Introduction to RF – Part I

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- Acknowledgements: F. Caspers, M. Betz, E. Jensen et al.
- Excellent online resource: http://www.microwaves101.com/

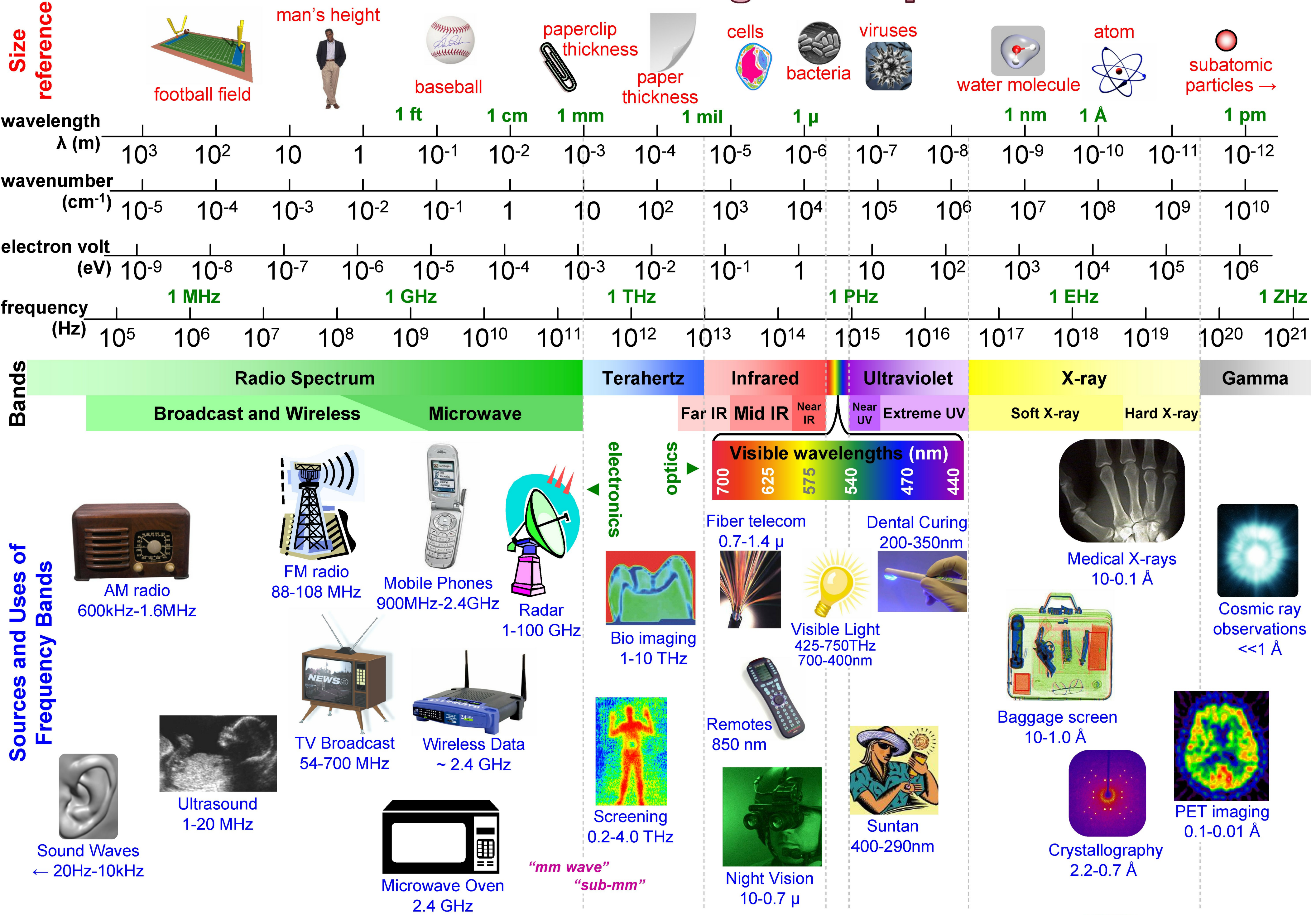
Introduction to RF – Part I

- Aim: Learn how high-frequency signals are measured

- Part I – time-domain vs. frequency domain (RF Detectors and Mixers)
  - Bode plots, power and dB(m, c, uV) definitions,
  - AM vs. FM vs. PM modulation
  - RF (Schottky) Diodes → RF Detectors (homodyne detection)
  - RF Mixers (heterodyne detection)
  - Introduction to: oscilloscopes, spectrum analyser, vector network analyser

- Laboratory:
  - some simple simulations using QUCS – guided/students
  - in || lab measurement with generator + spectrum analyser
  - some simple simulations using QUCS – guided/student
  - "Play" with BPM mock-up
  - Repeat in || previous day experience now with some active elements

Chart of the Electromagnetic Spectrum



Size reference

football field · man's height · baseball · paperclip thickness · paper thickness · cells · bacteria · viruses · water molecule · atom · subatomic particles →

wavelength λ (m)

1 ft · 1 cm · 1 mm · 1 mil · 1 μ · 1 nm · 1 Å · 1 pm

10^3 · 10^2 · 10 · 1 · 10^{-1} · 10^{-2} · 10^{-3} · 10^{-4} · 10^{-5} · 10^{-6} · 10^{-7} · 10^{-8} · 10^{-9} · 10^{-10} · 10^{-11} · 10^{-12}

wavenumber (cm^{-1})

10^{-5} · 10^{-4} · 10^{-3} · 10^{-2} · 10^{-1} · 1 · 10 · 10^2 · 10^3 · 10^4 · 10^5 · 10^6 · 10^7 · 10^8 · 10^9 · 10^{10}

electron volt (eV)

10^{-9} · 10^{-8} · 10^{-7} · 10^{-6} · 10^{-5} · 10^{-4} · 10^{-3} · 10^{-2} · 10^{-1} · 1 · 10 · 10^2 · 10^3 · 10^4 · 10^5 · 10^6

frequency (Hz)

1 MHz · 1 GHz · 1 THz · 1 PHz · 1 EHz · 1 ZHz

10^5 · 10^6 · 10^7 · 10^8 · 10^9 · 10^{10} · 10^{11} · 10^{12} · 10^{13} · 10^{14} · 10^{15} · 10^{16} · 10^{17} · 10^{18} · 10^{19} · 10^{20} · 10^{21}

Bands

Radio Spectrum · **Terahertz** · **Infrared** · **Ultraviolet** · **X-ray** · **Gamma**

Broadcast and Wireless · **Microwave** · **Far IR** · **Mid IR** · **Near IR** · **Near UV** · **Extreme UV** · **Soft X-ray** · **Hard X-ray**

◄ electronics · optics ►

Visible wavelengths (nm)

700 · 625 · 575 · 540 · 470 · 440

Sources and Uses of Frequency Bands

Sound Waves
← 20Hz-10kHz

Ultrasound
1-20 MHz

AM radio
600kHz-1.6MHz

FM radio
88-108 MHz

TV Broadcast
54-700 MHz

Mobile Phones
900MHz-2.4GHz

Wireless Data
~ 2.4 GHz

Microwave Oven
2.4 GHz

Radar
1-100 GHz

"mm wave"

"sub-mm"

Bio imaging
1-10 THz

Screening
0.2-4.0 THz

Fiber telecom
0.7-1.4 μ

Remotes
850 nm

Night Vision
10-0.7 μ

Visible Light
425-750THz
700-400nm

Dental Curing
200-350nm

Suntan
400-290nm

Medical X-rays
10-0.1 Å

Baggage screen
10-1.0 Å

Crystallography
2.2-0.7 Å

Cosmic ray observations
<<1 Å

PET imaging
0.1-0.01 Å

$\lambda = 3\times10^8/\text{freq} = 1/(\text{wn}*100) = 1.24\times10^{-6}/\text{eV}$

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Decibel (dB)

- Convenient logarithmic measure of a power ratio.
- A “Bel” (= 10 dB) is defined as a power ratio of 10^1.
- Consequently, 1 dB is a power ratio of $10^{0.1} \approx 1.259$
- If rdb denotes the measure in dB, we have:
  - N.B. 10 dB a factor 10 in power but factor 100 in amplitude!

$$r\ [dB] = 10 \cdot \log\left(\frac{P}{P_0}\right) = 10 \cdot \log\left(\frac{A^2}{A_0^2}\right) = 20 \cdot \log\left(\frac{A}{A_0}\right)$$

- Related: dBm (P_0=1 mW), dBµV (A_0=1 µV), dBc (P_0 being the carrier)

Decibel (dB) – Cont'd

- 10mW = 10dBm, 0dBm = 1mW
- -110 dBm = 10^{-11}mW = 0.00001nW
  - For a 50 ohm load : -110dBm ↔ 0.7 μV, i.e. not much!

- Rule of thumb:
  - Double the power = 3 dB increase
  - Half the power = 3 dB decrease

- Common ranges (assuming 50 Ohm load)
  - 0-30 dBm small-signal RF amplifiers
  - typ. RF-diode detection threshold: -50 dBm ↔ 0.01 μW ↔ 0.7 mV

Common Power Levels

W	dBW	dBm	V
	$10 \cdot \log_{10}(P)$	$10 \cdot \log_{10}(1000 \cdot P)$	$\sqrt{50P}$ (w/ 50Ω)
1.000	**0**	+30	7.071
0.032	-15	+15	1.257
0.020	-16.990	+13.010	**1.000**
0.010	-20	+10	0.707
0.003	-25	+5	0.397
0.001	-30	**0**	0.224
316.2µW	-35	-5	0.126
100µW	-40	-10	0.071
0.1nW	-100	-70	70.71µV
0.1pW	-130	-100	2.236µV
10fW	-140	-110	0.707µV
1fW	-150	-120	0.224µV
4.142E-21 (kT at 300K)	-203.8	-173.8	0.455nV

power RF
(typ. narrow-band)

small-signal RF
(wide- & narrow-band)

power RF
(typ. narrow-band)

Unassisted RF-diode detection threshold:
(-50 dBm ↔ 0.01 µW ↔ 0.7 mV)

thermal noise floor
(at room temperature)

Fourier Transform I

- An arbitrary signal g(t) can be expressed frequency-domain using the Fourier transform (FT):

$$\mathcal{F}\left\{g(t)\right\} = G(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} g(t)e^{+j\omega t}dt$$

$$\mathcal{F}^{-1}\left\{G(\omega)\right\} = G(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} G(\omega)e^{-j\omega t}d\omega$$

- Many problems dealing with linear-time-invariant (LTI) systems are much easier described in frequency domain

- FT Cousins:
  - Laplace Transform: jω → s, and integrating only from '0' rather than -∞
    - Strictly causal → used in system- and control theory
  - Z-Transform:
    - → z = $e^{j\omega T}$, with T being the sampling period
    - delays of $k \cdot T$ become z^{-k}

Fourier Transform II

$$\text{Linearity:} \quad \mathcal{F}\left(af(t)\right) = a\,\mathcal{F}\left(f(t)\right)$$

$$\text{Superposition:} \quad \mathcal{F}\left(a\ f(t)+b\ g(t)\right) = a\,F(\omega)+b\,G(\omega)$$

$$\text{Time } \mathit{Delay}: \quad \mathcal{F}\left(f(t-\Delta t)\right) = e^{-j\omega\Delta t}F(\omega)$$

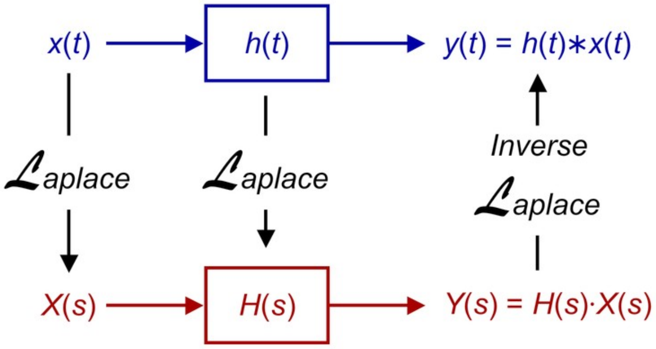
$$\text{Frequency } \mathit{Shift}: \quad \mathcal{F}\left(f(t)\cdot e^{-j\omega_{0t}}\right) = F(\omega-\omega_0)$$

$$\text{Time } \mathit{Scaling}: \quad \mathcal{F}\left(f(at)\right) = \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$$

$$\text{Differentiation:} \quad \mathcal{F}\left(d^{kf}\frac{(t)}{dt^k}\right) = (j\omega)^k F\left(\frac{\omega}{a}\right)$$

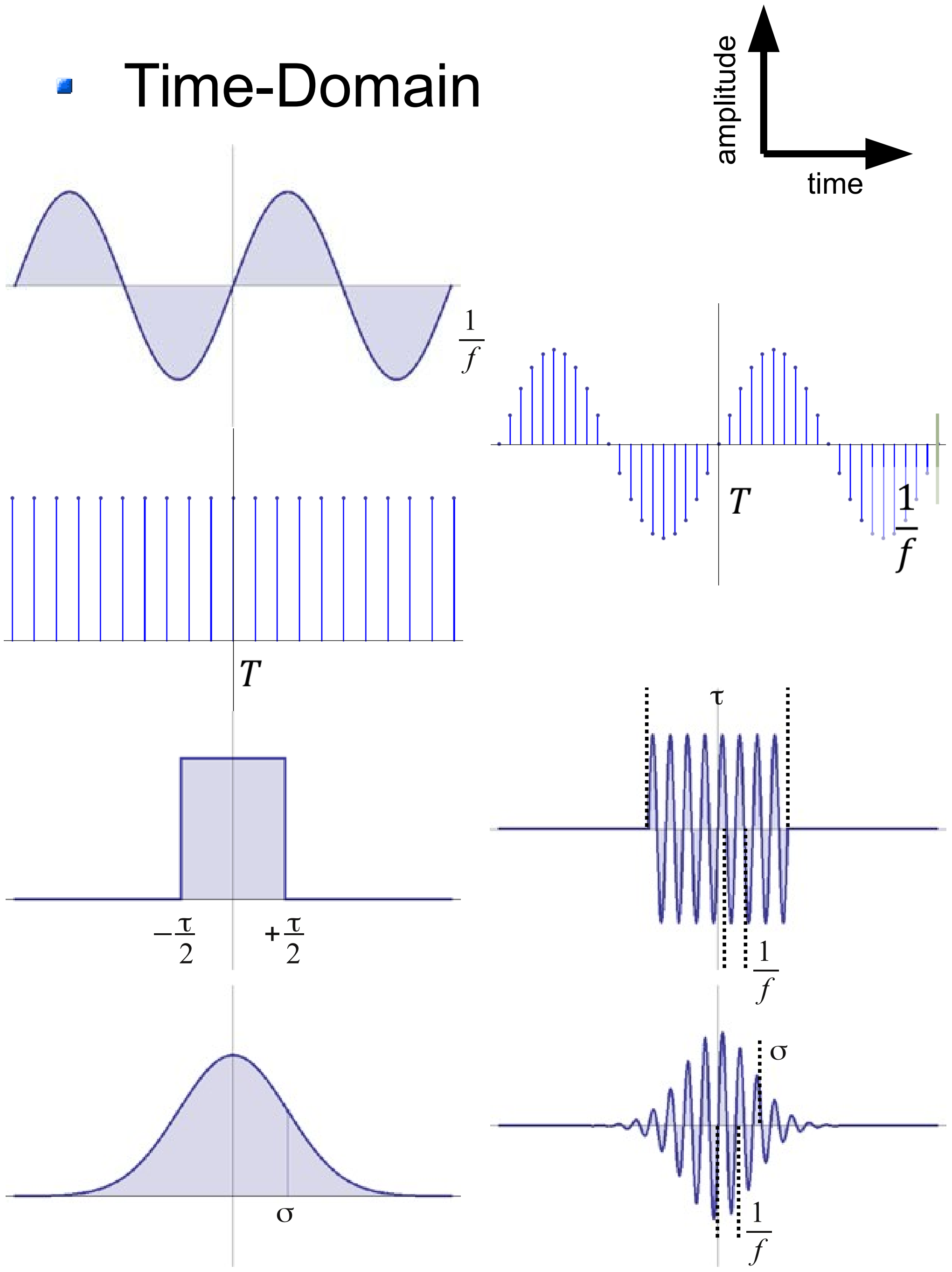
$$\text{Convolution:} \quad \mathcal{F}\left(f(t)\cdot g(t)\right) = F(\omega)*G(\omega), \quad \text{and}$$

