Introduction to Beam Diagnostics – Part I

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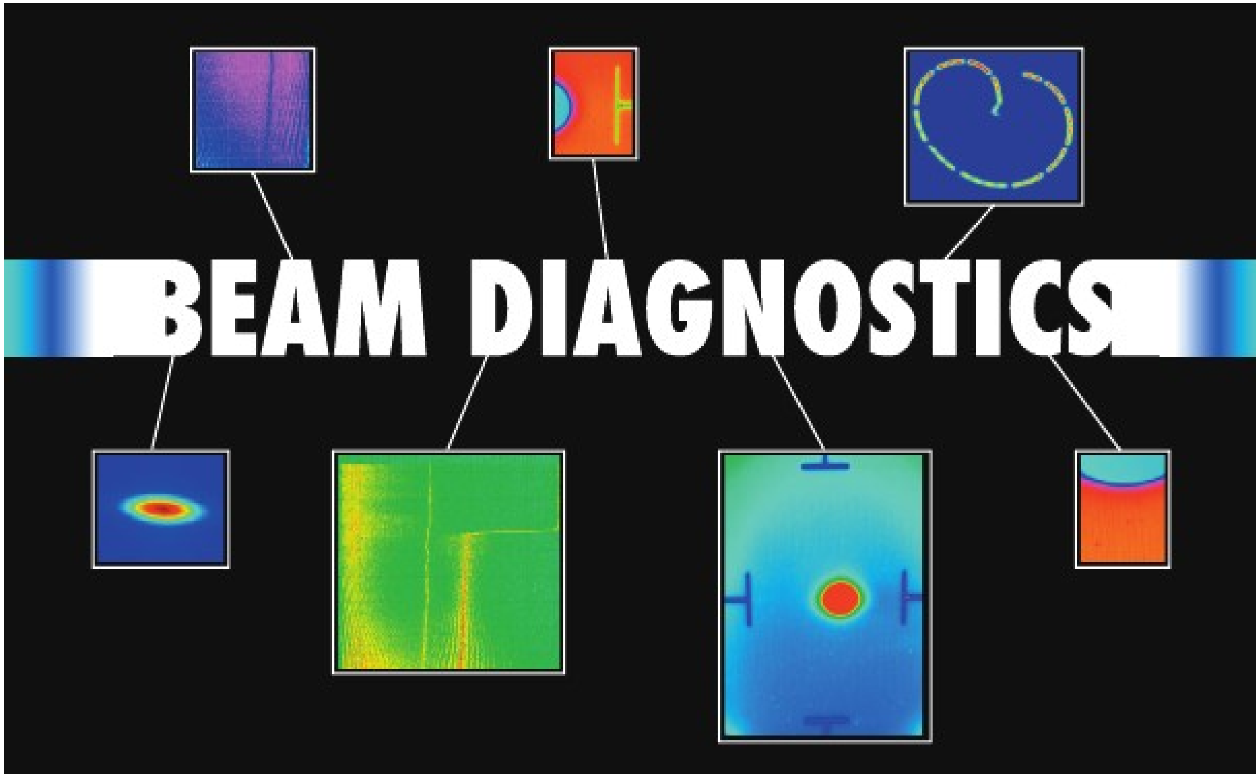


Outline

- Recap: What the .... is Q/Q', oscillations dampening
- Tune Diagnostics
  - Classic Fourier-Transform Based
  - Phase-Locked-Loop (PLL) Systems
- Classic Chromaticity Diagnostics
  - Momentum shift Δp/p based Q' tracking methods
- Longitudinal Tomography
- Advanced Topic → your choice

Beam Diagnostics

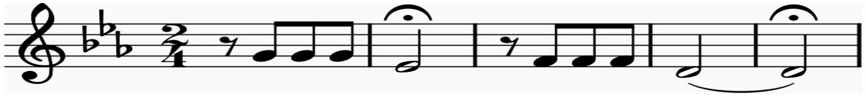
- … didn't we hear about this last week?

- Distinguish between
  - Beam Instrumentation:
    physical hardware provides a direct beam parameter measurement
    - e.g. Faraday cup → current → beam current
  - Beam Diagnostics:
    beam parameter is derived through a combination of beam instrumentation or procedure of beam parameter variation
    - Machine tune (Q) and chromaticity (Q')
    - Longitudinal phase-space distribution (long. tomography)

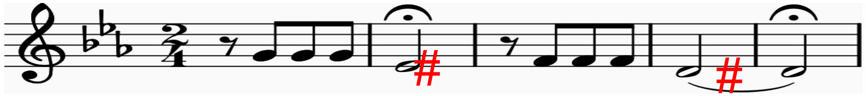
Tune Diagnostics - Primer

- Importance of tune:
  - defines beam life-time
  - strong impact on beam physics experiments:

Laymen/Musician's view
(Beethoven's 5th):
in tune (good):



off-tune (bad):



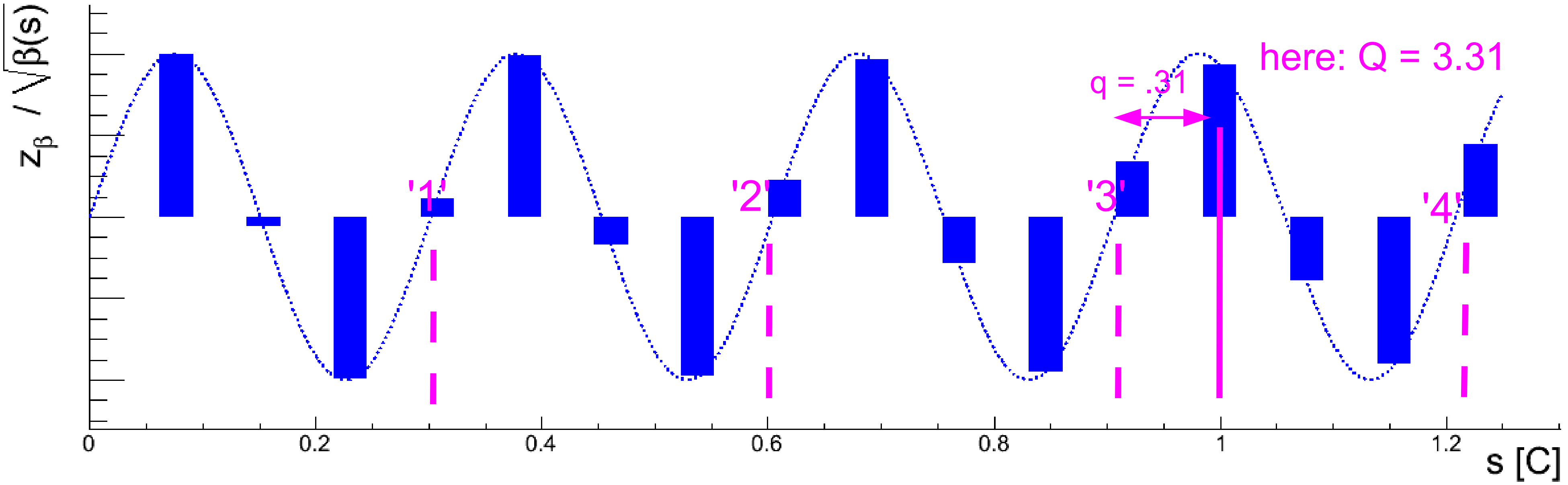
Audience will leave the concert
↔ Beam will leave the vacuum pipe

"I don't think we've quite repeated the experiment - last time we did it, the glass gave out a middle 'c'."

Recap: Transverse Beam Dynamics

- Free Betatron Oscillations:

$$z_\beta(s) = \sqrt{\epsilon_i \beta(s)} \cdot \sin(\mu(s) + \phi_i)$$



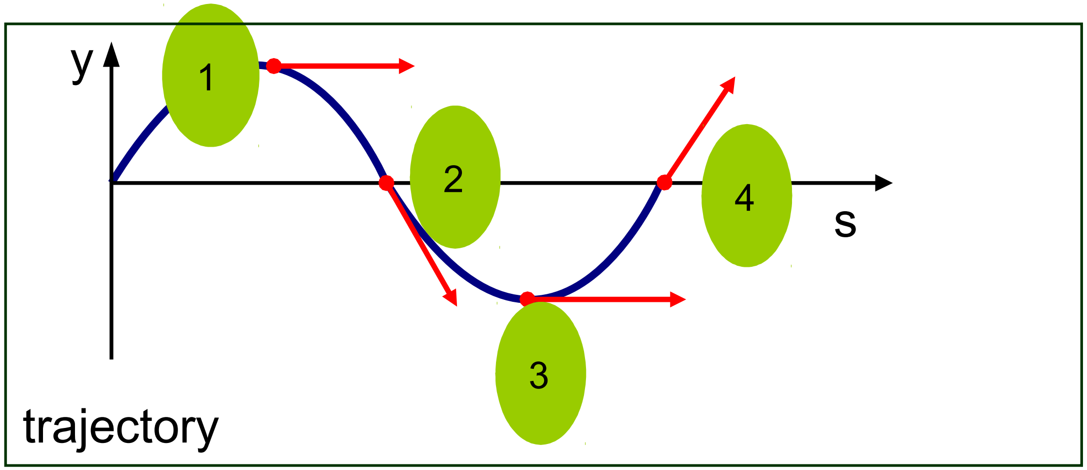
- Betatron Phase Advance: $\Delta\mu(s)$

