ C \quad (typ.\ ferrites)$$





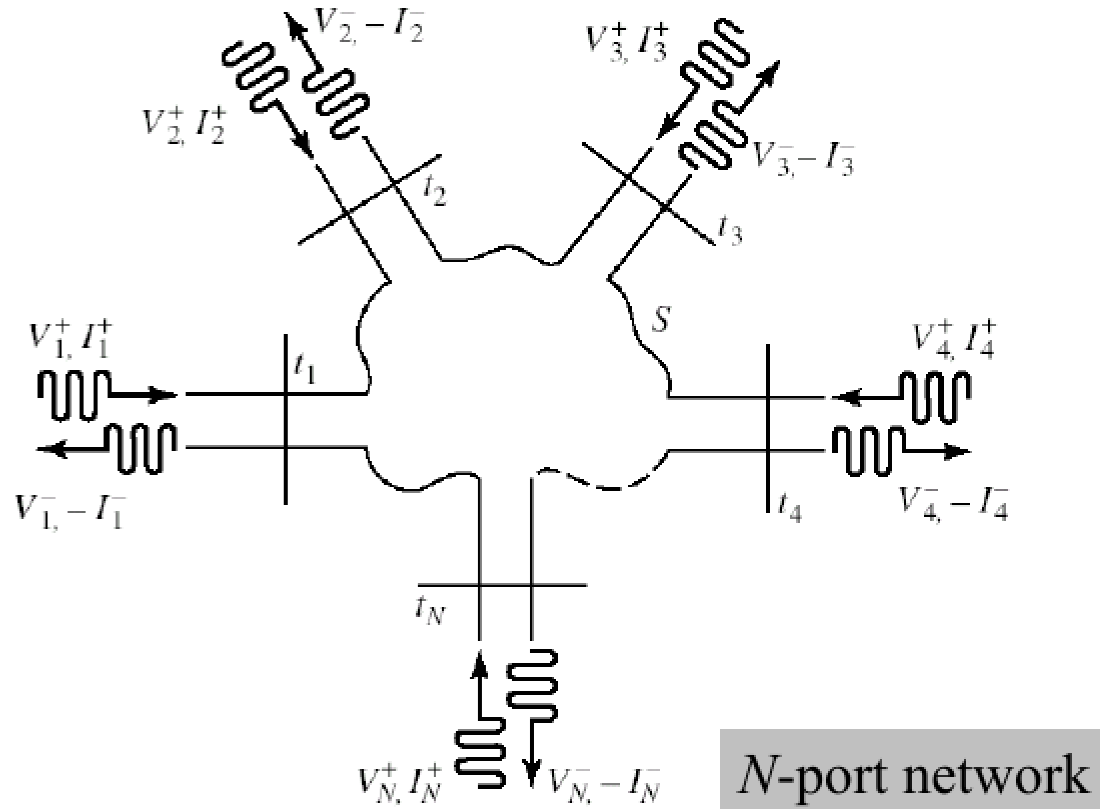
- Highly non-trivial and active research topic
  - N.B. PE melts at a very low temperature around 100 °C ↔ ~20 W/m power loss in cables

Microwave Network

- at each port incident and reflected voltage/current waves can be defined
- At the n-th port