$$\mathcal{F}\left(f(t)*g(t)\right) = F(\omega)\cdot G(\omega)$$

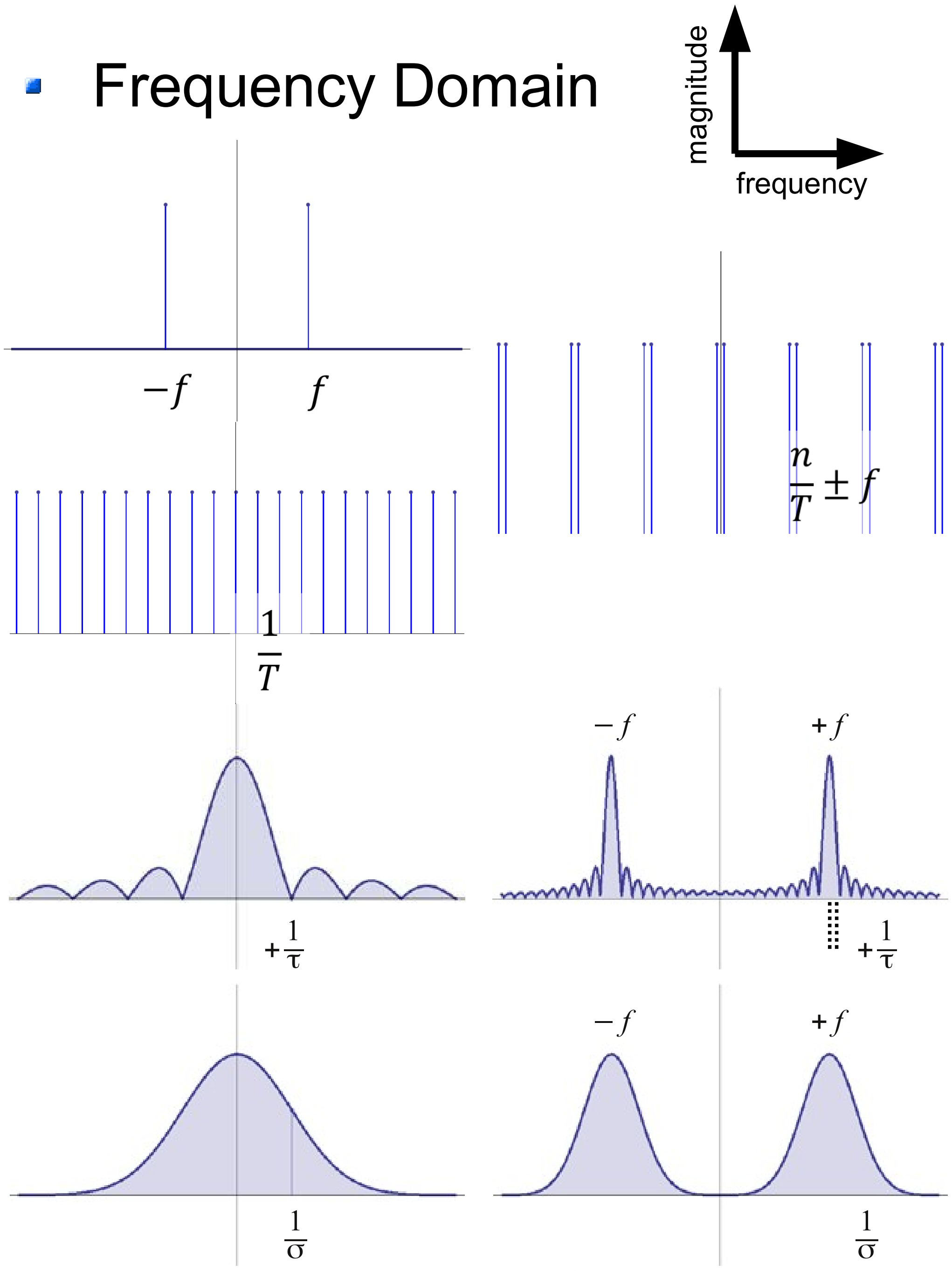


Fourier Transform III – Examples

- Time-Domain

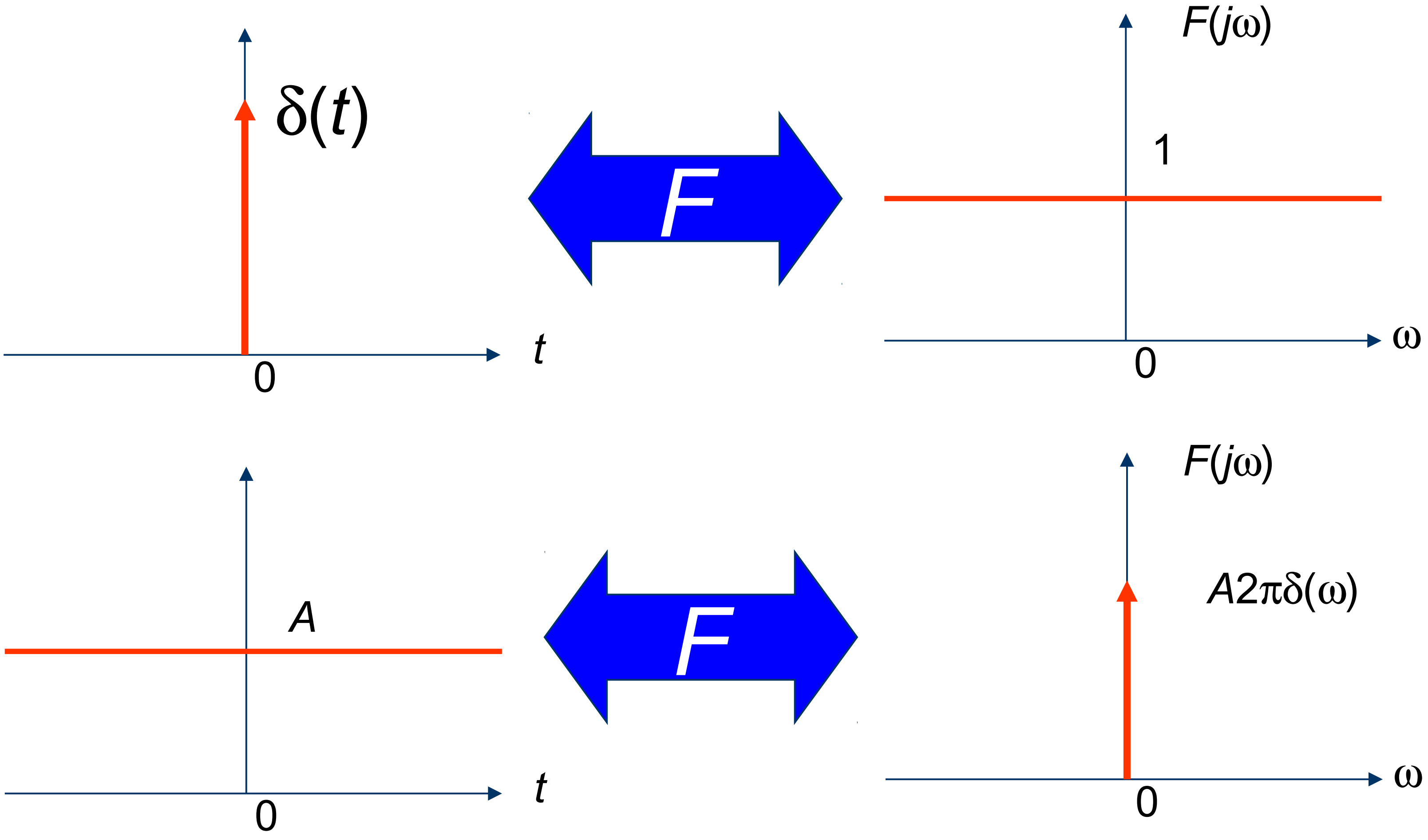


- Frequency Domain



Some important special Functions I/II

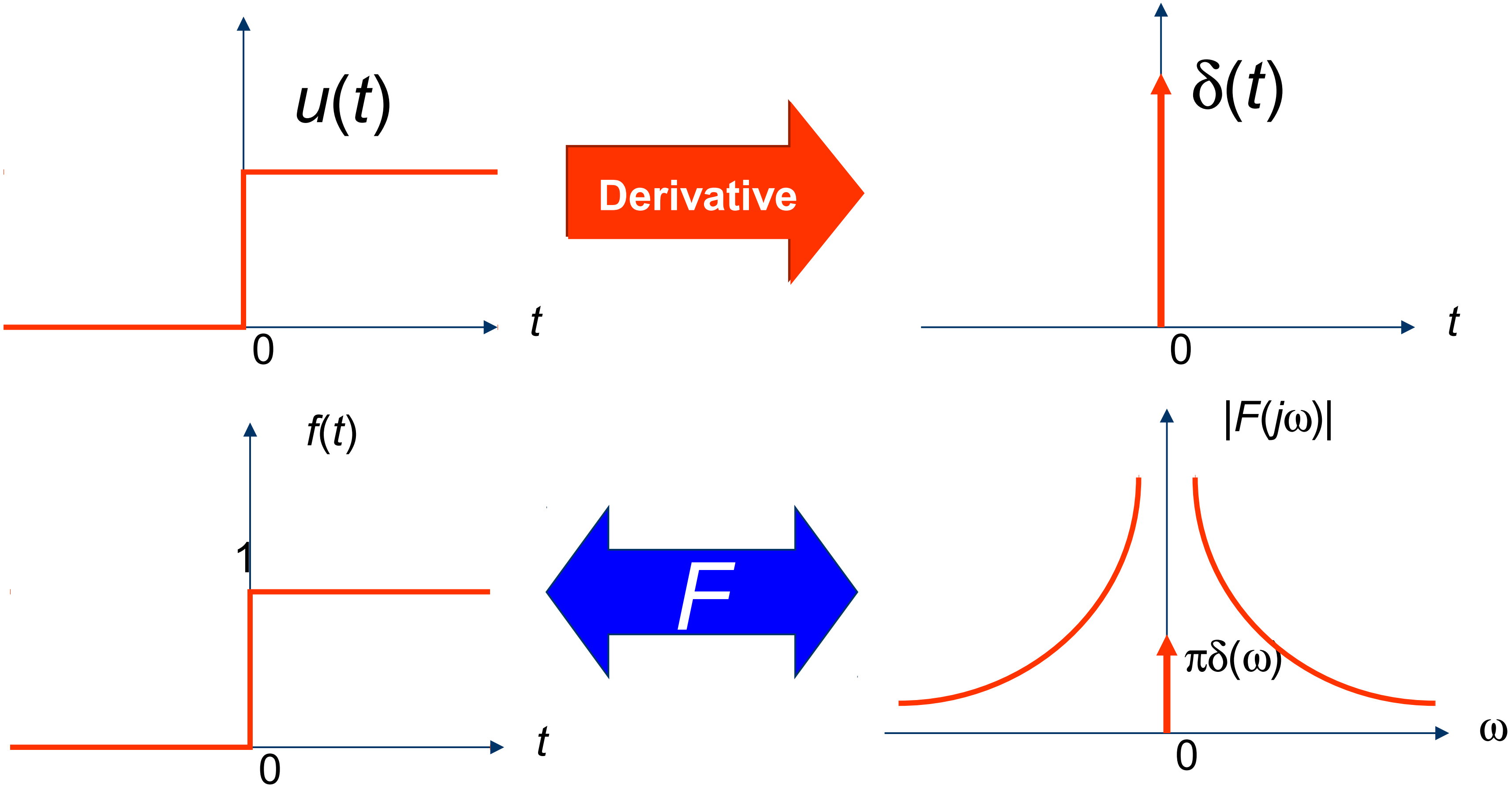
- Dirac delta function (theory) or 'impulse function' (praxis):



- Often used to decompose and analyse arbitrary wide-band signal, i.e. to measure system response

Some important special Functions II/II

- Heaviside step function:



- Equivalent to dirac delta-based analysis but in praxis more easy to use and produce experimentally

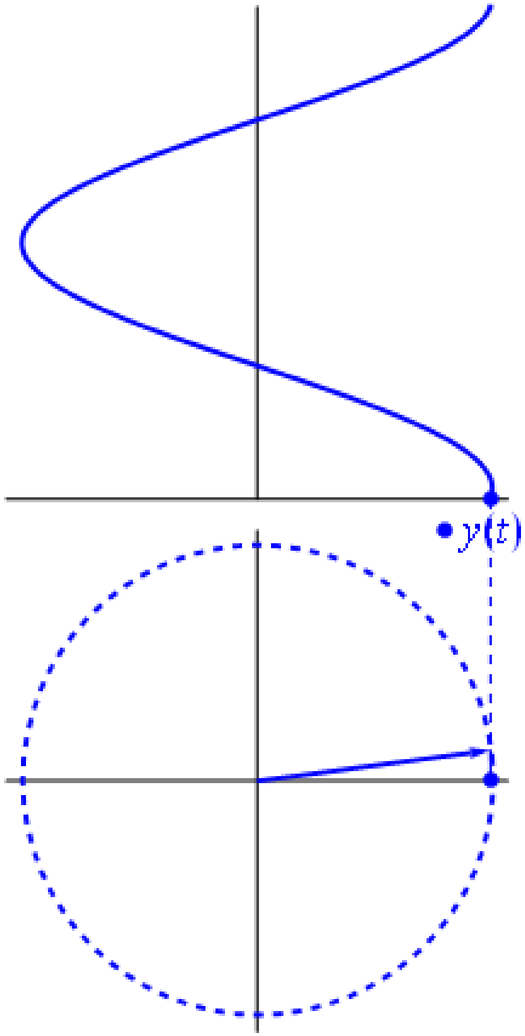
Fixed Frequency Oscillation & Phasor

- Steady-state frequency solution can be decomposed:

$$A\cos(\omega t - \varphi) = A\cos(\omega t)\cos(\varphi) + A\sin(\omega t)\sin(\varphi)$$