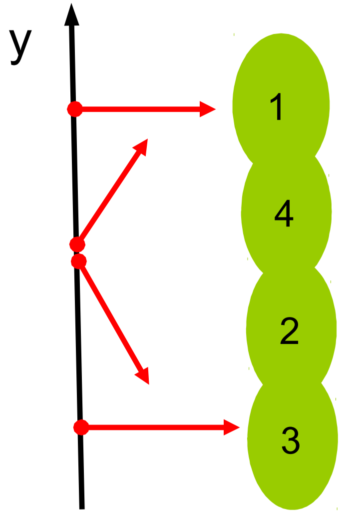
Tune defined as betatron phase advance over one turn:

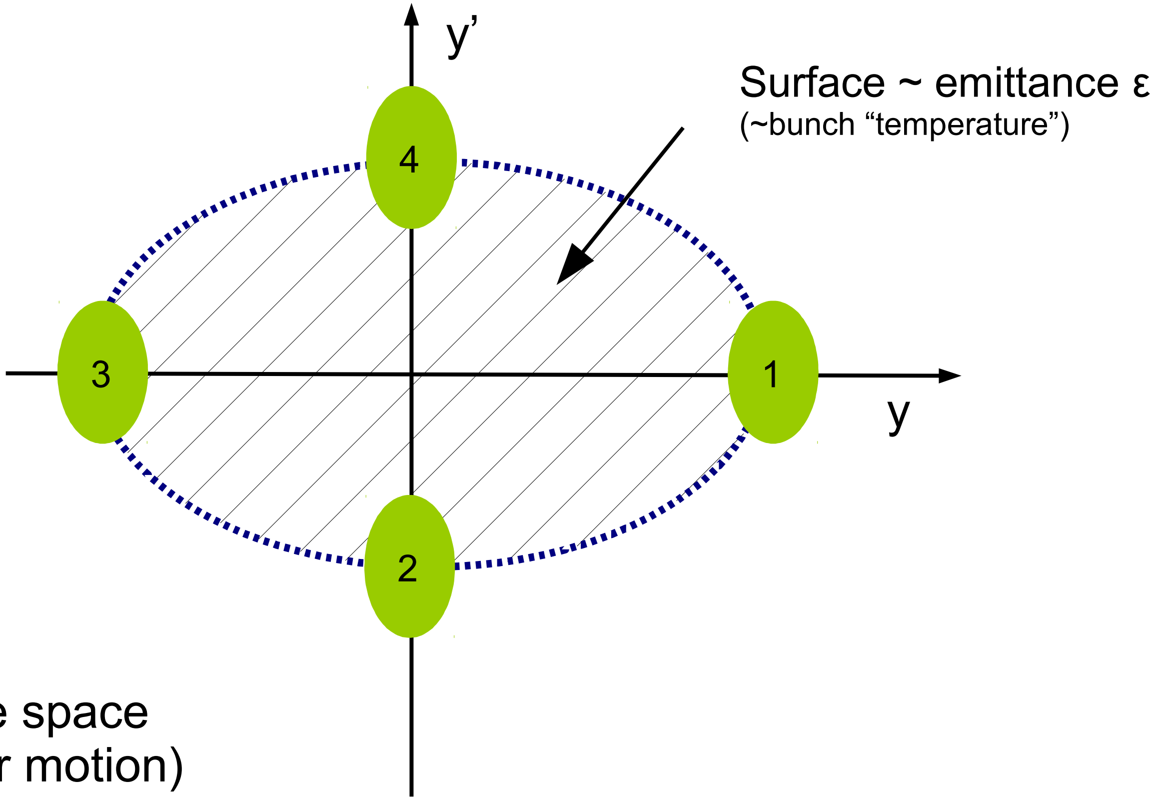
$$\boxed{Q := \frac{1}{2\pi} \oint_C \mu(s)\, ds}$$

common: $Q = \underbrace{Q_{int}}_{integer\ \ tune} + \underbrace{q_{frac}}_{fractional\ \ tune}$

Phase Space I/II







Surface ~ emittance ε
(~bunch “temperature”)

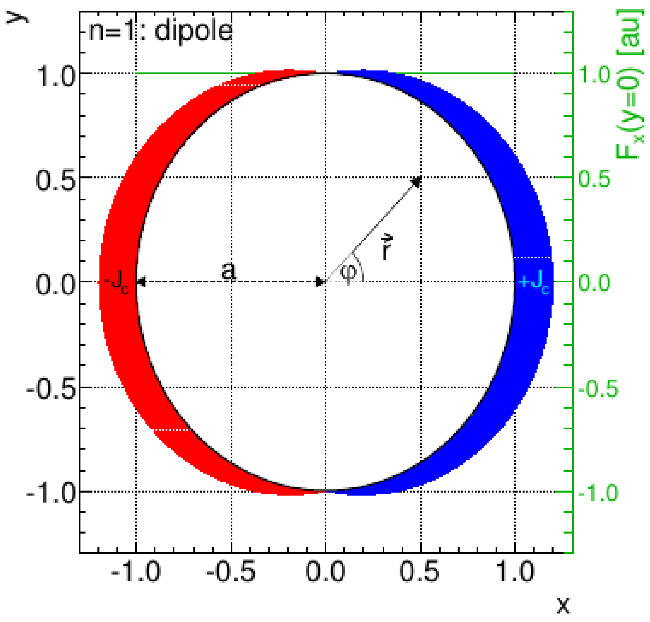
phase space
(linear motion)