$$\begin{cases} V_n = V_n^+ + V_n^- \\ I_n = I_n^+ - I_n^- \end{cases}$$

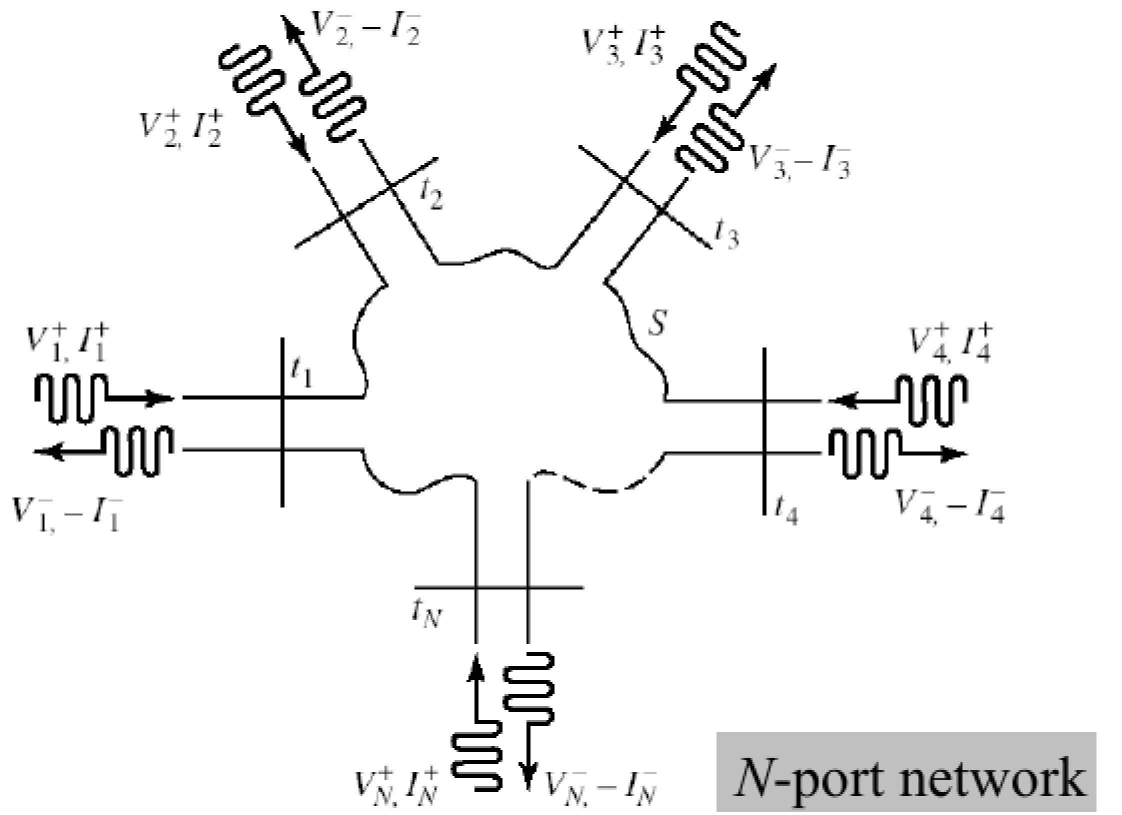
$$(n = 1, \ldots, N)$$



Scattering Matrix

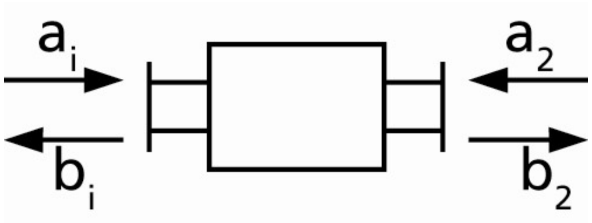
$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0 \text{ for all } k \neq j}$$

- Voltage waves:
  incident wave on port j and
  measure output at port i
  - N.B. assumes that all other ports are matched
    ↔ $V_k^+ = 0$ for all $k \neq j$



Scattering Matrix

- Two-port (4-pole)



$$(S)=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

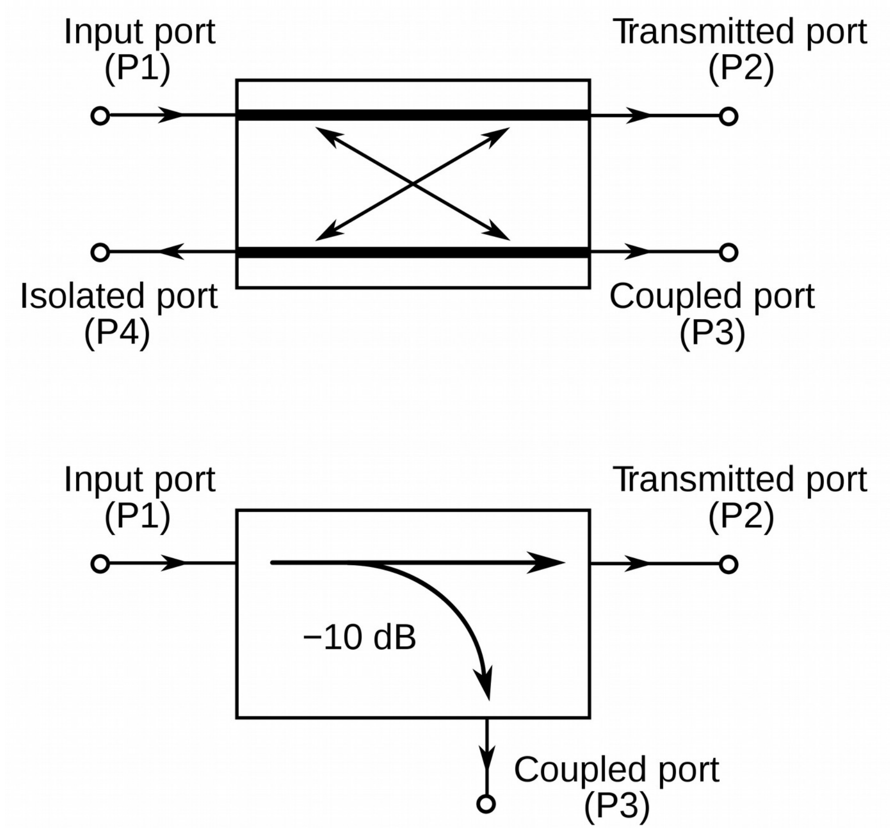
- A non-matched load present at port 2 with reflection coefficient Γ_{load} transfers to the input port as

$$\Gamma_{in} = S_{11} + S_{21}\frac{\Gamma_{load}}{1-S_{22}\Gamma_{load}}S_{12}$$