  - can be interpreted as the projection on the real axis of a circular motion in the complex plane.

$$\Re\left\{A\left[\cos(\phi) + j\sin(\phi)\right]\cdot e^{j\omega t}\right\}$$



- The complex amplitude is called “phasor”;

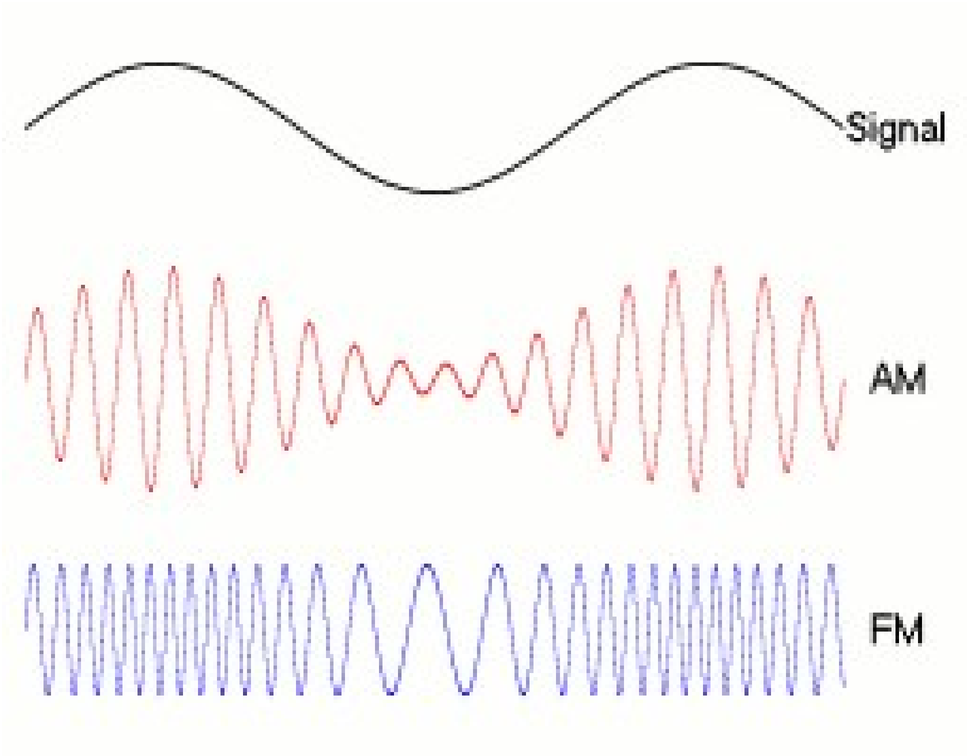
$$\tilde{A} = A\left[\cos(\phi) + j\sin(\phi)\right]$$

Amplitude, Frequency & Phase Modulation I/III

$$f_{AM}(t) = A_0 \cdot \left(1 + m\cos(\omega_s t)\right) \cdot \cos(\omega_c t)$$

$$f_{FM}(t) = A_0 \cdot \cos\left(\omega_c t + \underbrace{\frac{\Delta\omega}{\omega_m}}_{:=m} \cos(\omega_m t)\right)$$

- modulation index m: maximum relative amplitude resp. frequency deviation



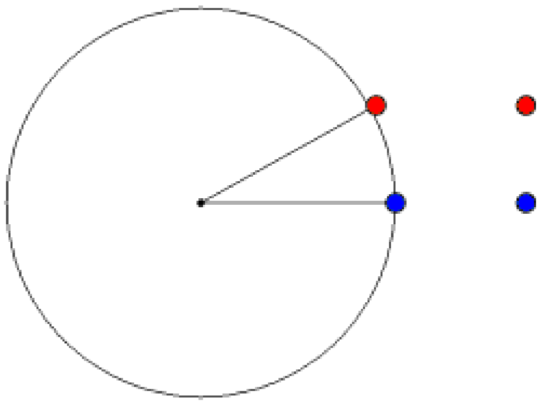
Amplitude, Frequency & Phase Modulation I/III

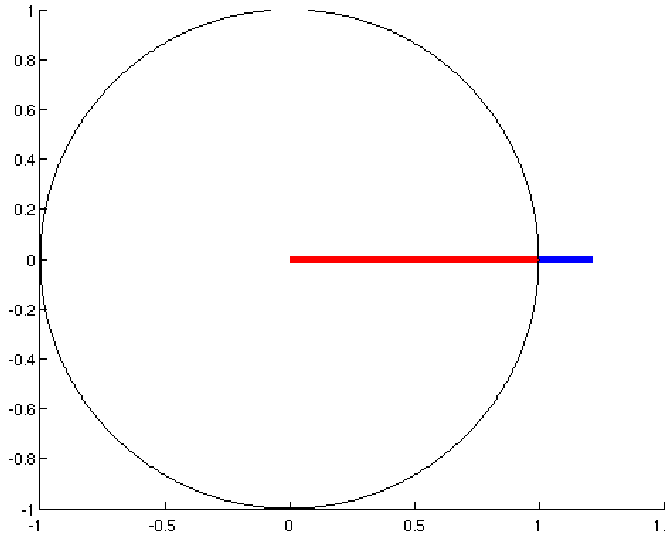
$$f_{AM}(t)=A_0\cdot\Re\left\{\left(1+\frac{m}{2}e^{+j\omega_m t}+\frac{m}{2}e^{-j\omega_m t}\right)e^{j\omega_c t}\right\}$$

$$f_{FM}(t)=A_0\cdot\Re\left\{e^{+j\,\omega_m t+m\sin(\omega_m t)}\right\}=A_0\cdot\Re\left\{\sum_{n=-\infty}^{n=+\infty}J_n(m)\,e^{+j(\omega_c t+n\cdot\omega_m)}\right\}$$

– modulation index m: maximum relative amplitude resp. frequency deviation

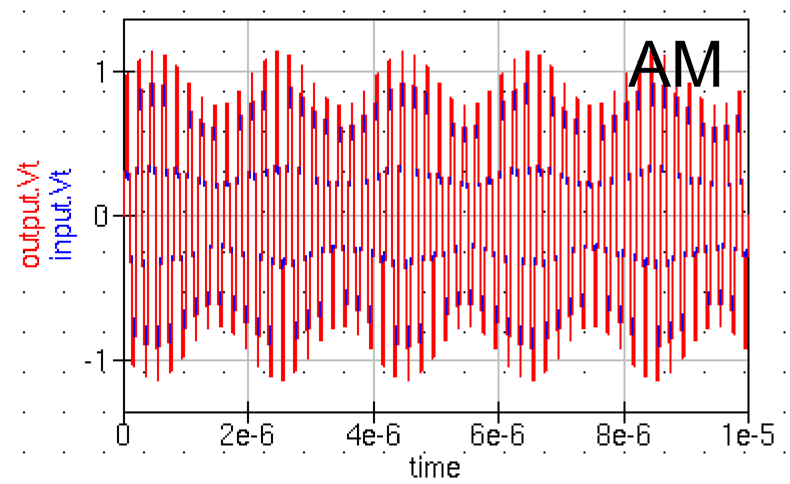
Image courtesy of whiteboard.ping.se

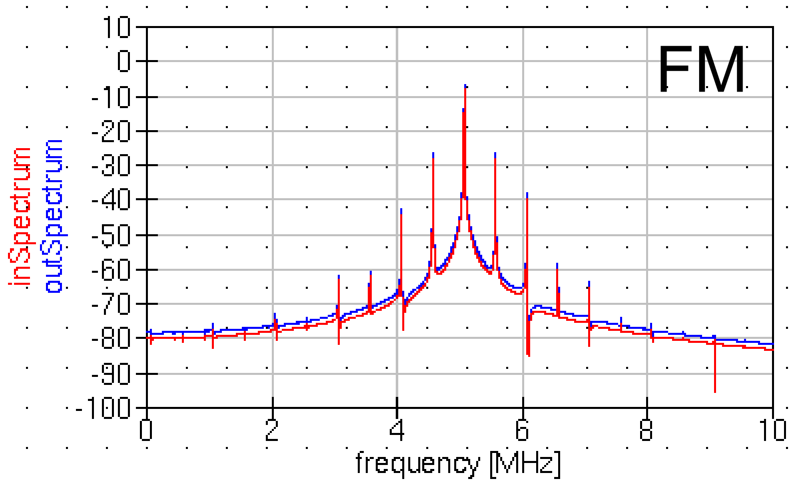


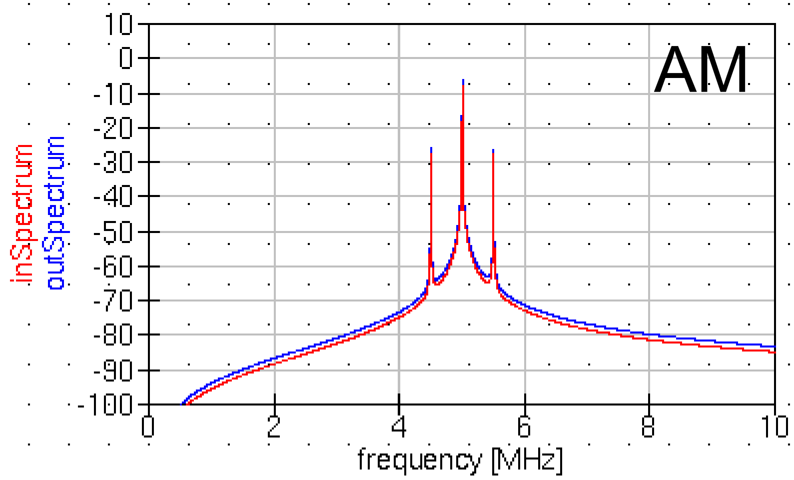


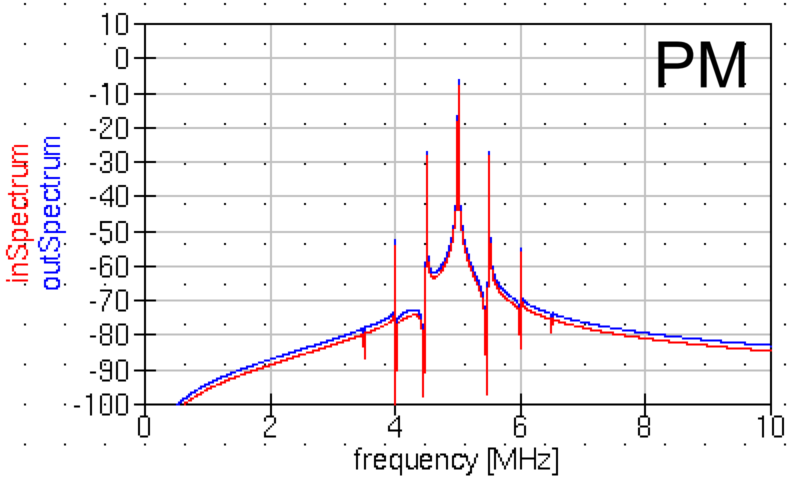
Amplitude, Frequency & Phase Modulation II/III

- Accelerator related examples
  - AM: turn-by-turn transverse beam position or betatron (Q) motion
  - FM: turn-by-turn longitudinal synchrotron motion (arrival time at pick-up)