Magnets – Basic Arsenal

- Hill's Equation

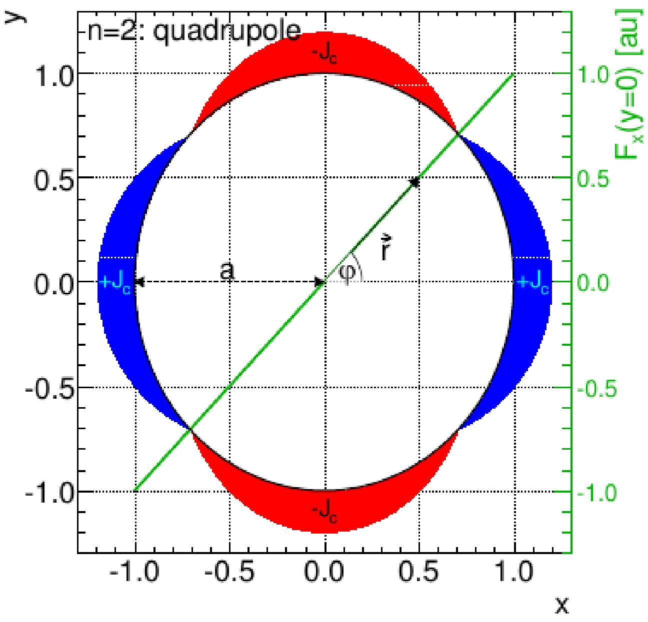
$$\boxed{z'' + k(s)\cdot z = f(s,t)}$$

Dipole:
constant field



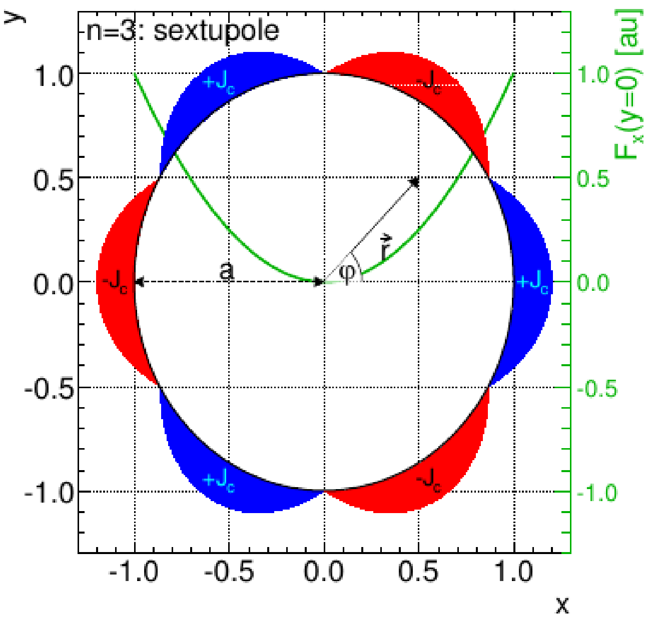
→ defines circular trajectory/orbit

Quadrupole:
linear field



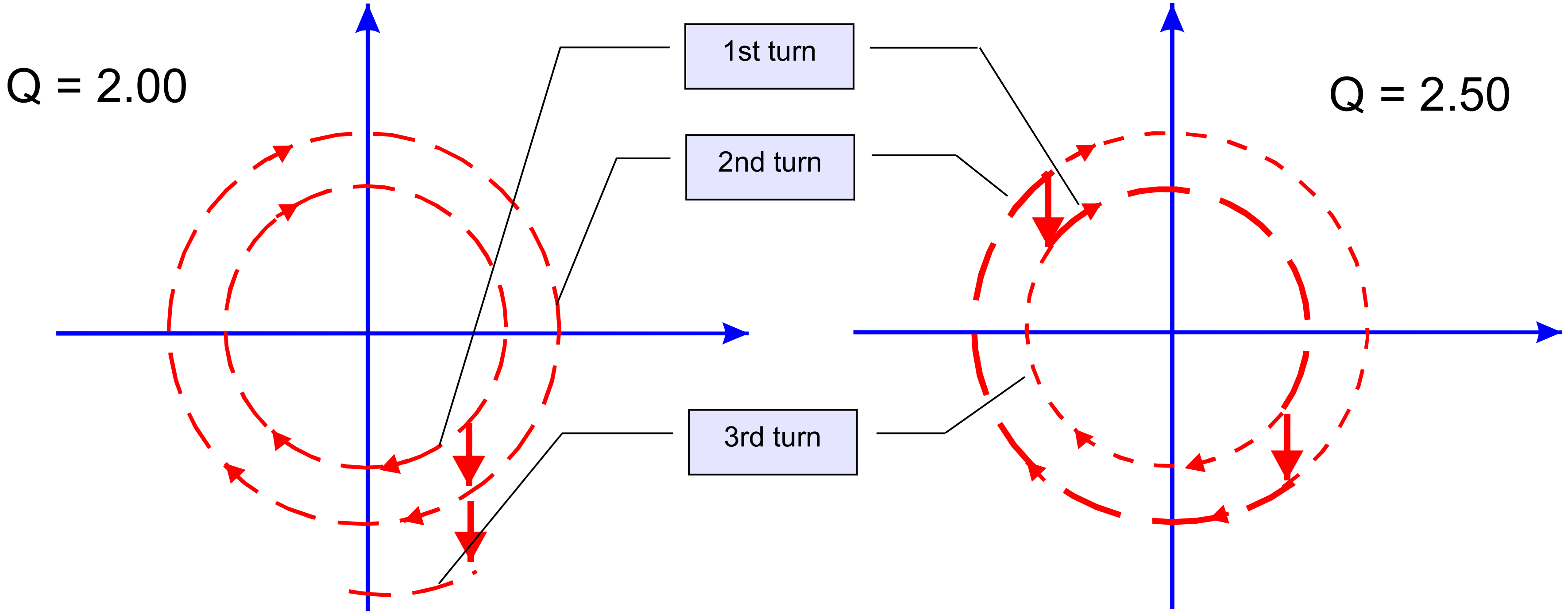
→ defines transverse focusing and periodic betatron oscillation

Sextupole:
quadratic field



→ corrects for non-linear /chromatic effects
→ defines dynamic aperture
LHC: up to 12 order

Non-Linear Dynamics – Dipolar Resonance

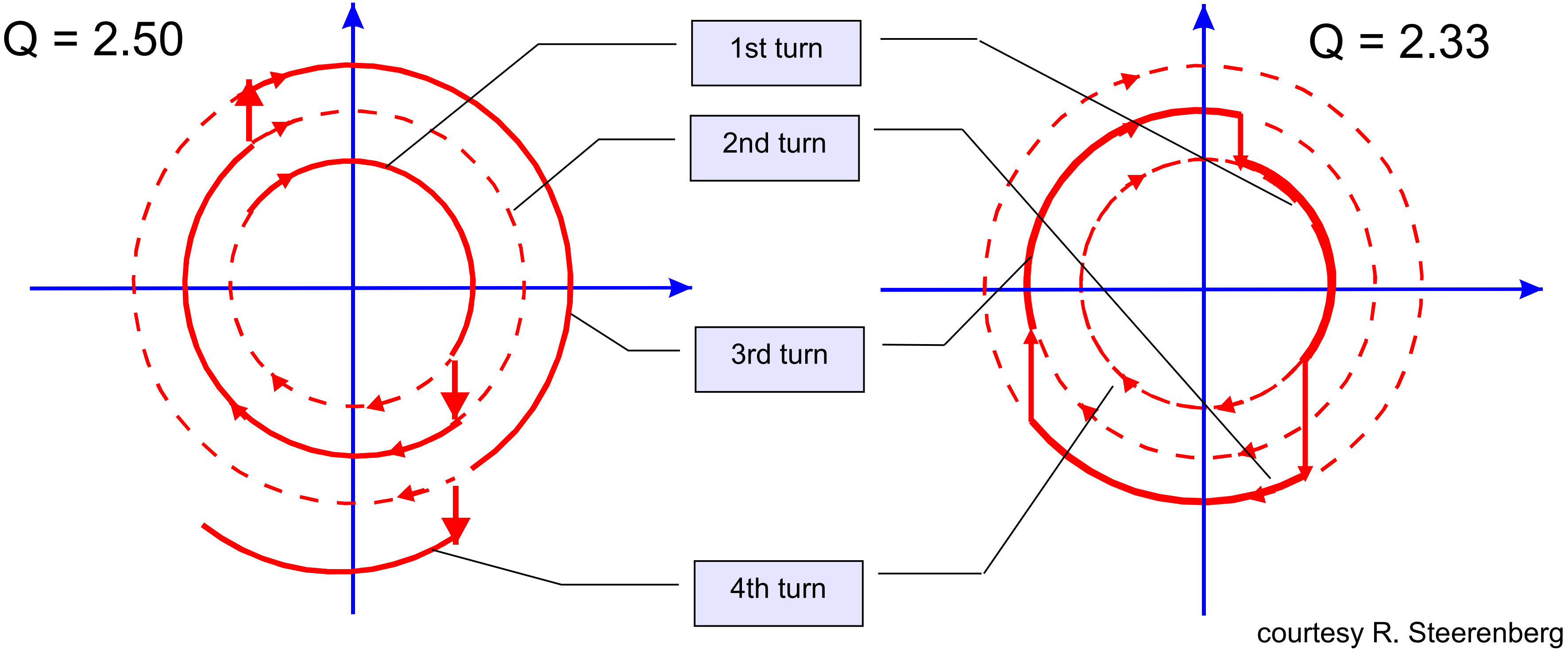


courtesy R. Steerenberg

- For Q = 2.00: Oscillation induced by the dipole kick grows on each turn and the particle is lost (1st order resonance Q = 2).
- For Q = 2.50: Oscillation is cancelled out every second turn, and therefore the particle motion is stable.