  – N.B. for a proper S-parameter measurement all ports of the Device Under Test (DUT) including the generator port must be terminated with their characteristic impedance in order to assure that waves travelling away from the DUT (b_n-waves) are not reflected back and convert into a_n-waves.

Directional Coupler

- Directional coupler $S_{14}=S_{23}$



$$\text{Coupling: } C = 10\log_{10}\frac{P_1}{P_3} = -20\log_{10}\underbrace{|S_{31}|}_{\beta} \text{ dB}$$

$$\text{Directivity: } D = 10\log_{10}\frac{P_3}{P_4} = 20\log_{10}\frac{|S_{31}|}{|S_{41}|} \text{ dB}$$

$$\text{Isolation: } I = 10\log_{10}\frac{P_1}{P_4} = -20\log_{10}|S_{41}| \text{ dB}$$

$$I = D + C, \text{ dB}$$

<u>the ideal coupler</u>

$$I \to \infty,\ D \to \infty$$

$$\mathbf{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

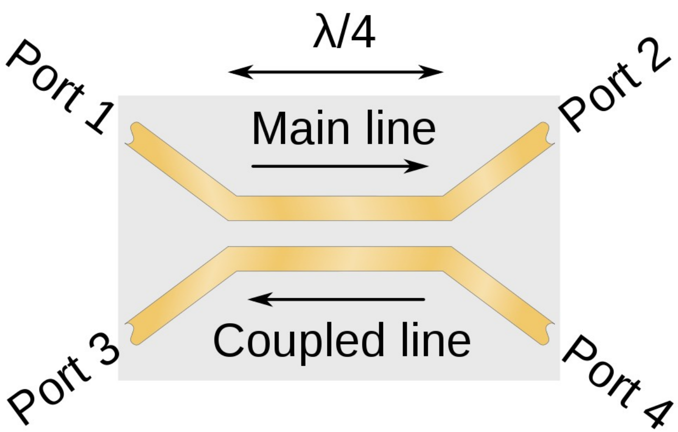
loss-less 90° hybrid

$$\mathbf{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

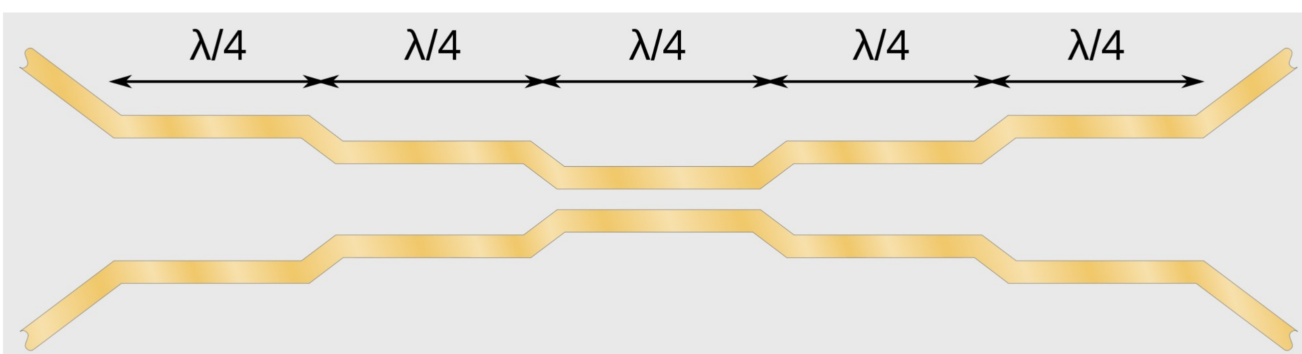
loss-less 180° hybrid

Directional Coupler

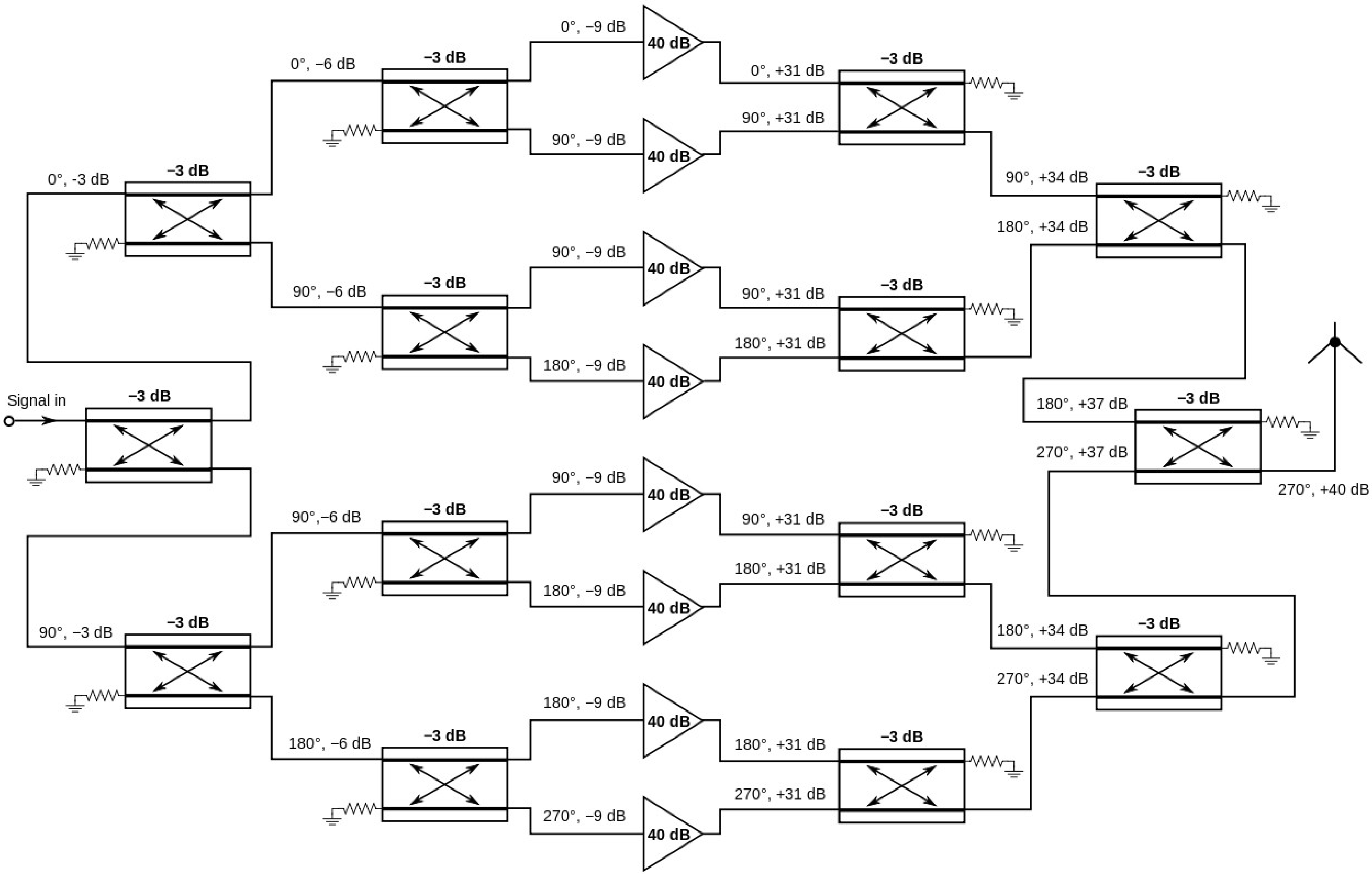
- Simple coupler – limited bandwidth



- Improved multi-stage coupler:



Application of High-Power Coupler



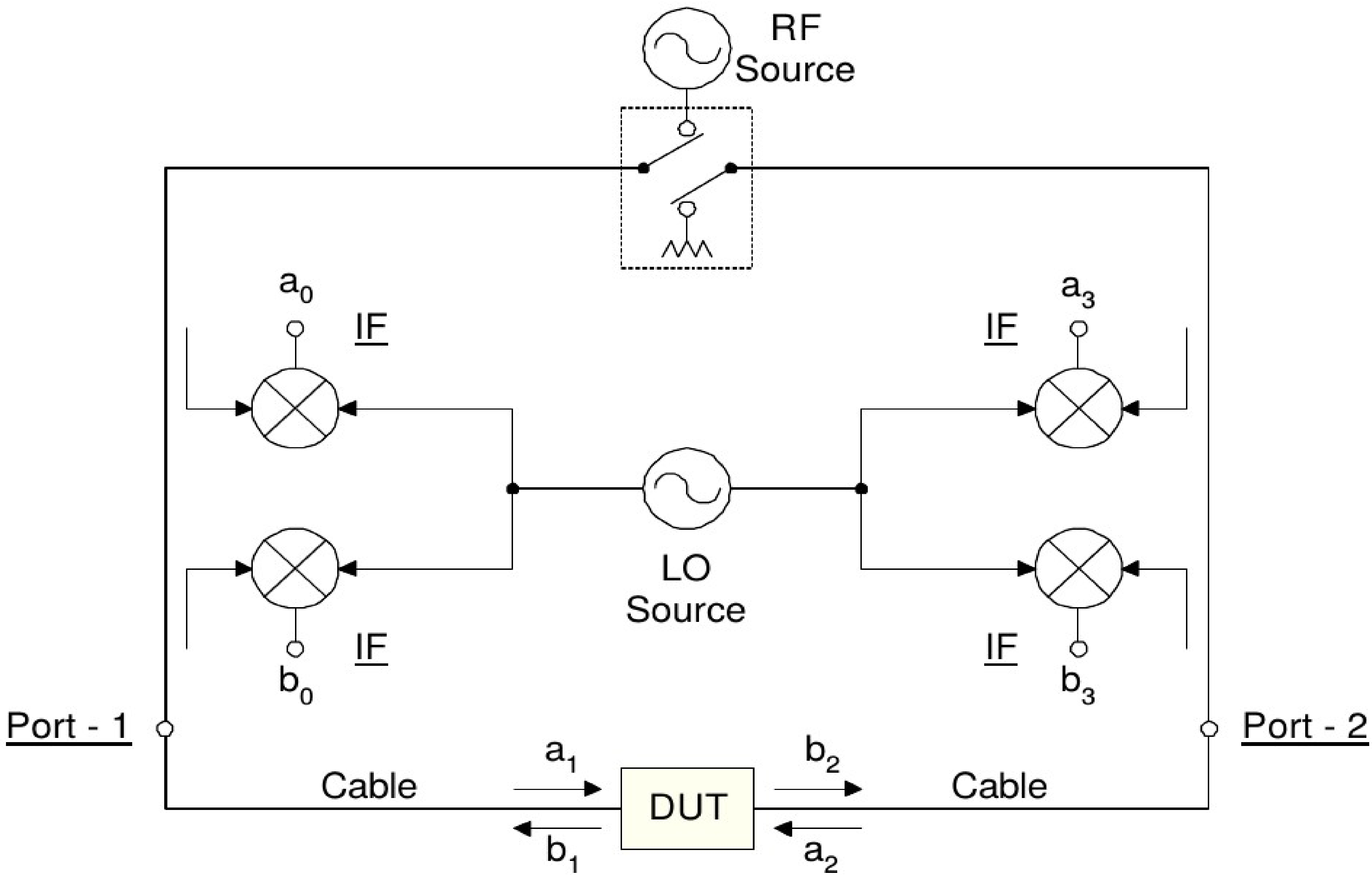
Measurement Methods

- Coaxial measurement line
  – old fashion method – no more in use but good for understanding of concept

- Network analyzer
  – Excites a network (circuit, antenna, amplifier or similar) at a given CW frequency and measures response in magnitude and phase → determines S-parameters
  – Covers a frequency range by measuring step-by-step at subsequent frequency points
  – Application: characterization of passive and active components, time domain reflectometry by Fourier transforming reflection response, etc.