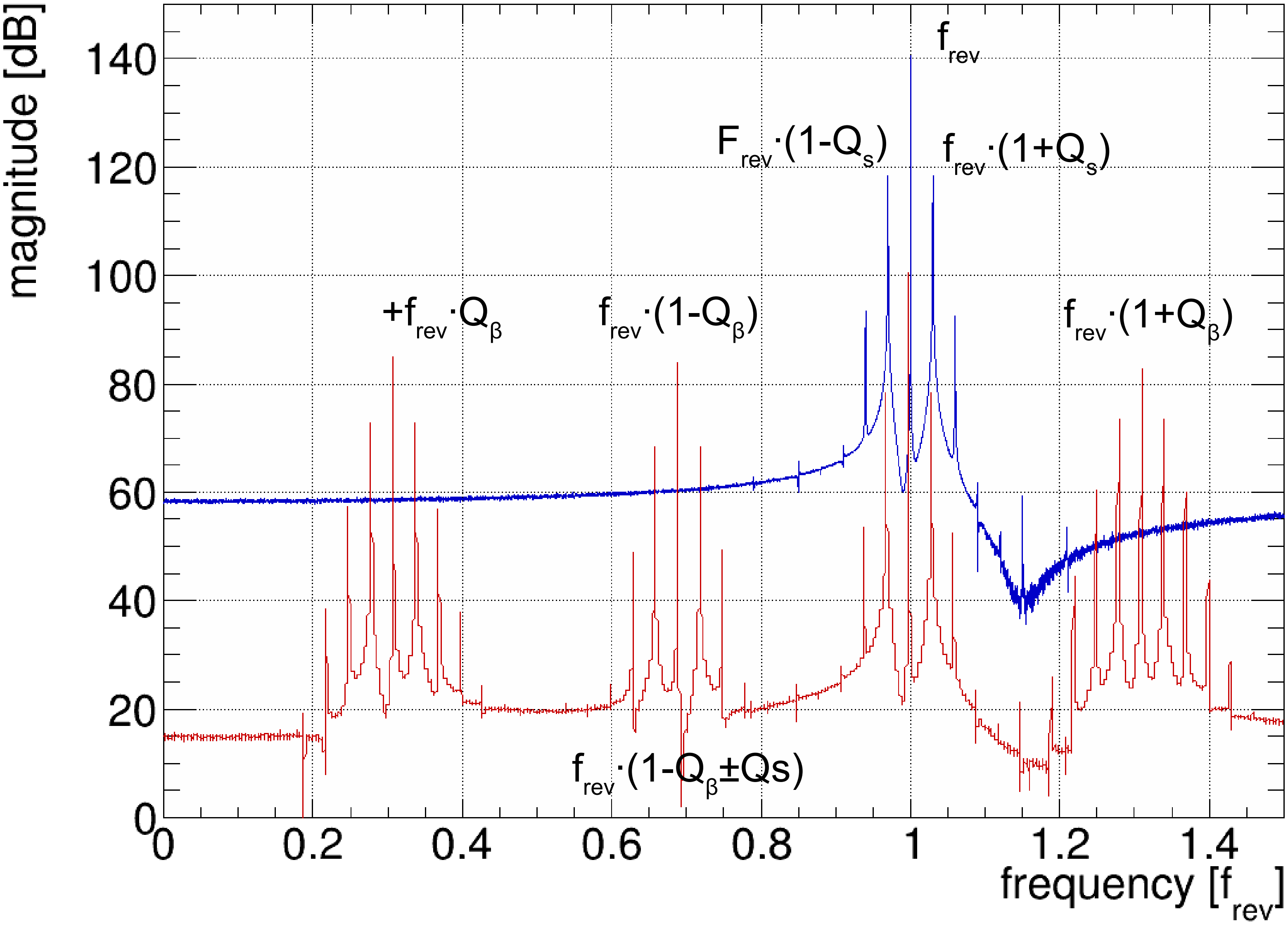






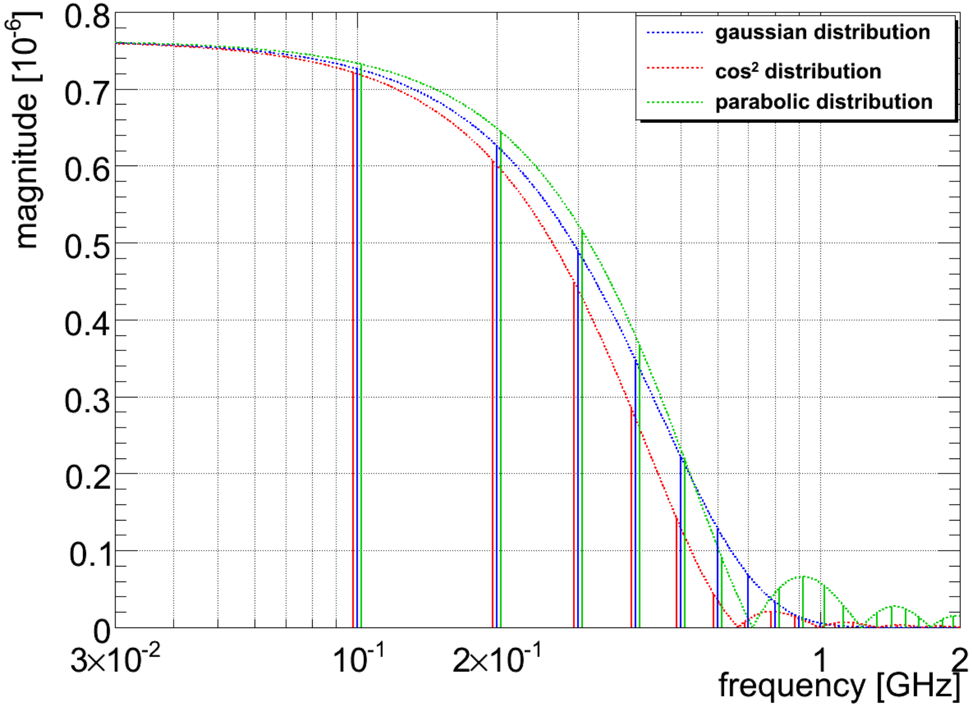
Amplitude, Frequency & Phase Modulation III/III

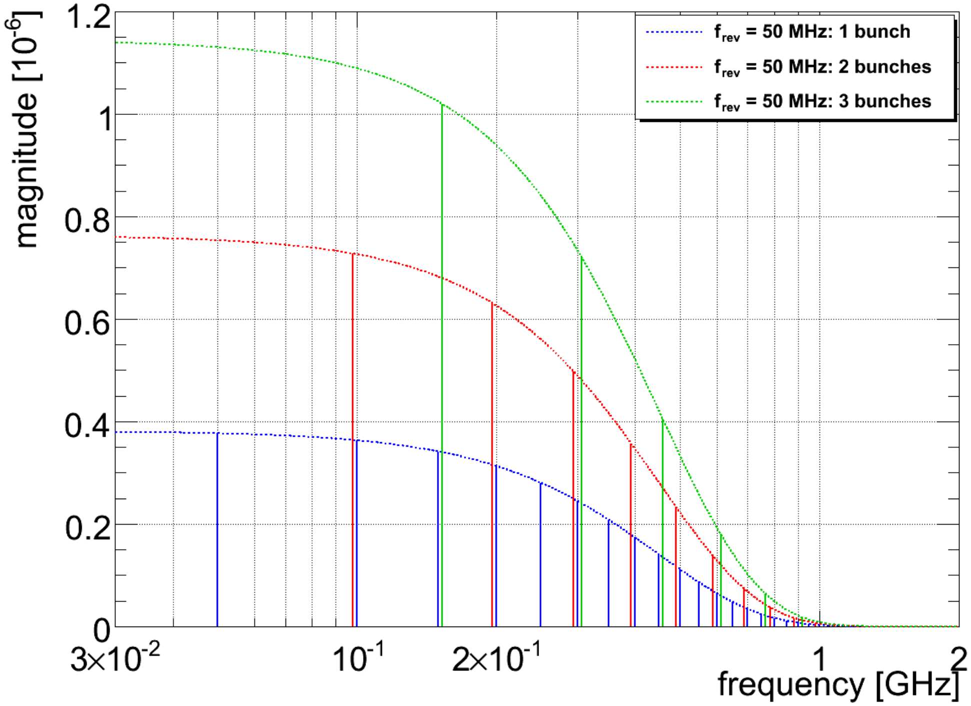
- A more complex bunch spectrum...



Beam Spectrum

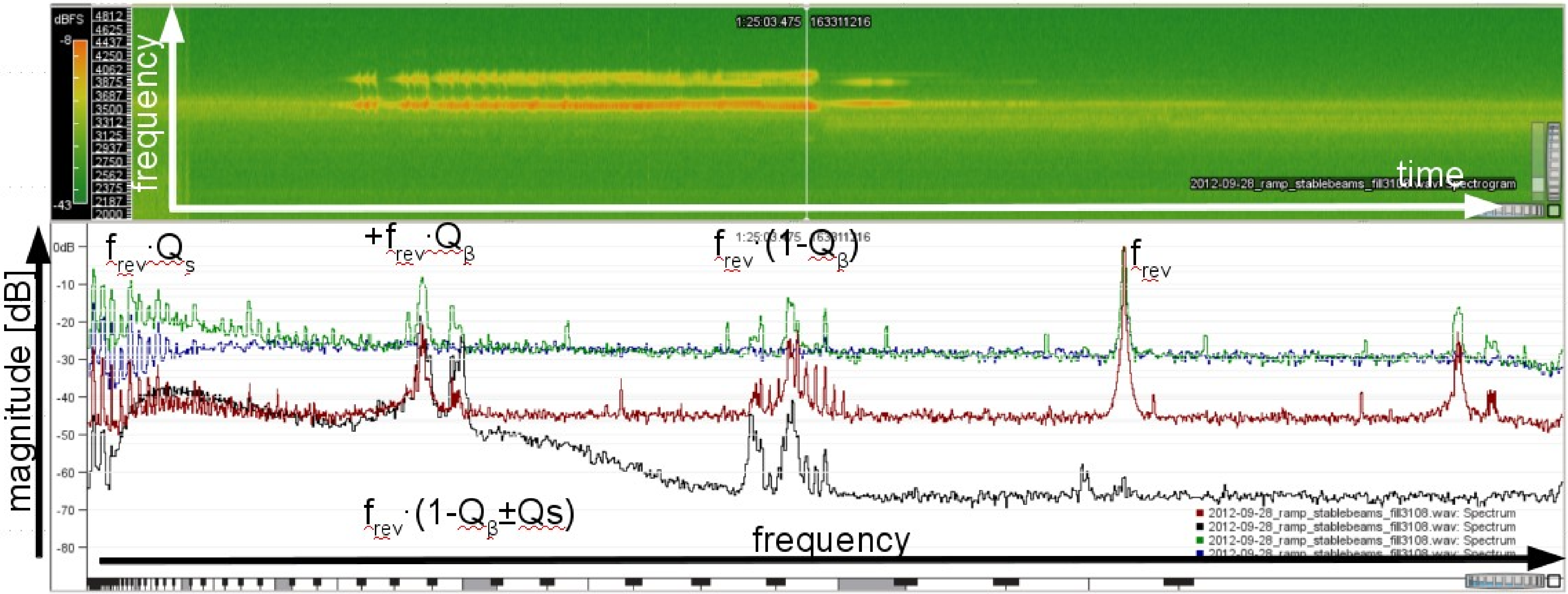
- Typical beam structure isn't a perfect sine but short bursts of particles contained inside the RF separatrix (aka. RF bucket)
  - Periodic dirac delta $\mathcal{F}\{\Sigma_n \delta(n_{turn}\cdot(t-\Delta t_{trun}))\} \rightarrow \Sigma_n \delta_\omega(n/\Delta t_{trun})$
  - beam spectrum: $\mathcal{F}\{ \rho_{longitudinal}(t)\cdot x_{transverse}(t) \} \rightarrow \rho_{longitudinal}(\omega) * X_{transverse}(\omega)$



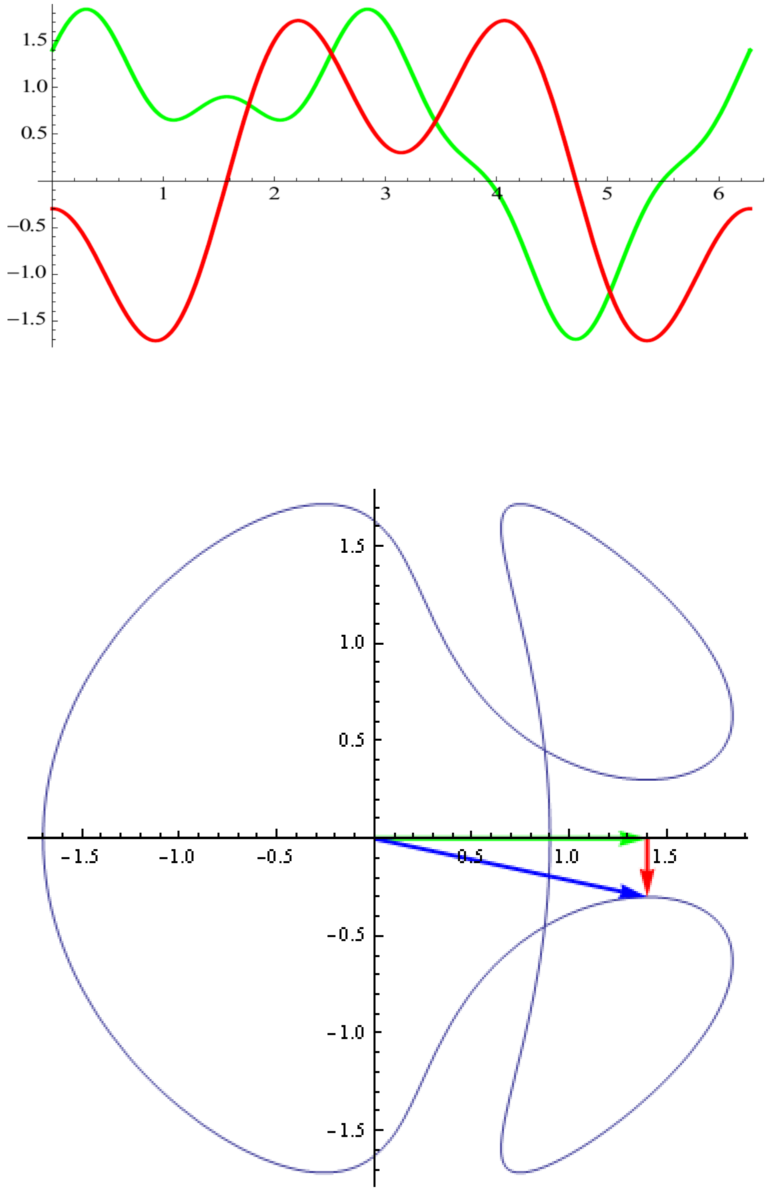


Transverse Tune Spectrum

- Still too easy?



IQ-Modulation



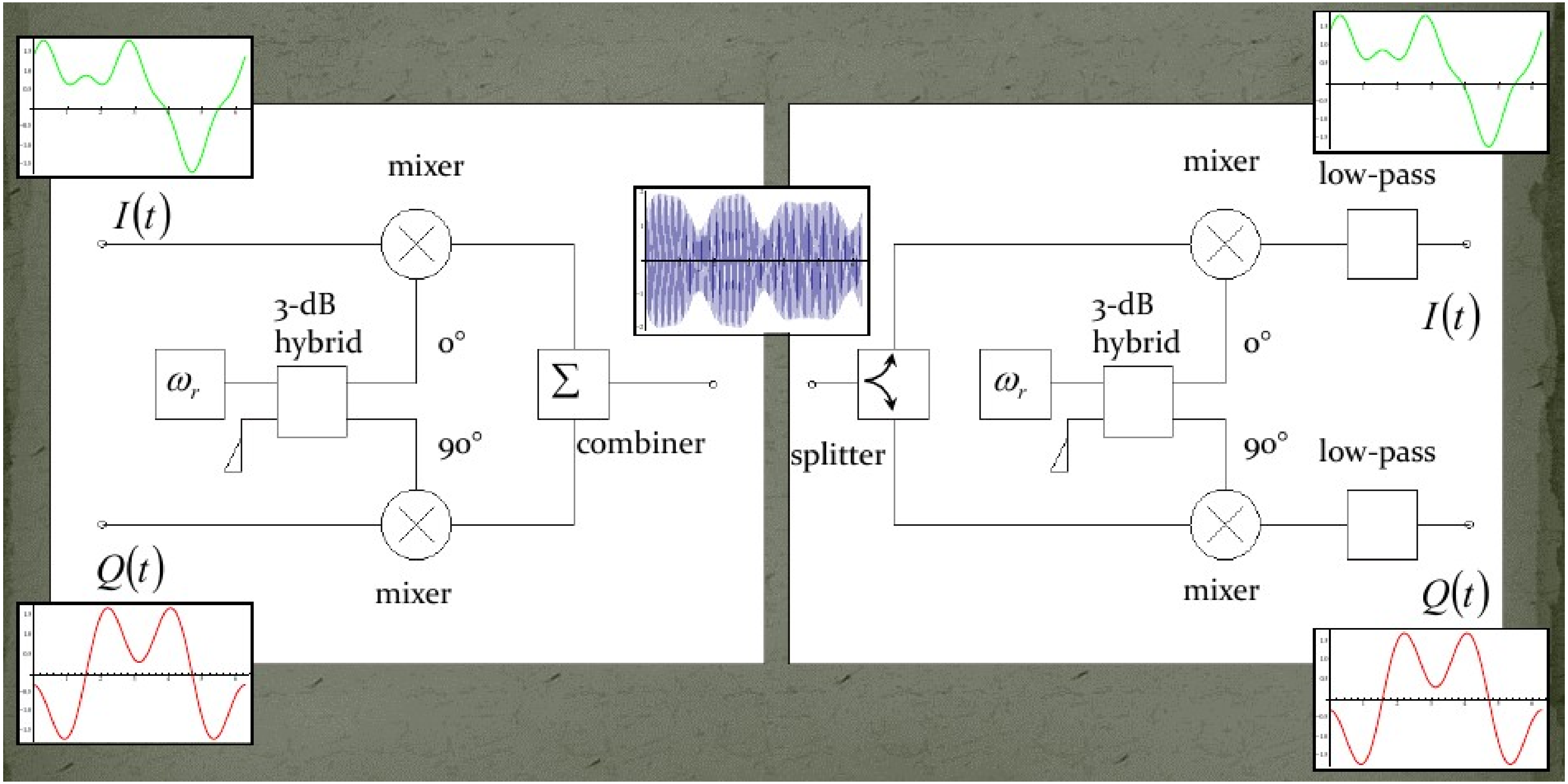
- More generally, a modulation can have both amplitude and phase modulating components. They can be described as the in-phase (I) and quadrature (Q) components in a chosen reference, cos(ω_rt).
- In complex notation, the modulated cos RF is:

$$\mathrm{Re}\left\{\left(I(t)+j\,Q(t)\right)e^{j\,\omega_r t}\right\} =$$
$$\mathrm{Re}\left\{\left(I(t)+j\,Q(t)\right)\left(\cos(\omega_r t)+j\sin(\omega_r t)\right)\right\} =$$
$$I(t)\cos(\omega_r t) - Q(t)\sin(\omega_r t)$$