courtesy R. Steerenberg

Non-Linear Dynamics – Quadrupolar Resonances

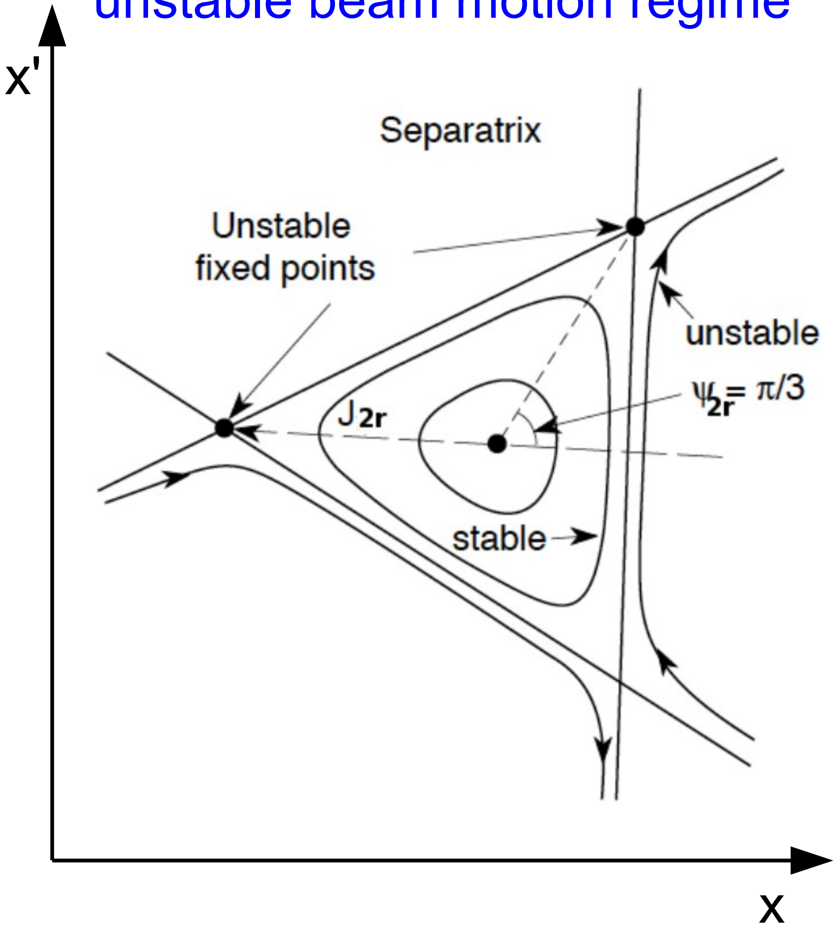


courtesy R. Steerenberg

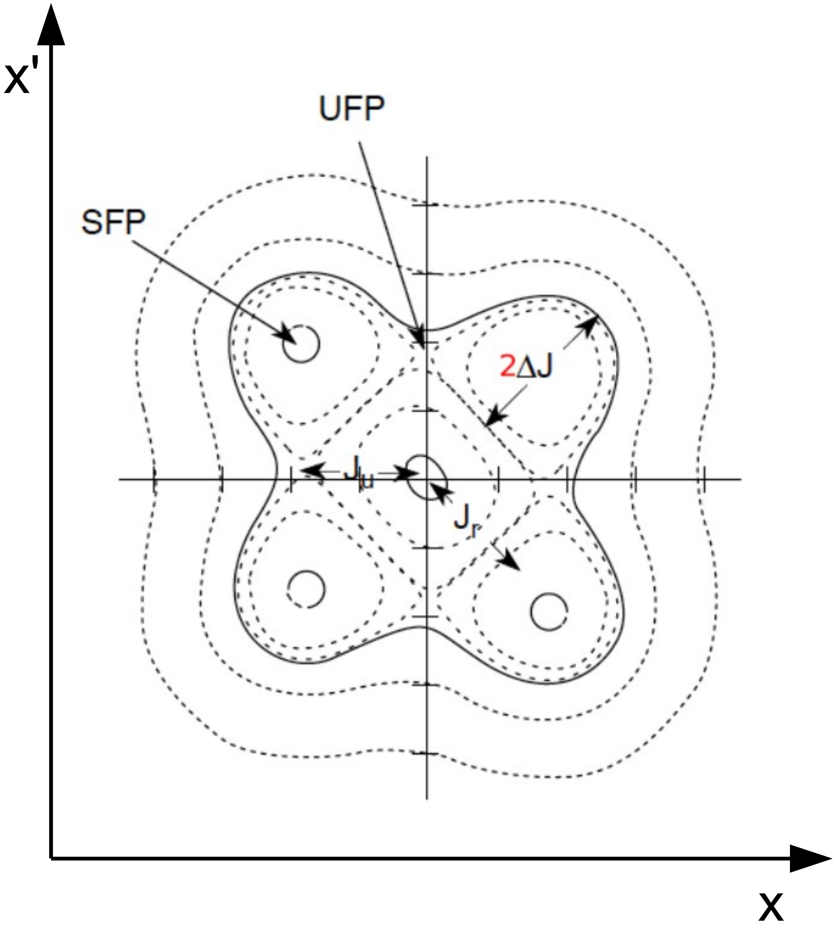
- For Q = 2.50: Oscillation induced by the quadrupole kick grows on each turn and the particle is lost (2nd order resonance 2Q = 5)
-
- For Q = 2.33: Oscillation is cancelled out every third turn, and therefore the particle motion is stable.

Phase Space II/II

- What happens if you add strong non-linear sextupole & octupole-components
  - 'separatrix' (aka. 'dynamic aperture') being the border between stable and unstable beam motion regime



sextupole resonance



octupole resonance

Tune Stability Requirements & Constraints

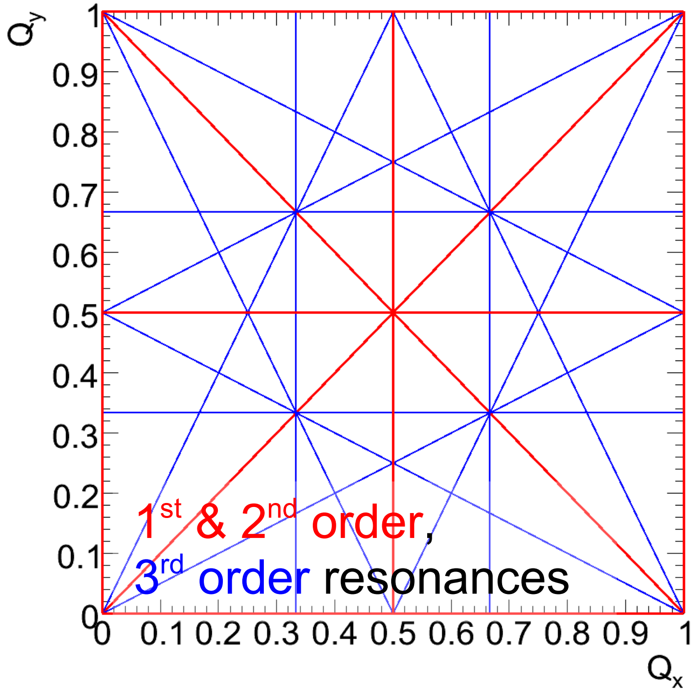
- Unstable particle motion if:

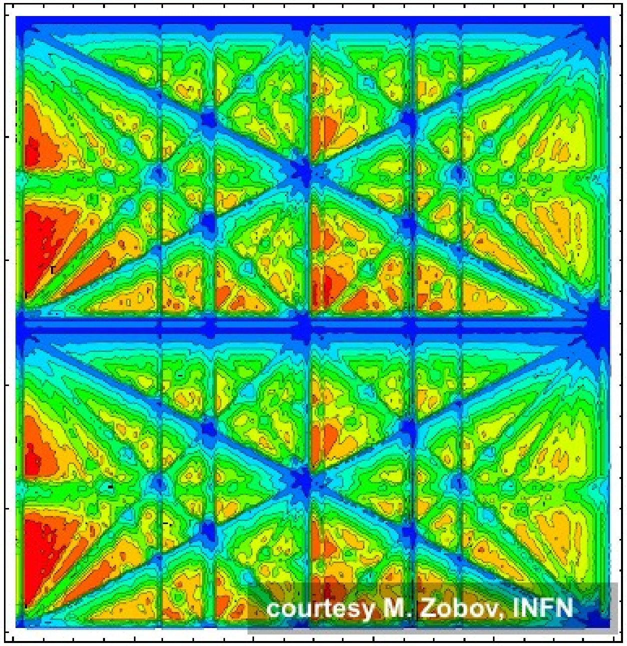
$$\boxed{p = m \cdot Q_x + n \cdot Q_y \quad \wedge \quad m, n, p \in \mathbb{Z}}$$

  - similar relation also in between Q_x & Q_s (important for lepton accelerators)

- Resonance order: $O = |m| + |n|$
  - Lepton accelerator: avoid up to ~ 3rd order
  - Hadron colliders:
    - negligible synchrotron radiation damping
    - need often to avoid up to the 12th order

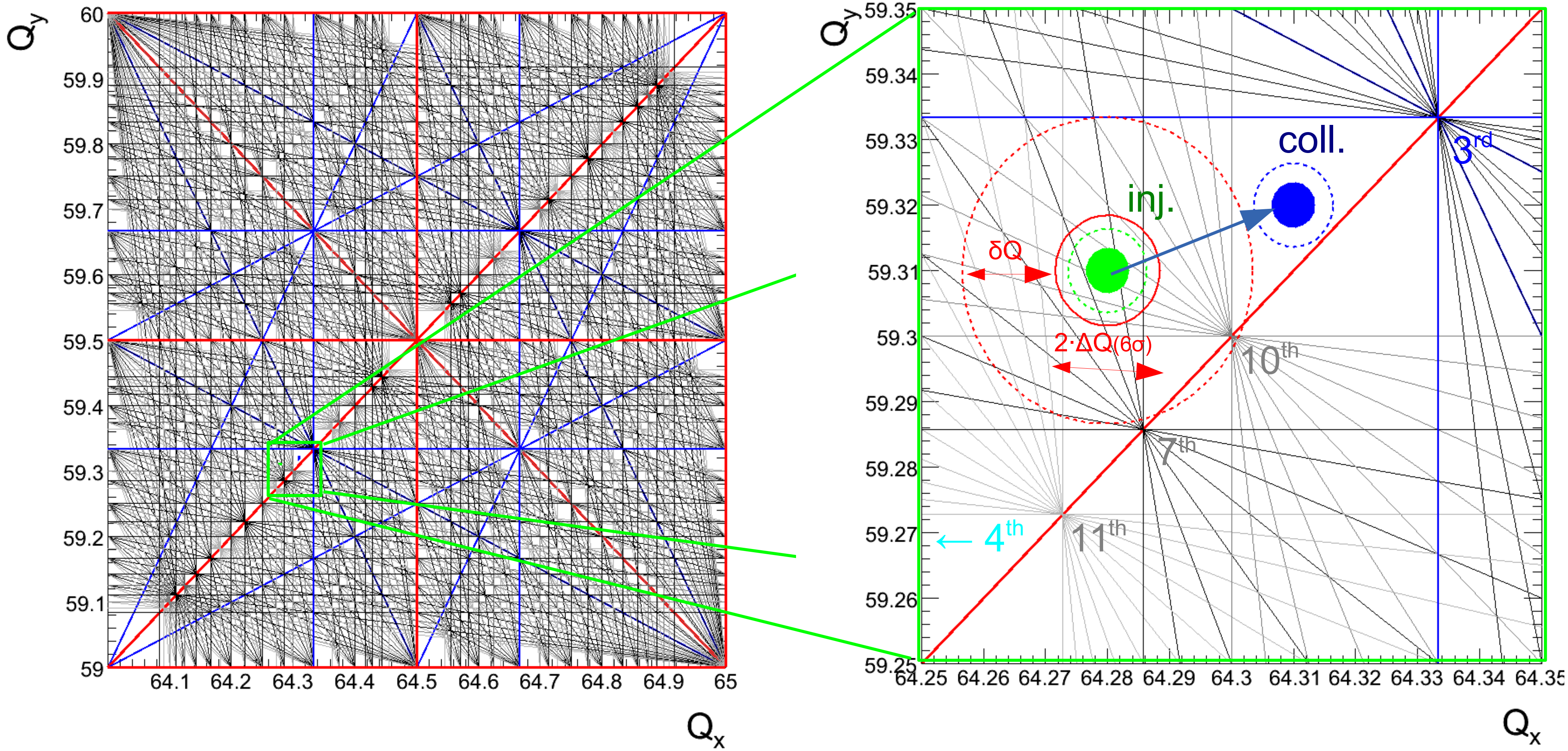
“Hadron beams are like elephants –
treat them bad and they'll never forgive you!”