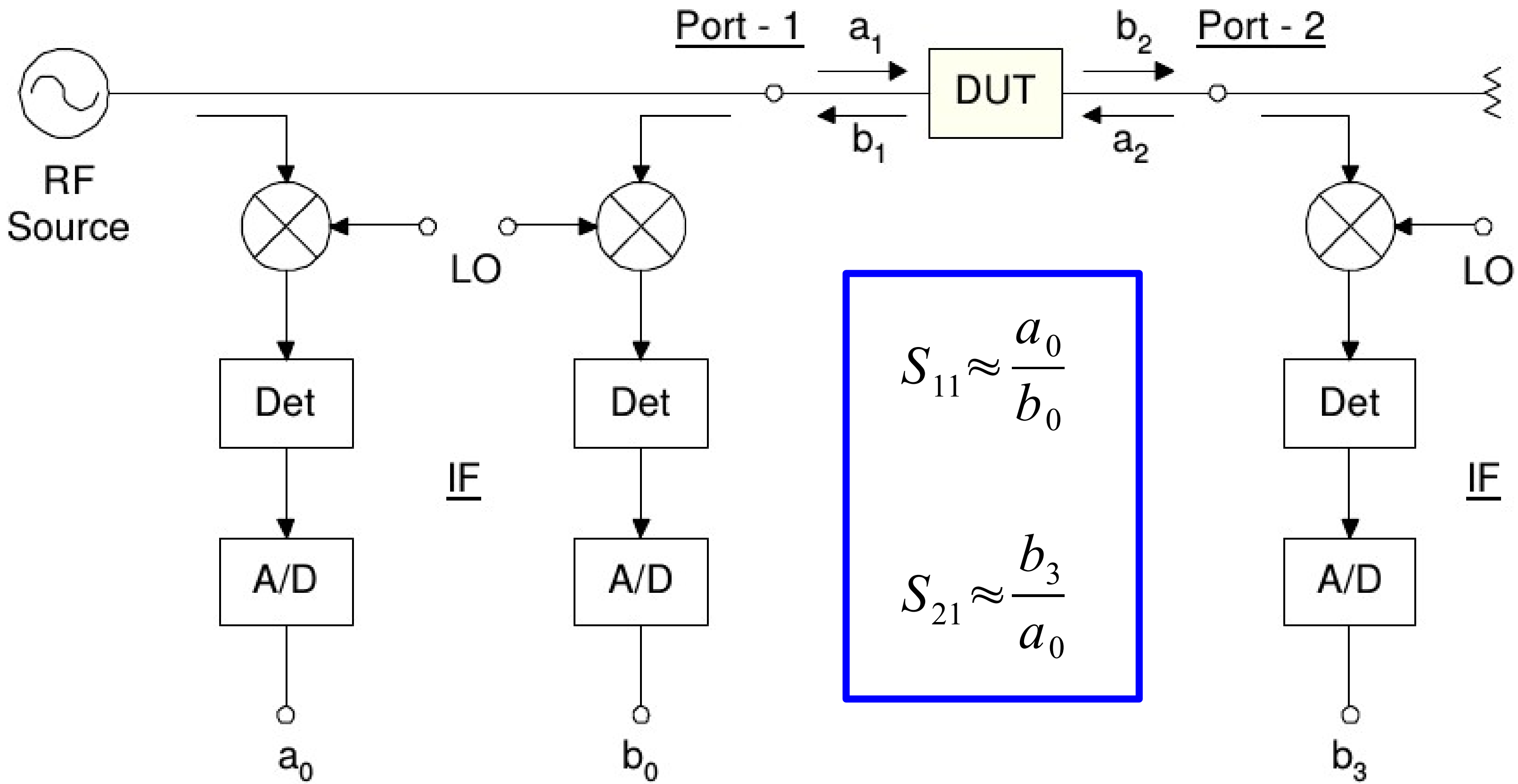
Calibration kit: – handle with great care!!
They are more worth than their weight in gold!

Vector-Network-Analyser Schematic I/II



Vector-Network-Analyser Schematic II/II

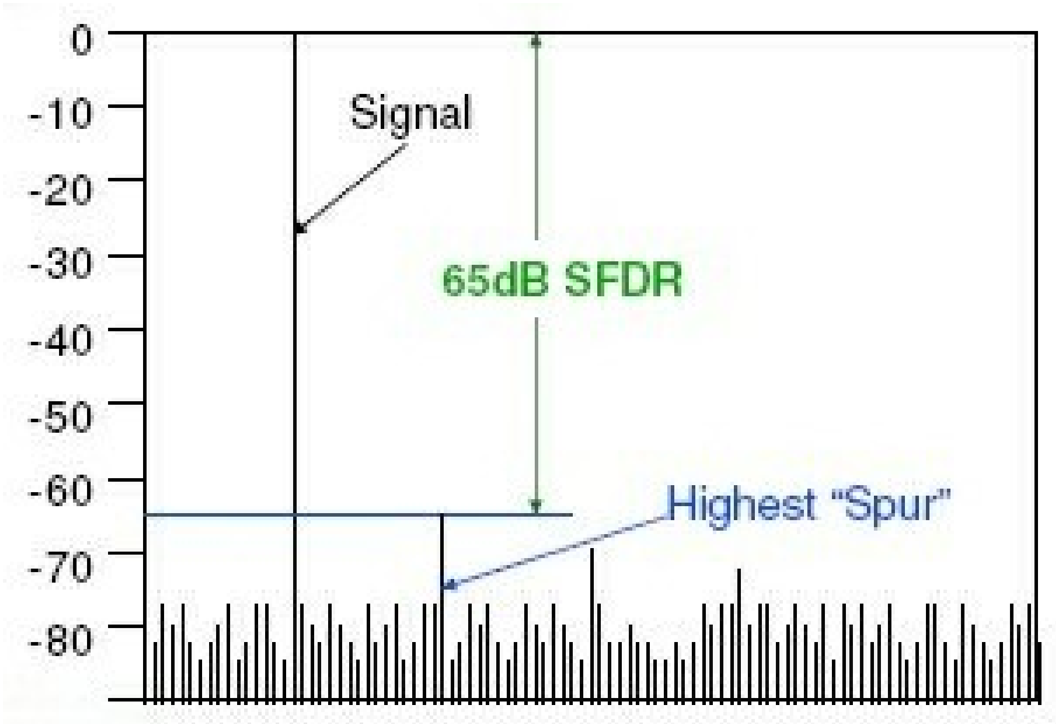
- Forward-direction only:



- VNA is based on relative power level measurements → needs calibration to equalise $a_0=a_1$, $b_0=b_1$ and $b_3=b_2$ → your laboratory exercise

Spurious Free Dynamic Range

- Spurious-Free Dynamic Range (SFDR) is the strength ratio of the fundamental signal to the strongest spurious signal in the output.
  - Used to describe and specify ADCs, i.e. effective number of bits ENOB

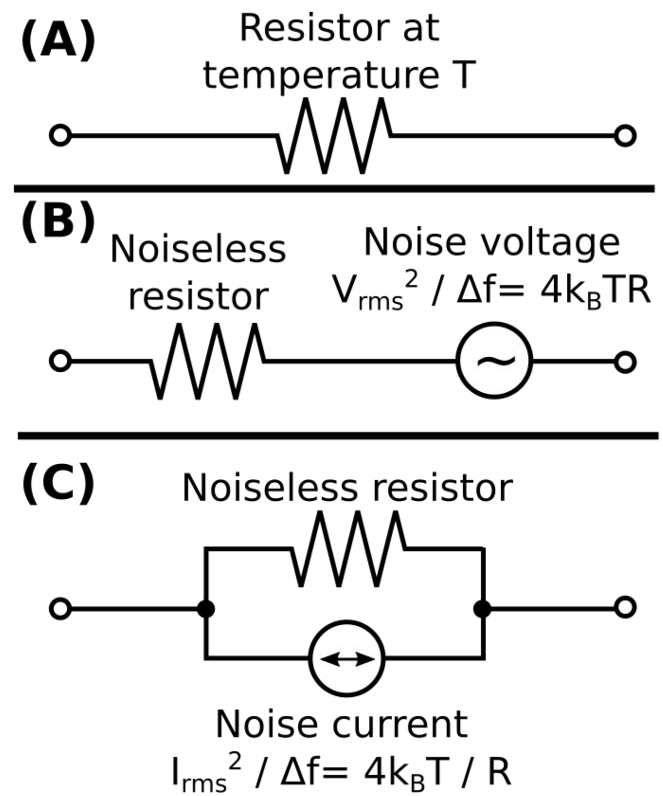


Thermal Noise

- John B. Johnsons' Bell Laboratory 1926

$$v_n = \sqrt{4 k_b T R \Delta f_{bw}}$$

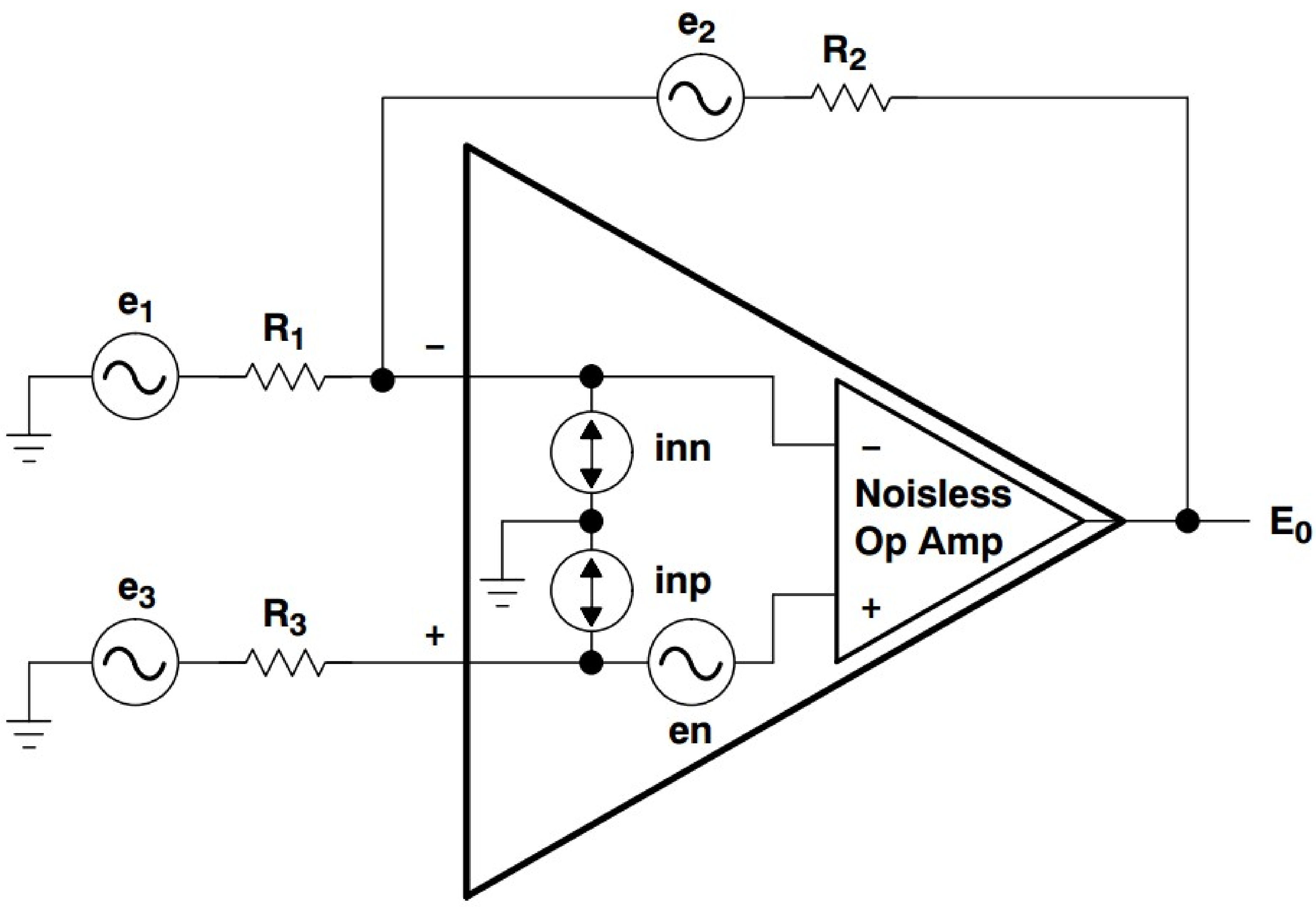
  - $k_b = 1.38 \cdot 10^{-23}$ J/K
  - T: temperature [K]
  - R: resistor value [Ohms]
  - Δf_{bw}: bandwidth
  - Examples:
    - R=1 kΩ → v_n=4.07 nV/√Hz.
    - R=50 Ω → v_n=0.2 nV/√Hz.



- Special case of R within an RC-filter
  - N.B. C itself doesn't have noise

$$v_n = \sqrt{\frac{k_b T}{C}}$$

Example Opamp



Definition of the Noise Factor/Figure

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_0B} = \frac{GN_i + N_R}{GkT_0B} = \frac{GkT_0B + N_R}{GkT_0B}$$

- F := noise factor
- S_i: available signal power at input
- $N_i=kT_0B$: available noise power at input
- T_0: absolute temperature of the source resistance
- N_o: available noise power at the output , including amplified input noise
- N_r: noise added by receiver
- G: available receiver gain
- B: effective noise bandwidth of the receiver
- → logarithmic definition: noise figure (NF)

$$NF = 10 \lg \frac{S_i / N_i}{S_o / N_o} dB$$

Amplifier Cascades & Attenuators

- Friis' formula:

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \frac{F_n - 1}{G_1 G_2 G_3 \ldots G_{n-1}} \qquad F = 1 + \frac{(L-1)T}{T_0}$$