- I and Q are the Cartesian coordinates in the complex “Phasor” plane, where amplitude and phase are the corresponding polar coordinates.
  - I(t) = A(t) cos(φ)
  - Q(t) = A(t) sin(φ)

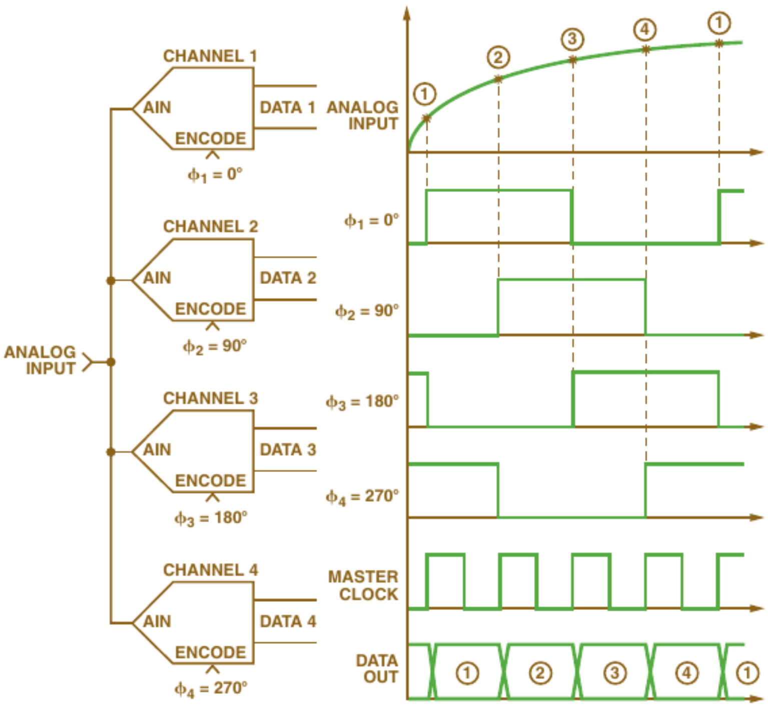
IQ-Modulator and Demodulator

- Combination of AM- and FM- modulation
  - Basically the analog equivalent of the sin-cos Fourier transform definition



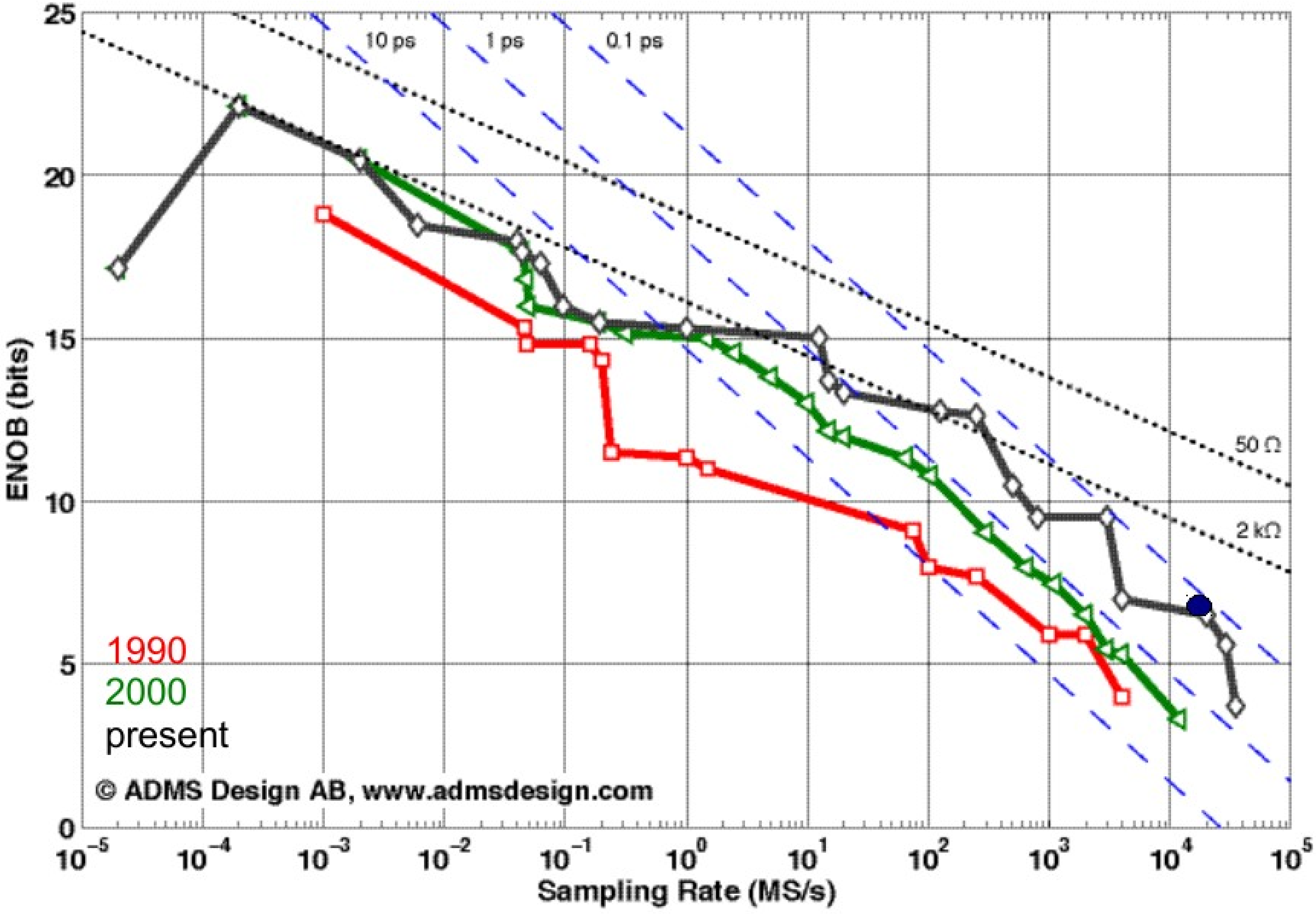
Direct Time-Domain Observation

- Oscilloscope (or. 'scope'):
  direct sampling in time domain
  - periodic signals, burst signal
  - typ. application: e.g. direct observation of pick-up signal
  - post-processing: e.g. fourier transform → wide-band spectrum analyser
  - sampling speeds and bandwidths: 500 MHz → >50 GHz
    - N.B. price range: few k\$ → >200 k\$
  - Limited maximum instantaneous voltage range $U_{max}/U_{min} \approx 100$



Limits of direct time-domain digitization

- ADCs performance levels out and approaching fundamental physical limits



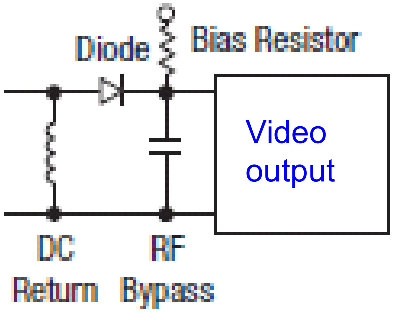
The RF Detector Diode I

- We are not discussing the generation of RF signals here, just the detection
- Basic tool: fast RF diode (= Schottky diode)
  - In general, Schottky diodes are fast but still have a voltage dependent junction capacity
    (metal – semi-conductor junction)

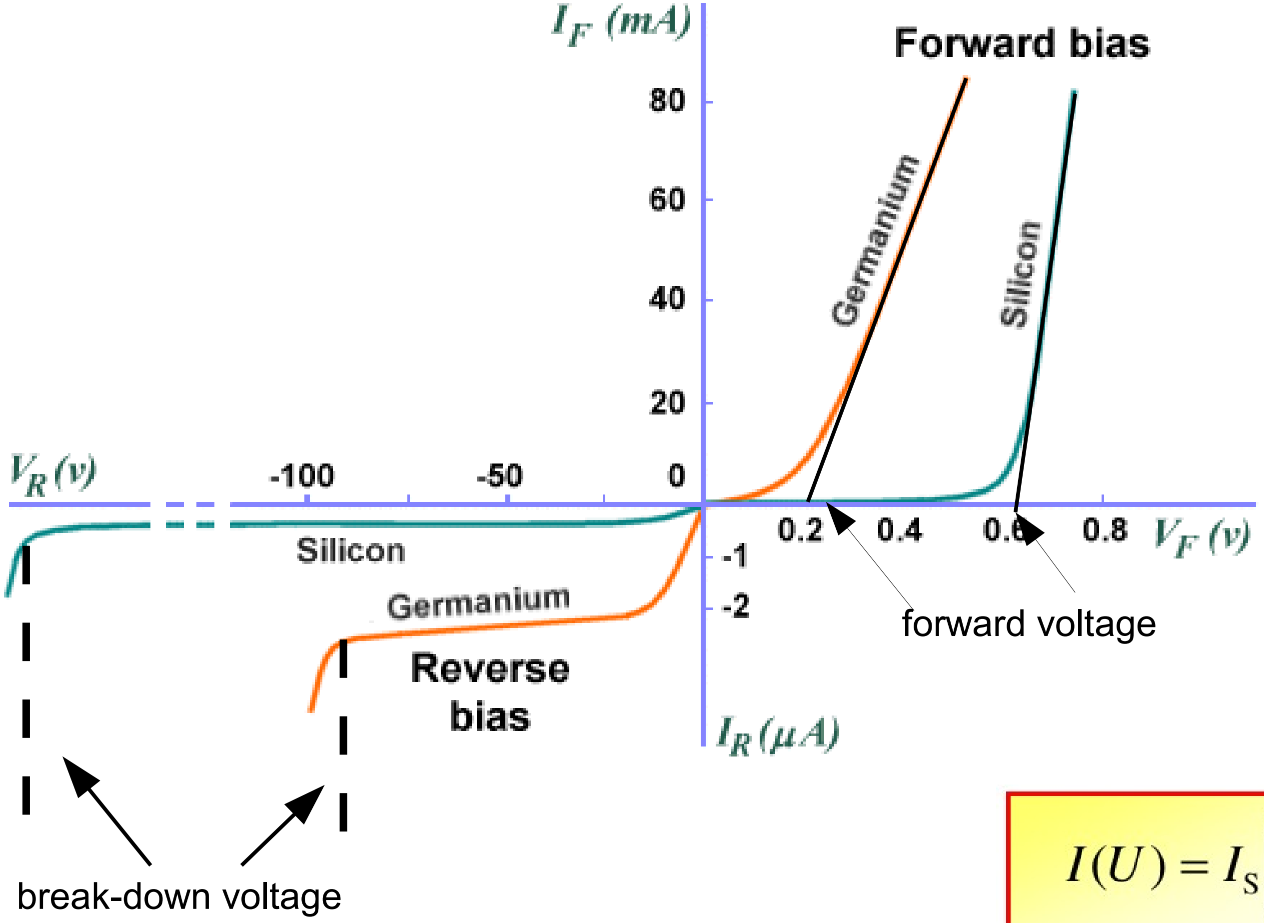
A typical RF detector diode

Try to guess from the type of the connector which side is the RF input and which is the output

- Equivalent circuit:



The RF Detector Diode II



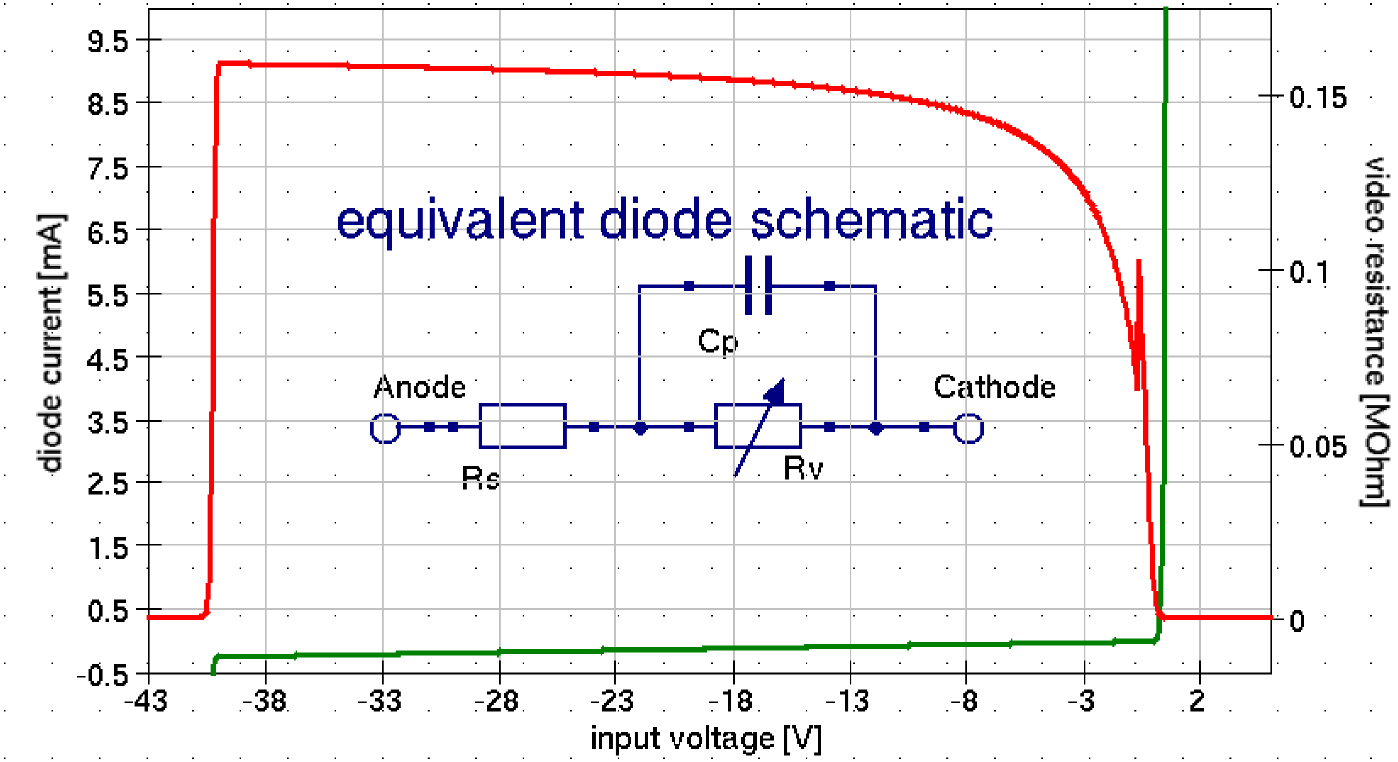
$$
\begin{aligned}
I(U) &= I_S\left(\exp\left(\frac{eU}{mkT}\right)-1\right)\\
&= I_S\left(\exp\left(\frac{U}{mU_T}\right)-1\right)
\end{aligned}
$$