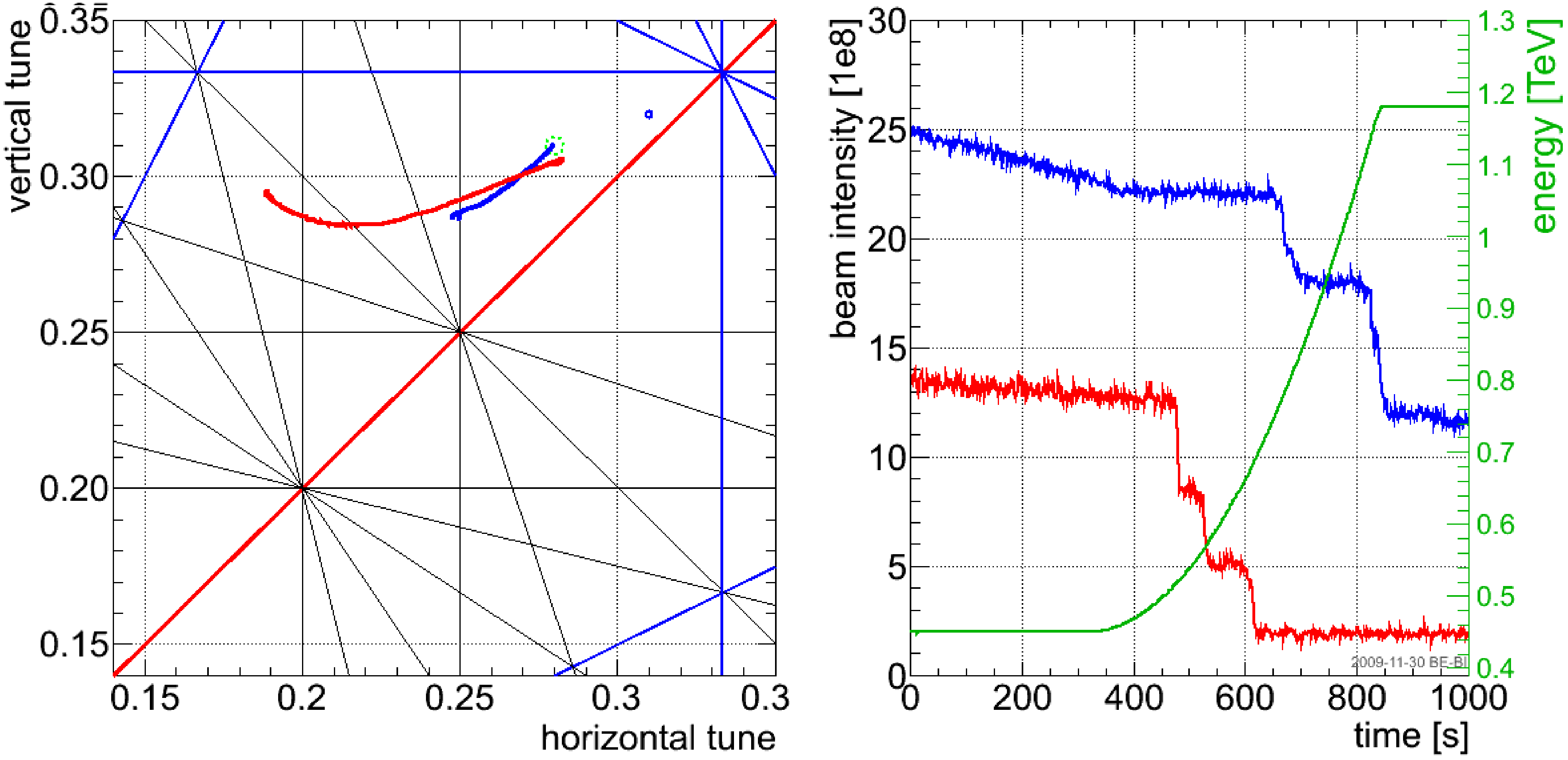


Tune Stability Requirements & Constraints

- Example LHC: stability requirement: ΔQ ≈ 0.001 vs. exp. drifts ~ 0.06



Example: Tune During LHC Ramp



Tune Diagnostics Principle

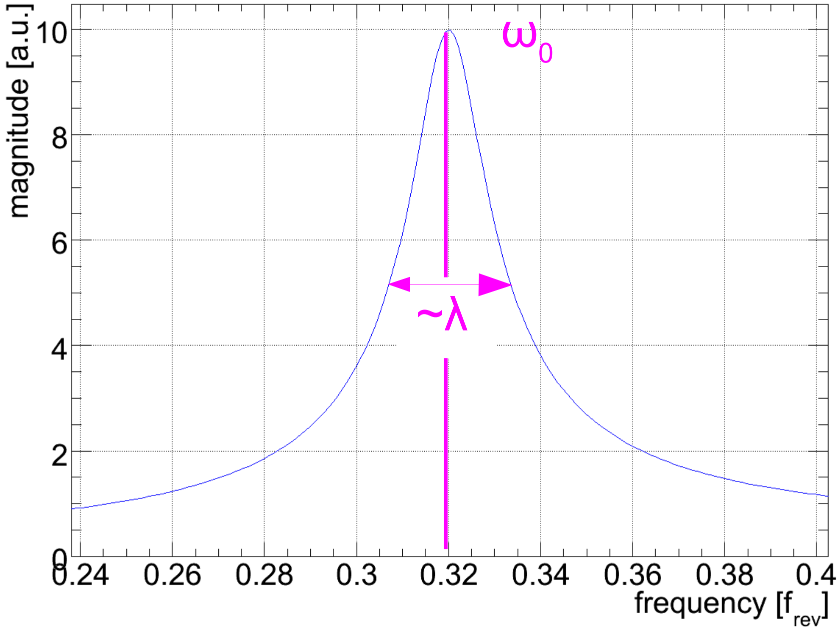
- Control Theory → System Identification

- Example (first order) beam response ≈ damped harmonic oscillator resonance

  (ω_0: resonant frequency (Q), λ: tune resonance width (σ_Q),

  ω: driving frequency)

$$\left|G(\omega)\right| := \left|\frac{X(s)}{E(s)}\right| \approx \frac{\omega_0}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+\left(2\lambda\omega_0\omega\right)^2}}$$

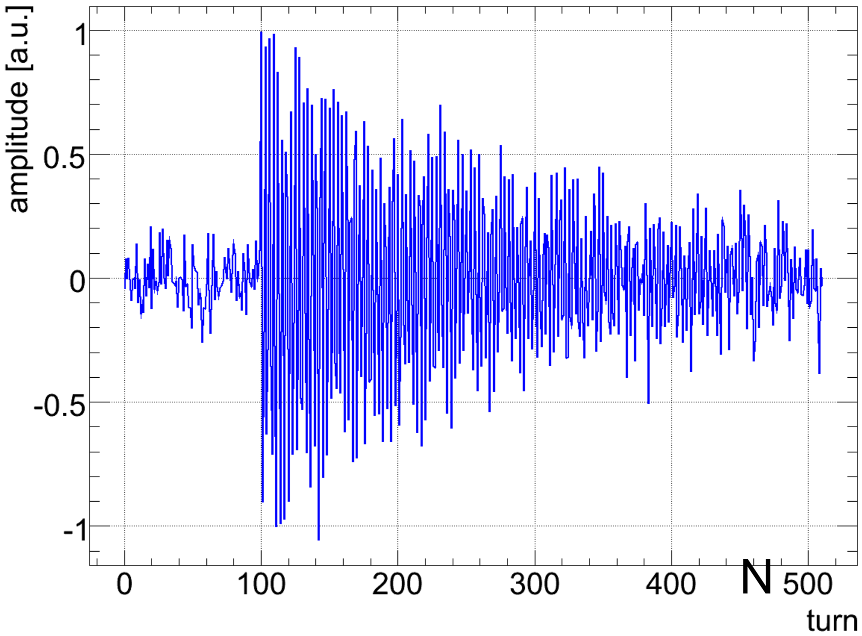


- Excitation choices:
  - White or remnant noise
    - no information on signal phase
  - Single-turn transverse kick (classic)
  - Frequency Sweep aka. 'Chirp'
    - focuses excitation power on frequency range of interest → less ε-blow-up, constant power
  - Phase-Locked-Loop Systems = resonant excitation on the Tune