- Take-homes:
  - Overall noise is dominated by the noise from first amplifier/device in the chain
  - In order to minimise the overall noise, chose the first amplifier gain G_1 as large as reasonably possible

- Discussed thermal Johnson noise, lots of other noise sources:
  - Flicker-noise, shot noise, burst noise

"Someone's noise is somebody else's signal"

Introduction to RF – Part II

- Aim: learn how high-frequency signals are transmitted

- Part II – RF Transmission, S-Parameter & Noise
  - Signal transmission and reflection → S-parameters
  - Low-, band- and high-pass filters
  - Amplification and noise figure

- Laboratory:
  - In || lab measurements with VNA and various RF components (hair-pin filter, strip-lines, diplexer, attenuator, low-pass, etc.)
  - Measure cable response (terminated, open,short) – see defects
  - Cantenna + VNA → building their own radar
  - demonstrate equivalence between 'scope+pulse generator' and VNA
  - RF mixer (DIY radar or FM modulation generation, check with SA and RFD)

Additional Slides

Impedance and Admittance Matrix

- Related to the total voltage and current at the port
- Impedance or Z-matrix:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N2} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k=0 \text{ for all } k \neq j}$$

all ports except port *j* are open-circuited

- Admittance or Y-matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{N2} & \cdots & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k=0 \text{ for all } k \neq j}$$

all ports except port *j* are short-circuited

$$\boldsymbol{Y} = \boldsymbol{Z}^{-1}$$