- Material properties
  - Silicon: $I_s = 10^{-11}$ A, mU_T=0.03V
  - Germanium: $I_s = 10^{-7}$ A, mU_T=0.03V

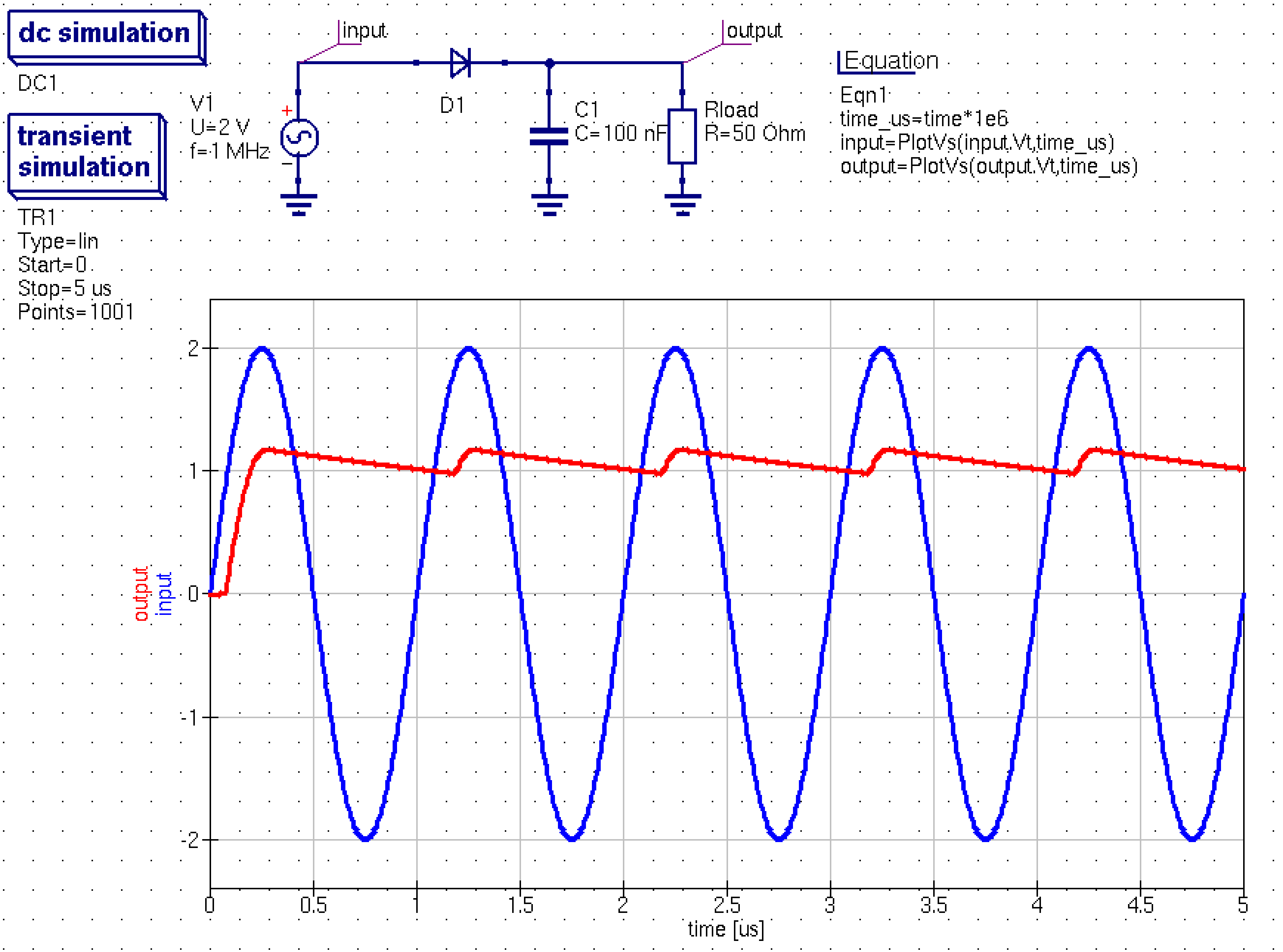
$$U_T = \frac{kT}{e} = 25.5\text{ mV}$$

The RF Detector Diode III

- Equivalent diode circuit
  - R_s: series resistance (Schottky: Rs< few Ω)
  - C_p: junction/parasitic capacitance (Schottky: C_p< 1 pF ↔ limits bandwidth)
  - R_v: video resistance → similar to a voltage controlled variable resistor
    - Very small resistance for positive voltage
    - Very large resistance for negative voltages



The RF Detector Diode IV – Rectifier



dc simulation

DC1

transient simulation

TR1
Type=lin
Start=0
Stop=5 us
Points=1001

V1
U=2 V
f=1 MHz

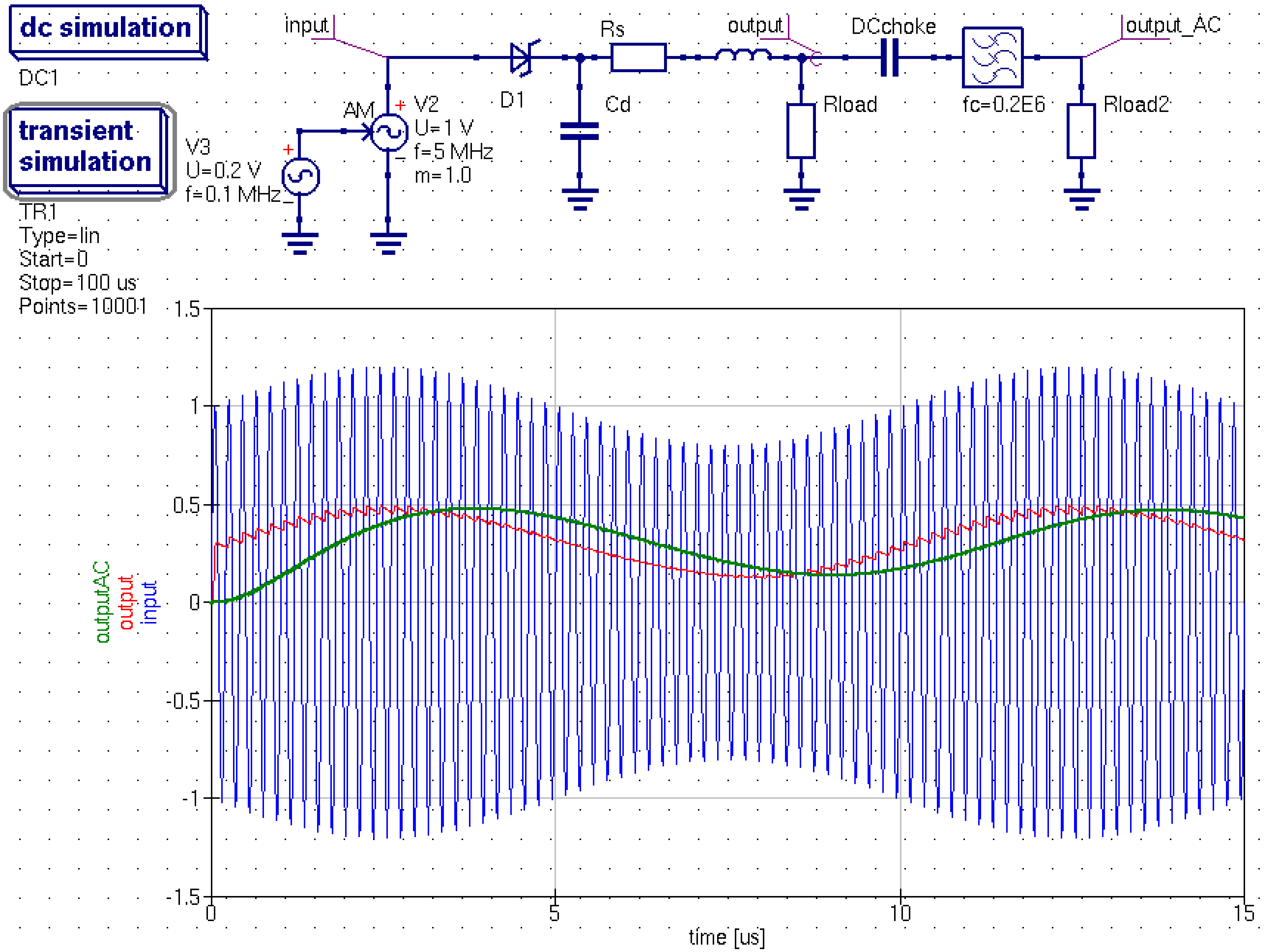
D1

C1
C=100 nF

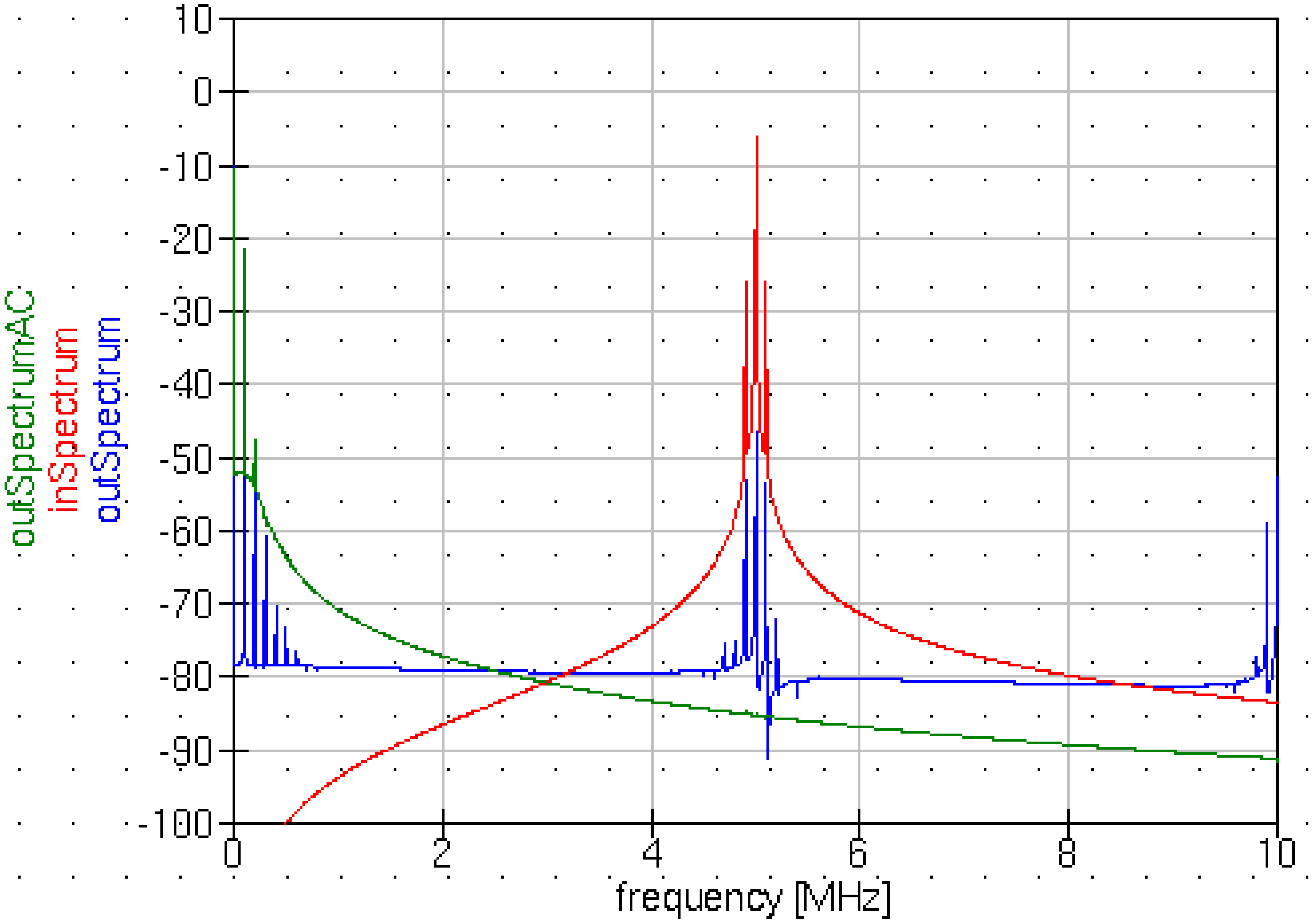
Rload
R=50 Ohm

Equation
Eqn1
time_us=time*1e6
input=PlotVs(input.Vt,time_us)
output=PlotVs(output.Vt,time_us)

Simple RF Detector – Time Domain



Simple RF Detector – Frequency Domain



- This detection scheme is also referred to as 'homodyne detection'

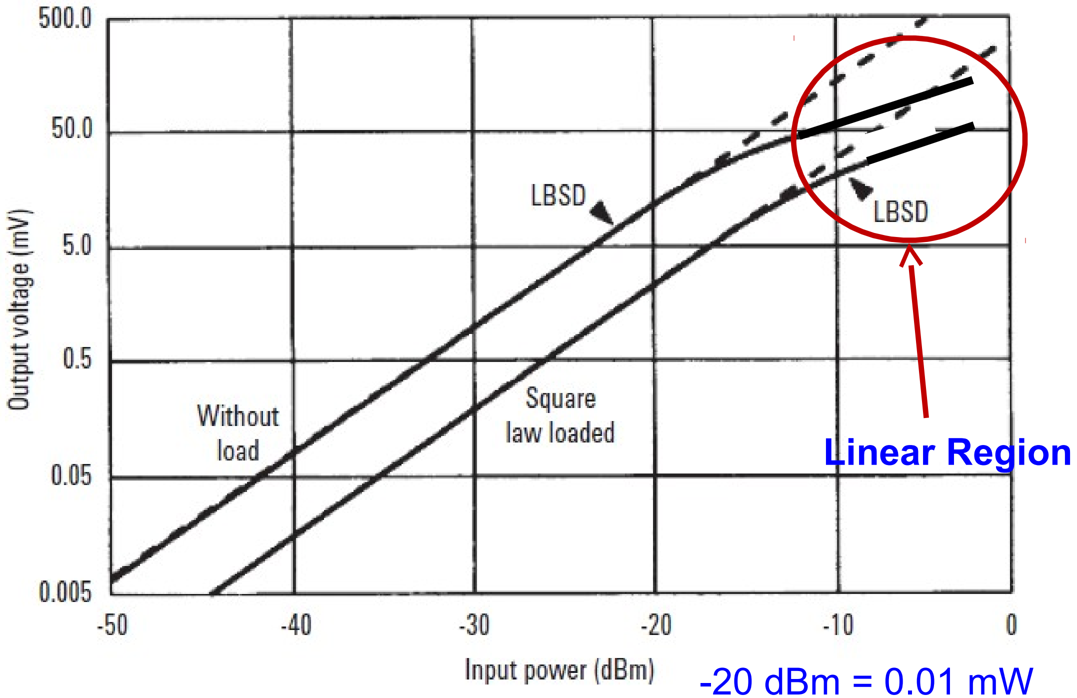
RF Diode – Square-Law

- This diagram depicts the so called square-law region where the output voltage (V_{Video}) is proportional to the input power

Since the input power is proportional to the square of the input voltage (V_{RF2}) and the output signal is proportional to the input power, this region is called square- law region.