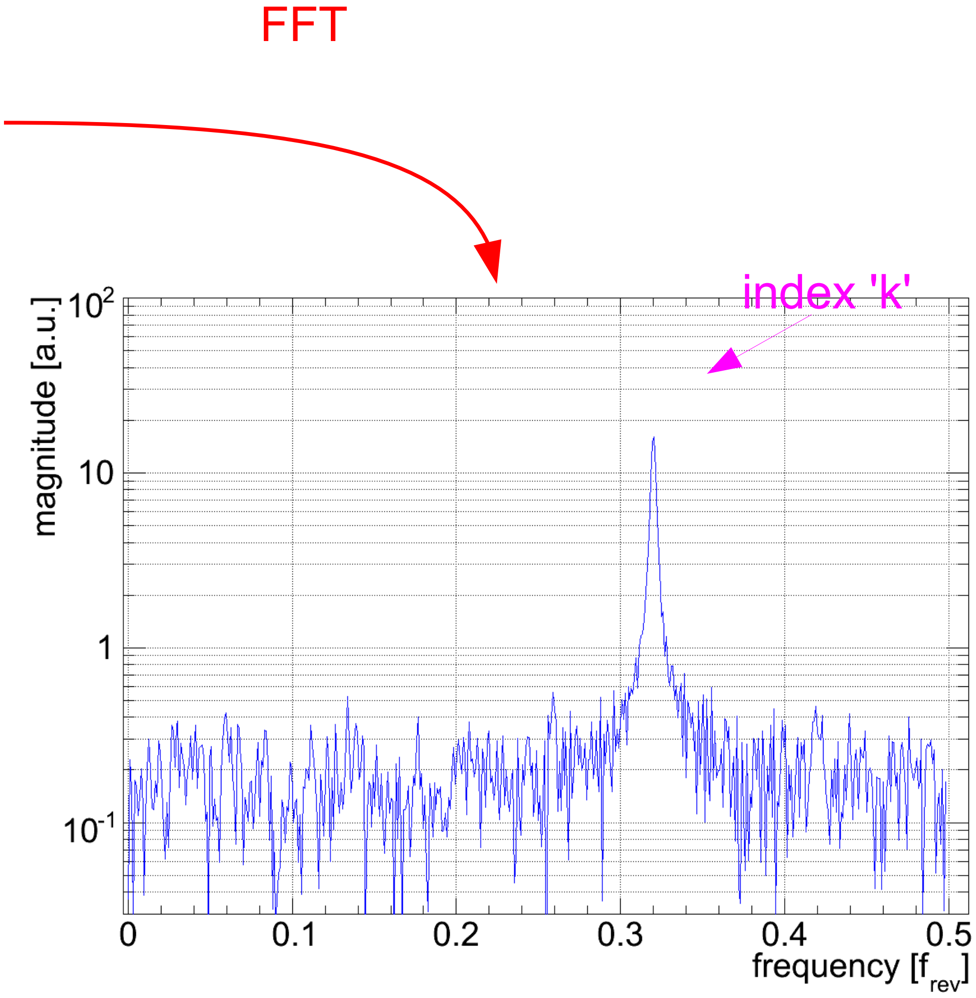
- Note: Exciter and pickup have additional non-beam related responses!

Classic BPM based Tune Diagnostics

- .... how an kick-induced beam oscillation typically looks like

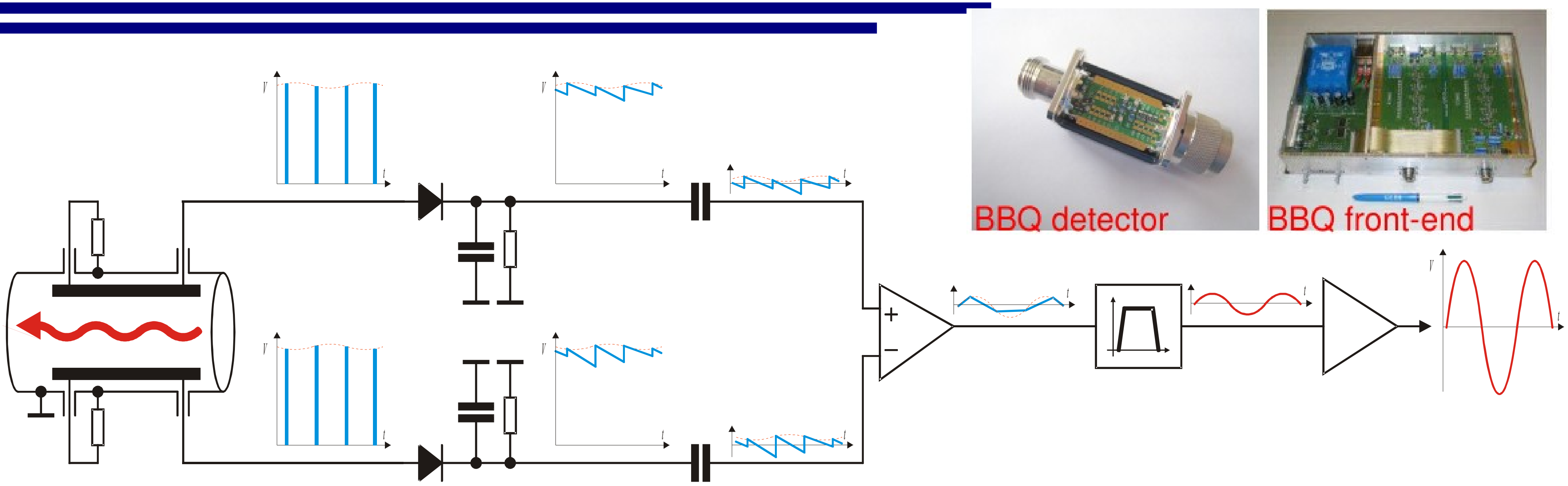


$$q_{frac} \approx \frac{k}{N}$$



- Fourier analysis of turn-by-turn data:
  - magnitude peaks at q_{frac}
  - N.B. no information on Q_{int} !

Tune Instrumentation – Direct-Diode-Detection



- Basic principle: AC-coupled RF diode peak detector[1]
  - intrinsically down samples spectra: ... GHz → kHz (independent on filling pattern)
    - thus 'Base-Band-Tune Meter' (aka. BBQ)
    - Base-band operation: very high sensitivity/resolution ADC available
    - Measured resolution estimate: < 10 nm → ε blow-up is a non-issue
  - AC-coupling removes common-mode → only relative changes play a role
    - capacitance keeps the “memory” of the to be rejected signal
  - no saturation, self-triggered, no gain changes to accommodate single vs. multiple bunches or low vs. high intensity beam

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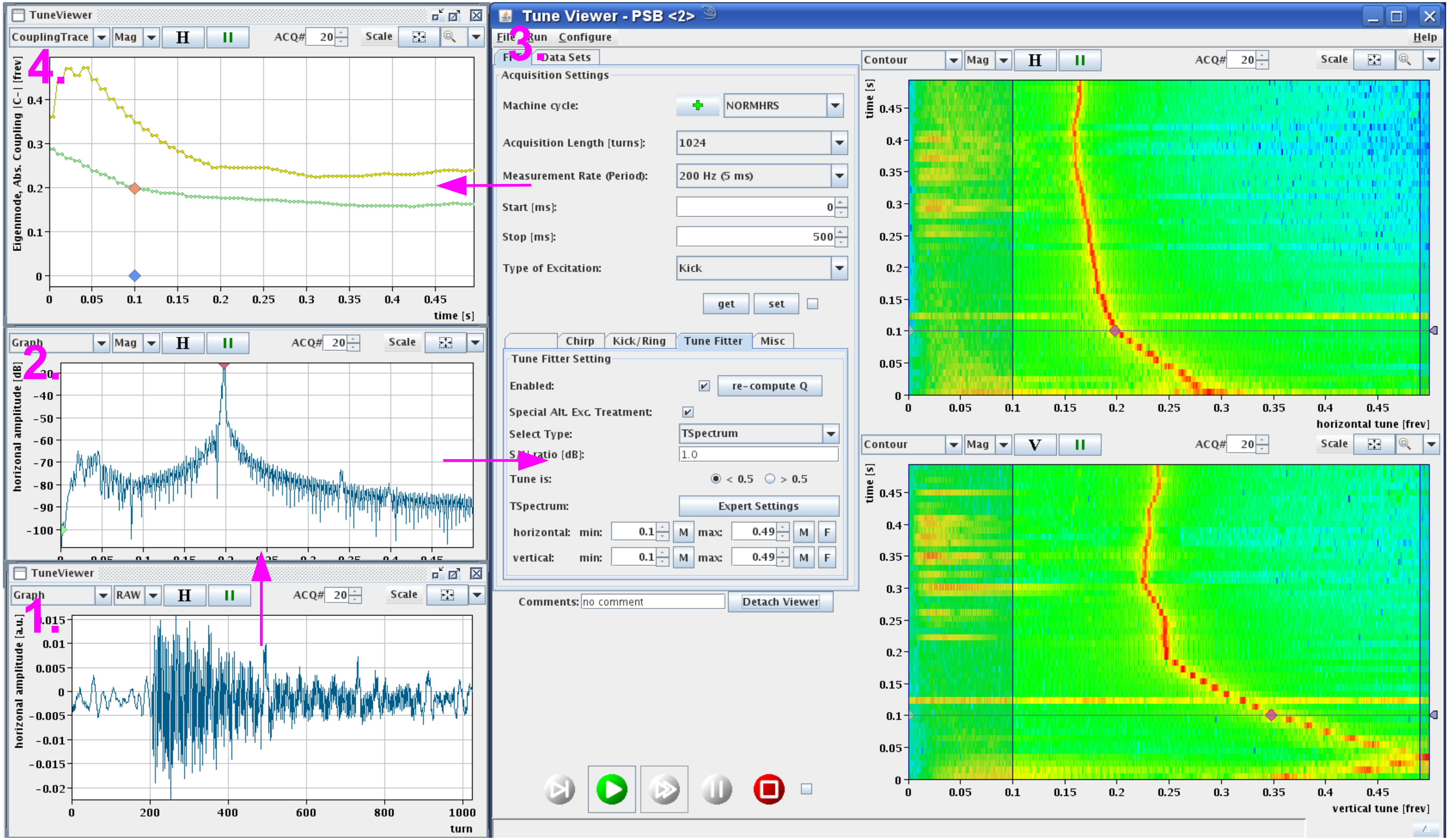
[1] M. Gasior, *“The principle and first results of betatron tune measurement by direct diode detection”*, CERN-LHC-Project-Report-853, 2005

Example: BBQ Spectra CERN-PSB, $f_{rev} \approx 2$ MHz



Acquisition Settings

Machine cycle: NORMHRS

Acquisition Length [turns]: 1024

Measurement Rate (Period): 200 Hz (5 ms)

Start [ms]: 0

Stop [ms]: 500

Type of Excitation: Kick

Tune Fitter Setting

Enabled: ☑

Special Alt. Exc. Treatment: ☑

Select Type: TSpectrum

S/N ratio [dB]: 1.0

Tune is: ● < 0.5 ○ > 0.5

TSpectrum:

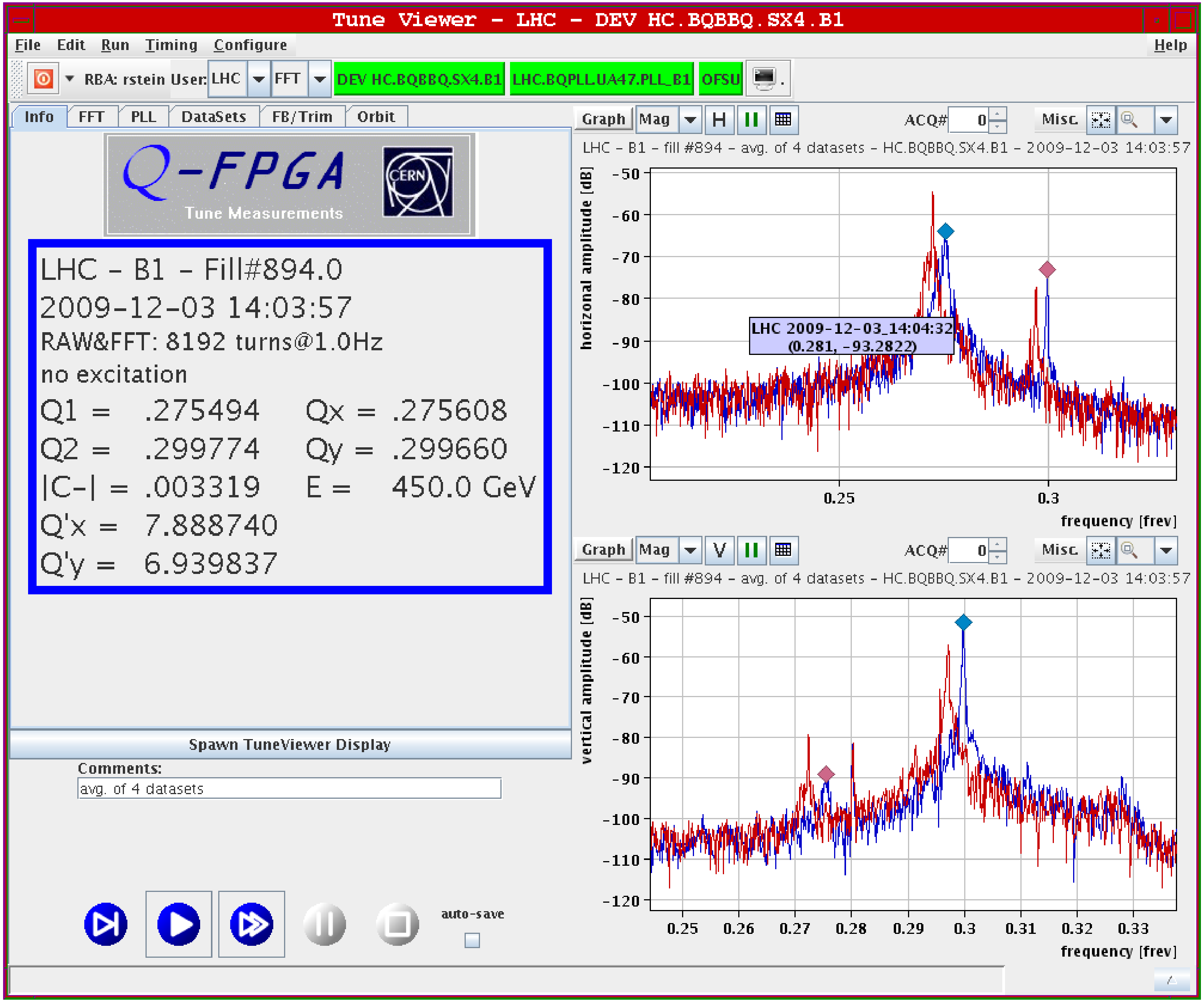
horizontal: min: 0.1 max: 0.49

vertical: min: 0.1 max: 0.49

Comments: no comment

- BBQ → fast ADC → FPGA based digital signal processing chain, FFTs @ 500 – 1 kHz!

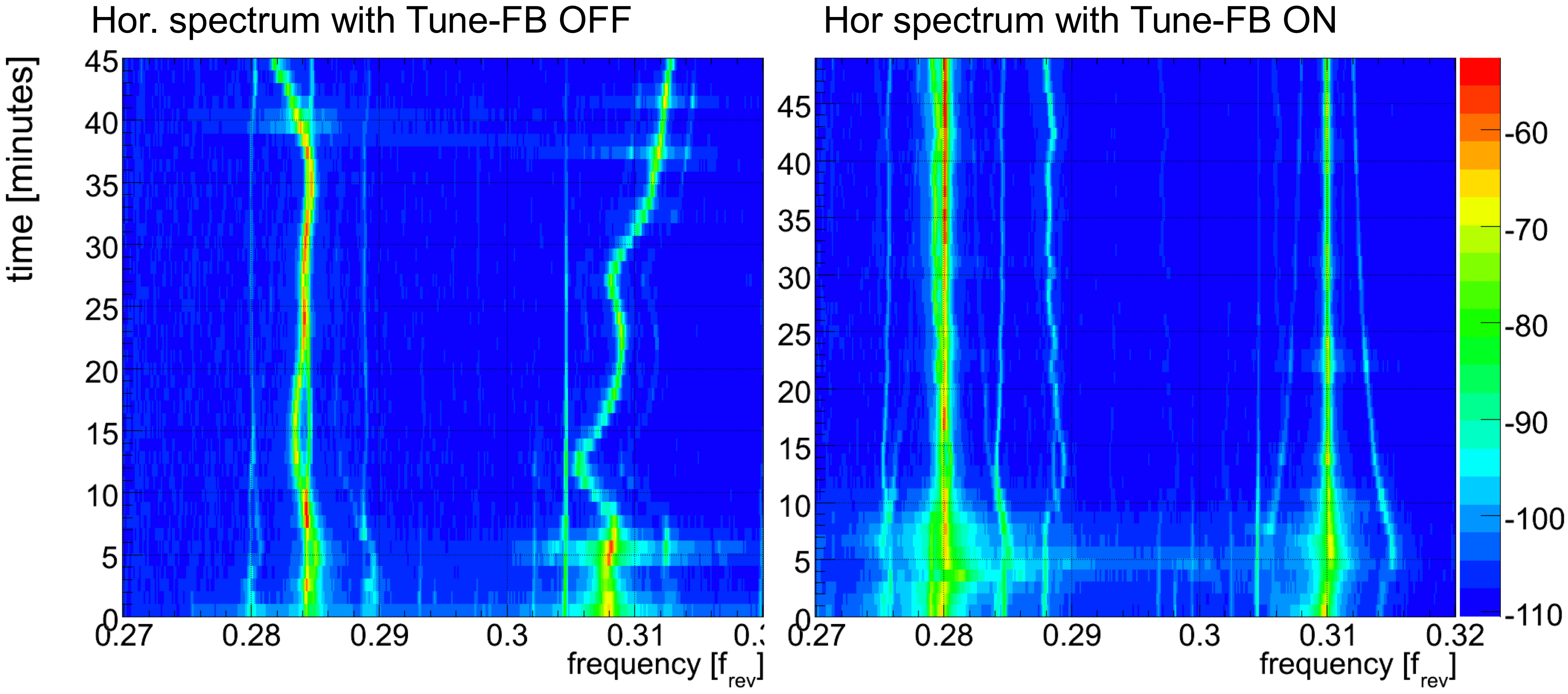
Example: LHC Q/Q' Diagnostics



LHC – B1 – Fill#894.0
2009-12-03 14:03:57
RAW&FFT: 8192 turns@1.0Hz
no excitation
Q1 = .275494 Qx = .275608
Q2 = .299774 Qy = .299660
|C-| = .003319 E = 450.0 GeV
Q'x = 7.888740
Q'y = 6.939837