$V_{Video} \sim (V_{RF})^2$

$>$ -20 dBm: $V_{Video} \sim V_{RF}$



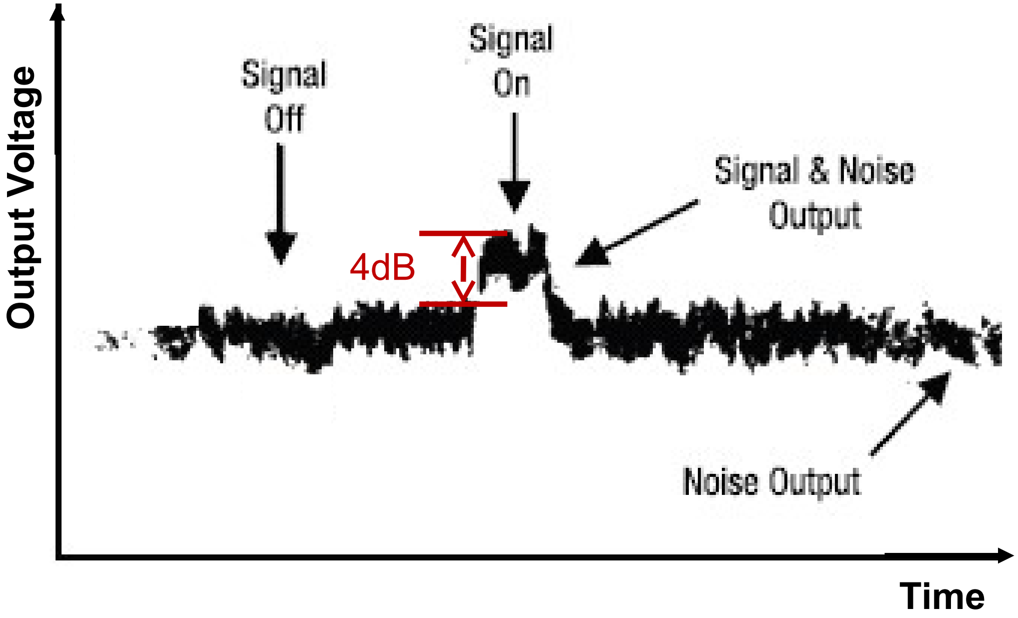
-20 dBm = 0.01 mW

- The transition between the linear region and the square-law region is typically between -10 and -20 dBm RF power.
- Why does it matter ?
  → this is being used as and limits the fundamental property of RF mixers

RF Diode – Tangential Signal Sensitivity

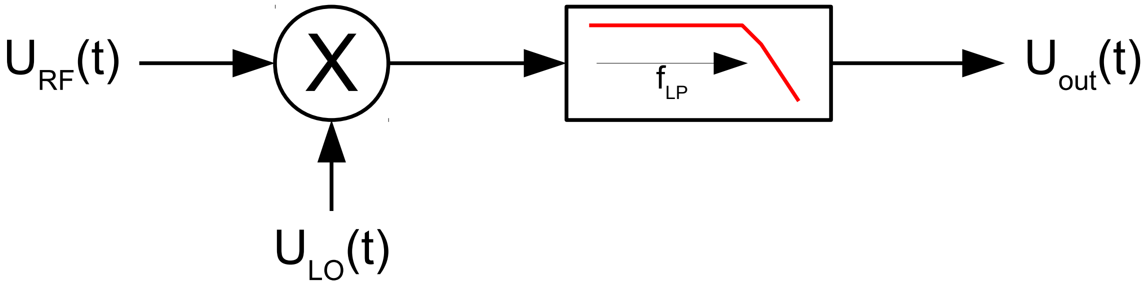
- Due to the square-law characteristic we arrive at the thermal noise region already for moderate power levels (-50 to -60 dBm) and hence the V_{Video} disappears in the thermal noise

- This is described by the term
  - tangential signal sensitivity (TSS) where the detected signal is 4 dB over the thermal noise floor (Observation BW, usually 10 MHz)



- Passive RF detector are very robust but require that the carrier signal is above -60...-50 dBm ie. 250 mV

Classic Ideal RF (Down-)Mixer



$$\begin{aligned}
U_{out}(t) &= LP\left(U_{RF}(t)\cdot U_{LO}(t)\right)\\
&= LP\left(U_1\cdot\cos(2\pi f_{RF})\cdot U_2\sin(2\pi f_{LO})\right)\\
&= \frac{U_1U_2}{2}\sin(2\pi(f_{RF}-f_{LO})) + \cancel{\frac{U_1U_2}{2}\sin(2\pi(f_{RF}+f_{LO}))}
\end{aligned}$$

removed by low-pass filter

- input/output port assignment is arbitrary but the low-pass is often implicit and defines the down-conversion port
  - RF: to be measured input
  - LO: local oscillator input (larger power, constant reference frequency)
  - Replacing the low- with a high-pass makes this an up-converter/mixer

Closer to Real-World Mixer

$$I(U) = I_S\left(\exp\left(\frac{eU}{mkT}\right)-1\right)$$
$$= I_S\left(\exp\left(\frac{U}{mU_T}\right)-1\right)$$

- From Shockley equation:

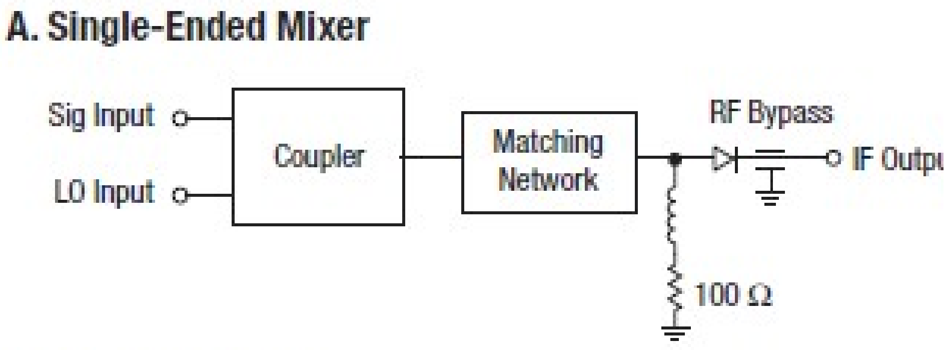
$$\left(\exp(x)-1\right) = x + \frac{1}{2}x^2 + h.o.$$

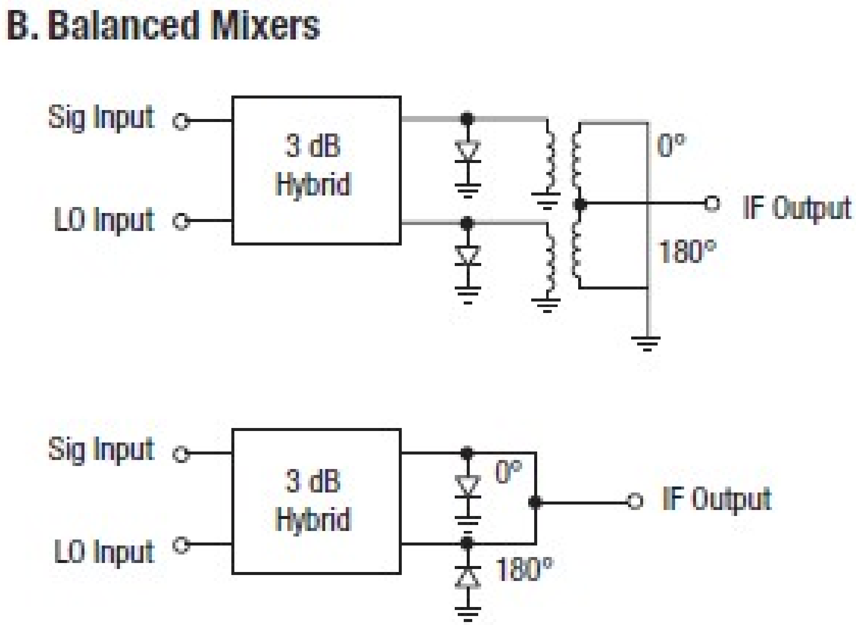
- So if we sum two voltages prior to the diode

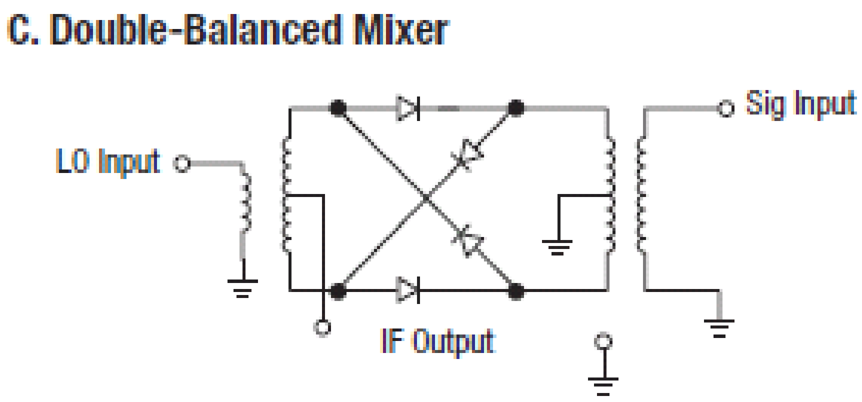
$$U_0 = \underset{\text{removed by LP}}{\cancel{(U_1+U_2)}} + \underbrace{\frac{1}{2}(U_1+U_2)^2}_{\underset{\text{removed by LP}}{\cancel{\frac{1}{2}U_1^2}}+U_1\cdot U_2+\underset{\text{removed by LP}}{\cancel{\frac{1}{2}U_2^2}}} + \underset{\text{mostly removed by LP}}{\cancel{h.o.}}$$

→ our homodyne RF diode detector is basically a mixer with the RF and LO port tight together (U_1=U_2)

Real-World RF Diode Mixer Topologies

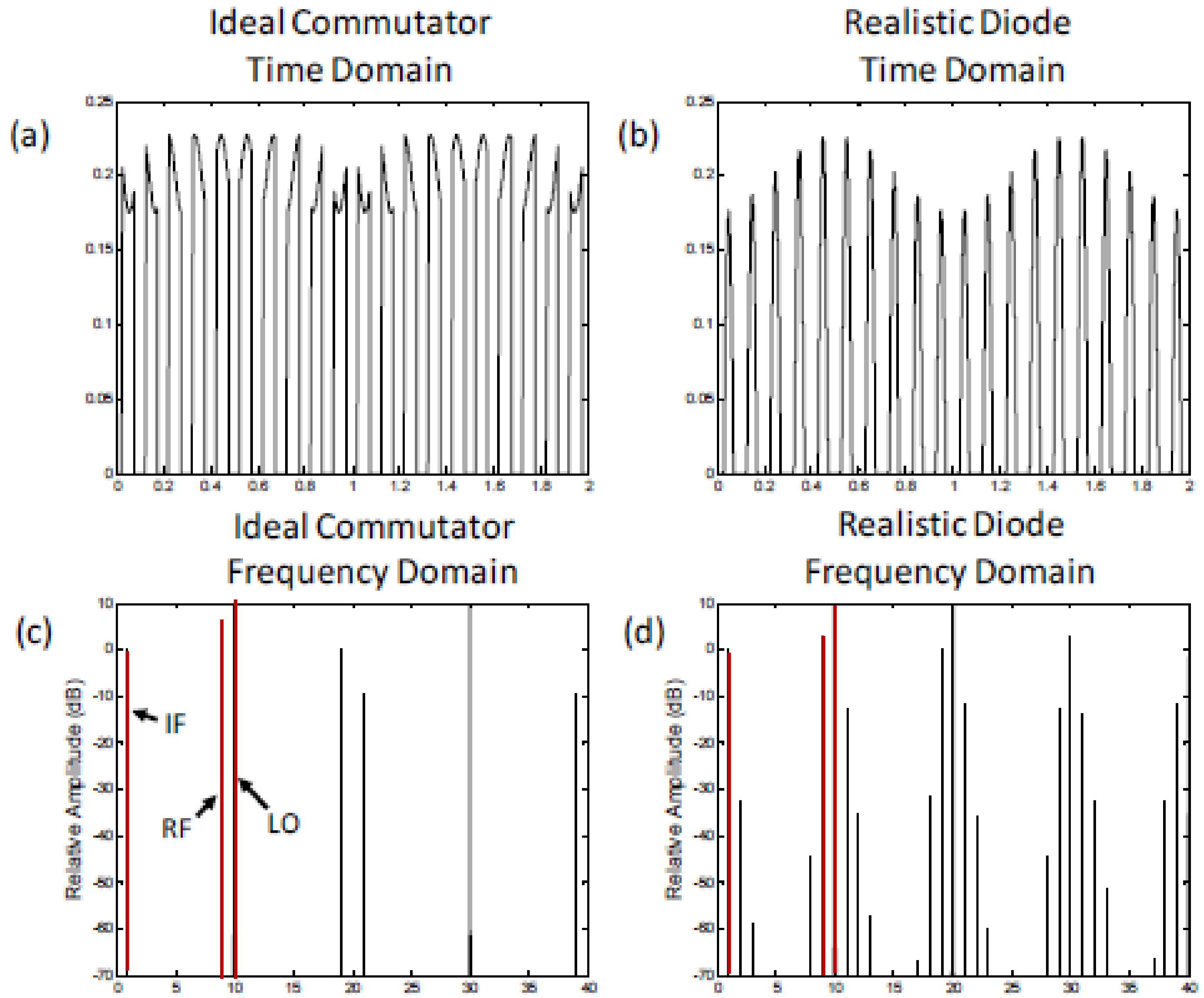






A typical coaxial mixer (SMA connector)

RF Mixer Time and Frequency Response


Ideal Commutator Time Domain

Realistic Diode Time Domain

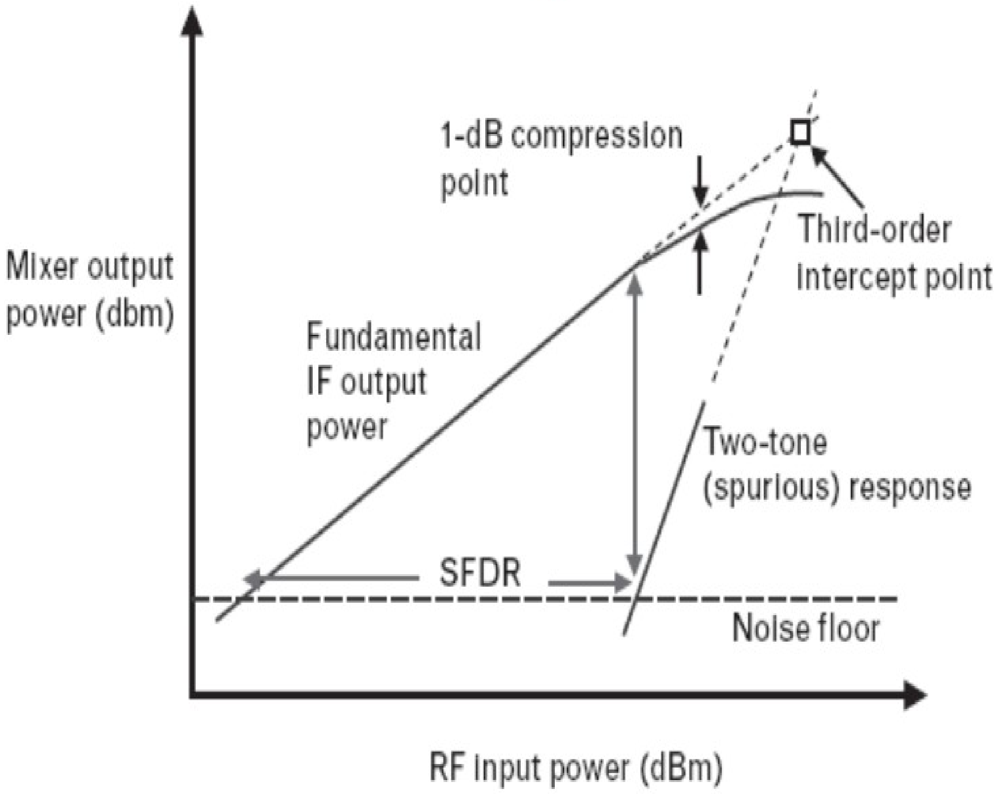
Ideal Commutator Frequency Domain

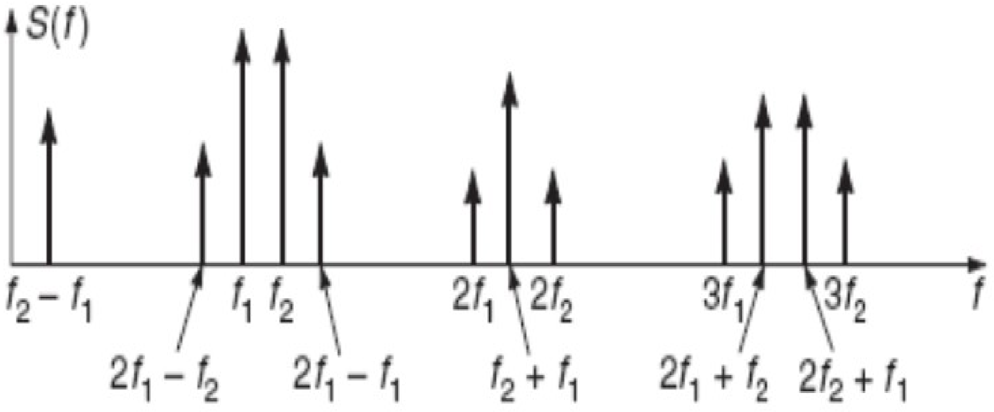
Realistic Diode Frequency Domain

- Input signals here: LO = 10 MHz & RF = 8 MHz
  - Mixing products at 2 and 18 MHz are higher order terms

RF mixer – Dynamic range and IP3

- 3^{rd}-order Input Interception Point (IP3): RF input power at which the unwanted intermodulation products equal desired IF output.

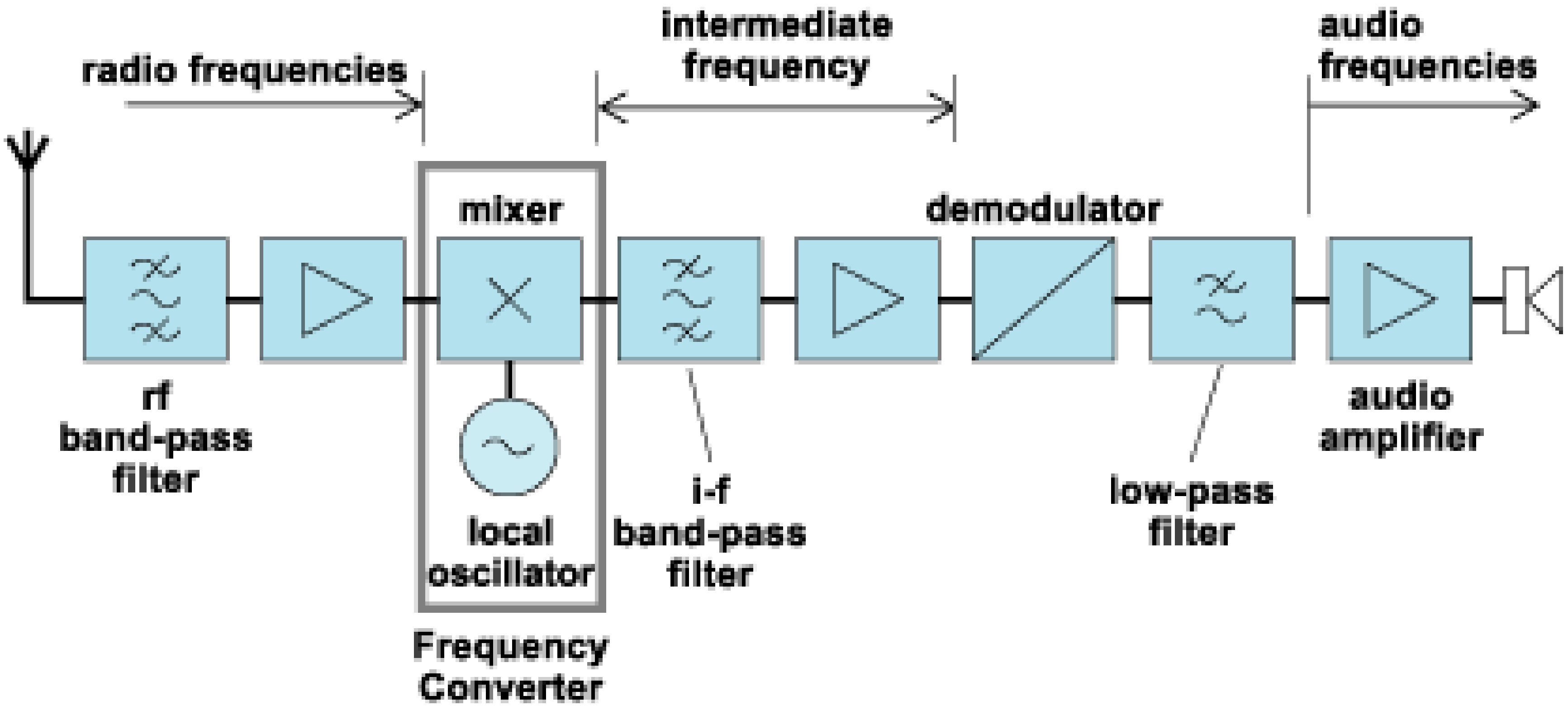




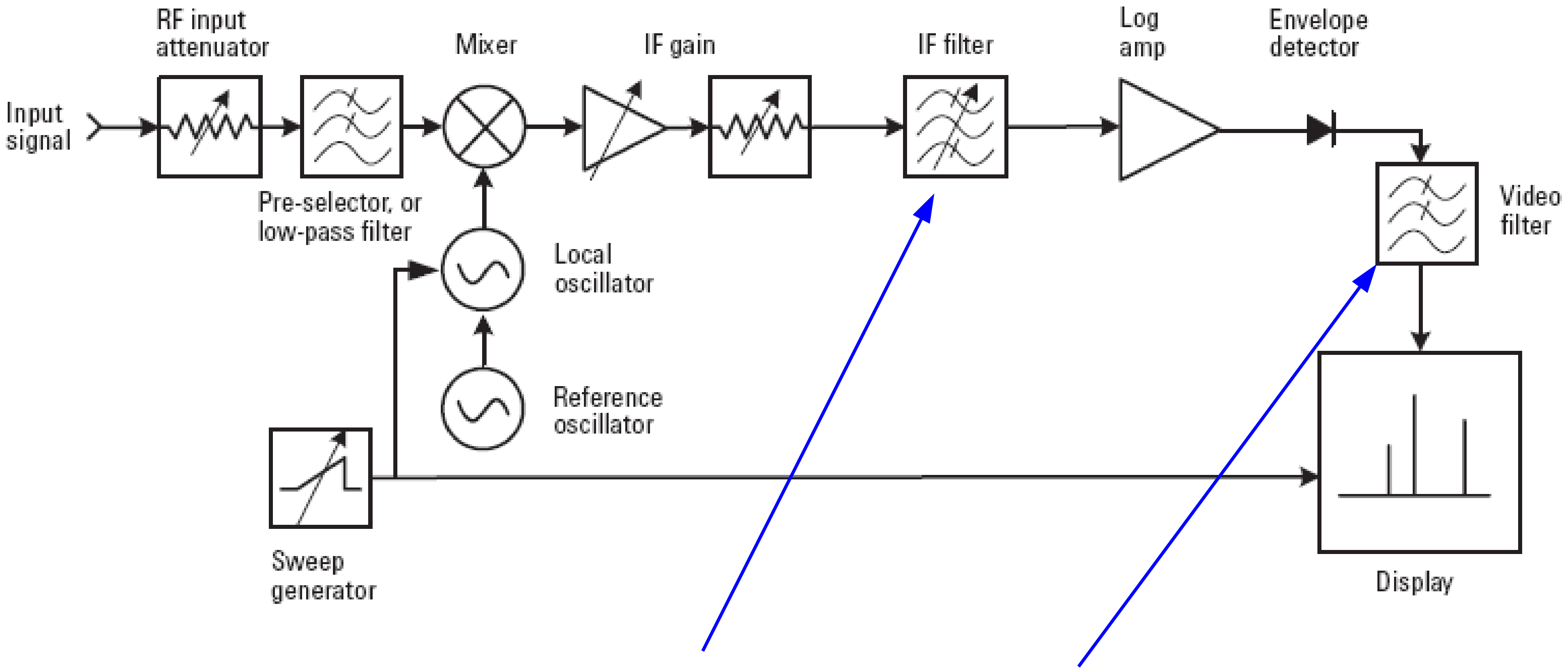
  – N.B. this intersection point is usually extrapolated from lower power levels to not overload and damage of the DUT.

Super-Heterodyne Concept

- 'Super-' (*latin: over*) 'hetero-' (greek: different) -dyne (greek:“power” or “force”): Superimposes a strong local-oscillator (LO) signal with a (usually weaker) radio-frequency (RF) signal
→ mixing produces intermediate (IF) frequency signal



Super-Heterodyne Application: Spectrum Analyser



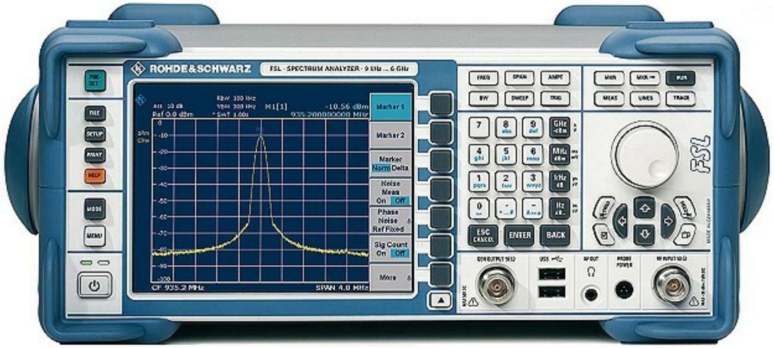
The center frequency is fixed, but the bandwidth of the IF filter can be modified.

The video filter is a simple low-pass with variable bandwidth before the signal arrives to the vertical deflection plates of the cathode ray tube.

Agilent, 'Spectrum Analyzer Basics,' Application Note 150, page 10 f.

Spectrum Analyser

- Spectrum Analyser (SA): observation in time domain
  - application: observation of beam spectra or spectrum emitted from an antenna, etc.
  - Classic version: sweeps through a given frequency range point by point with a very narrow bandwidth
  - Modern version: Dynamic signal analyzer (aka. Real-Time FFT/spectrum analyzer): Acquires signal in time domain by fast sampling with wider bandwidth (typ. 20 … 200 MHz)
    - Contrary to the SPA: non-repetitive signals and transients can be observed
    - Further numerical treatment in digital signal processors (DSPs)
    - Spectrum calculated using Fast Fourier Transform (FFT)
    - Combines features of a scope and a spectrum analyzer: signals can be looked at directly in time domain or in frequency domain
    - Application:
      - Observation of tune sidebands,
      - transient behaviour of a phase locked loop, etc.



Introduction to RF – Part I

- Aim: Learn how high-frequency signals are measured

- Part I – time-domain vs. frequency domain (RF Detectors and Mixers)
  - Bode plots, power and dB(m, c, uV) definitions,
  - AM vs. FM vs. PM modulation
  - RF (Schottky) Diodes → RF Detectors (homodyne detection)
  - RF Mixers (heterodyne detection)
  - Introduction to: oscilloscopes, spectrum analyser, vector network analyser

- Laboratory:
  - some simple simulations using QUCS – guided/students
  - in || lab measurement with generator + spectrum analyser
  - some simple simulations using QUCS – guided/student
  - “Play” with BPM mock-up
  - Repeat in || previous day experience now with some active elements