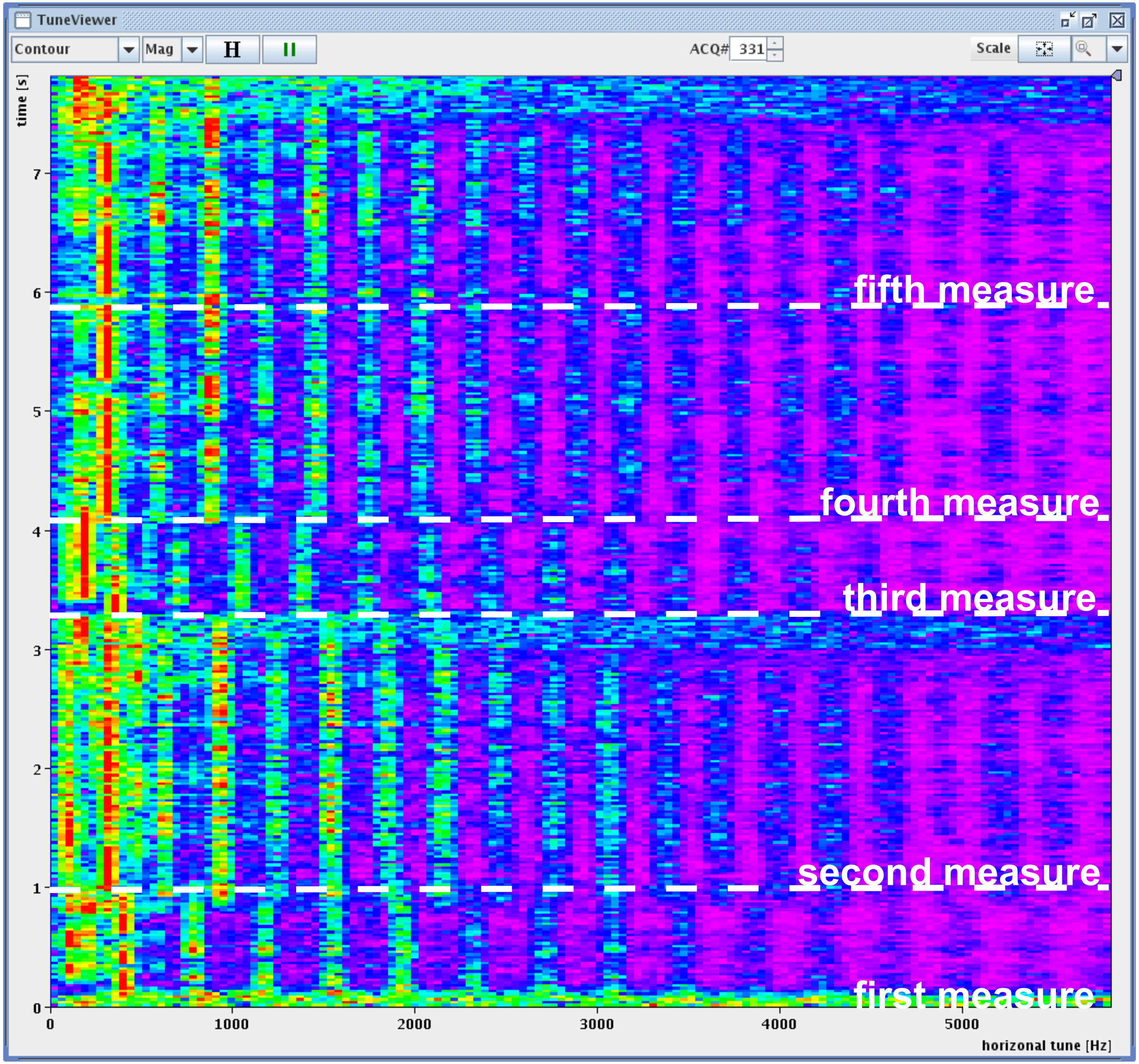
Comments: avg. of 4 datasets

Tune Feedback in the LHC

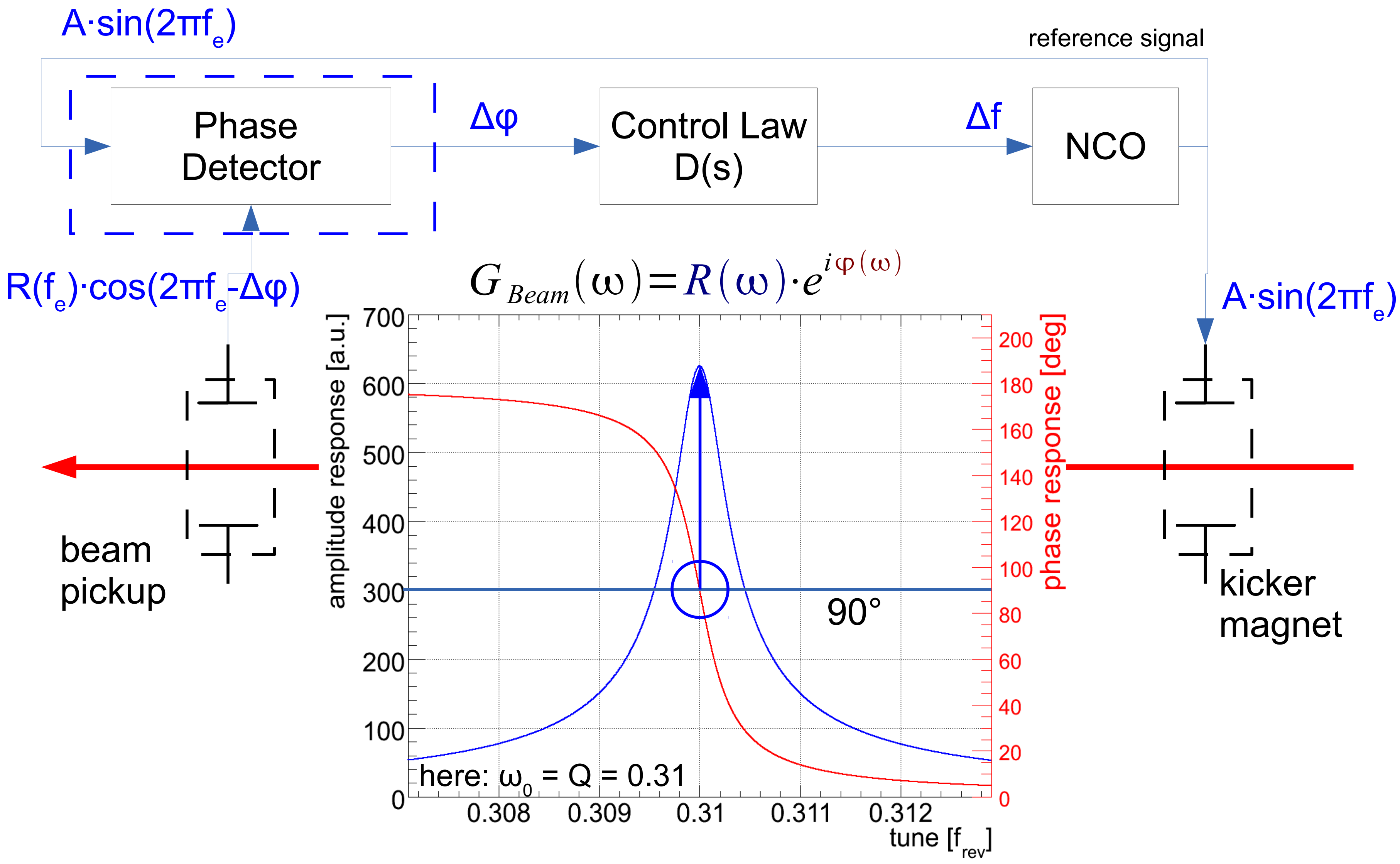


- With full pre-cycling the fill-to-fill stability is now typically 2-3×10^{-3}
- Variations frequently increase up to 0.02
  - Due to partial or different magnet pre-cycles after e.g. access or sector trips
- Tune-FB routinely used for physics ramps to compensate these effects
  - Using peak fit on FFT with 0.1..0.3 Hz Bandwidth

Reference Spectra
Beethoven's 5th, First Five Measures

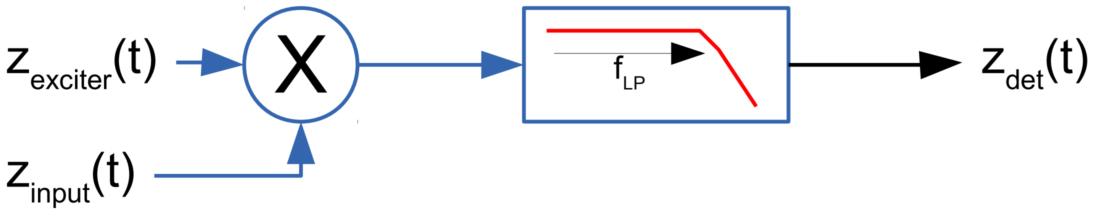


Classic Phase-Locked-Loop Tune Diagnostics



- BTF provides also information on collective effects
  - (landau damping→ spread distribution) impedance, stability diagram, lattice non-linearities (Q', Q''), etc.

Classic PLL Detector



$$z_{det}(t) = LP\left(z_{input}(t)\cdot z_{exciter}(t)\right)$$

$$= LP\left(R(f_e)\cdot\cos(2\pi f_e - \Delta\varphi(t))\cdot A\sin(2\pi f_e)\right)$$

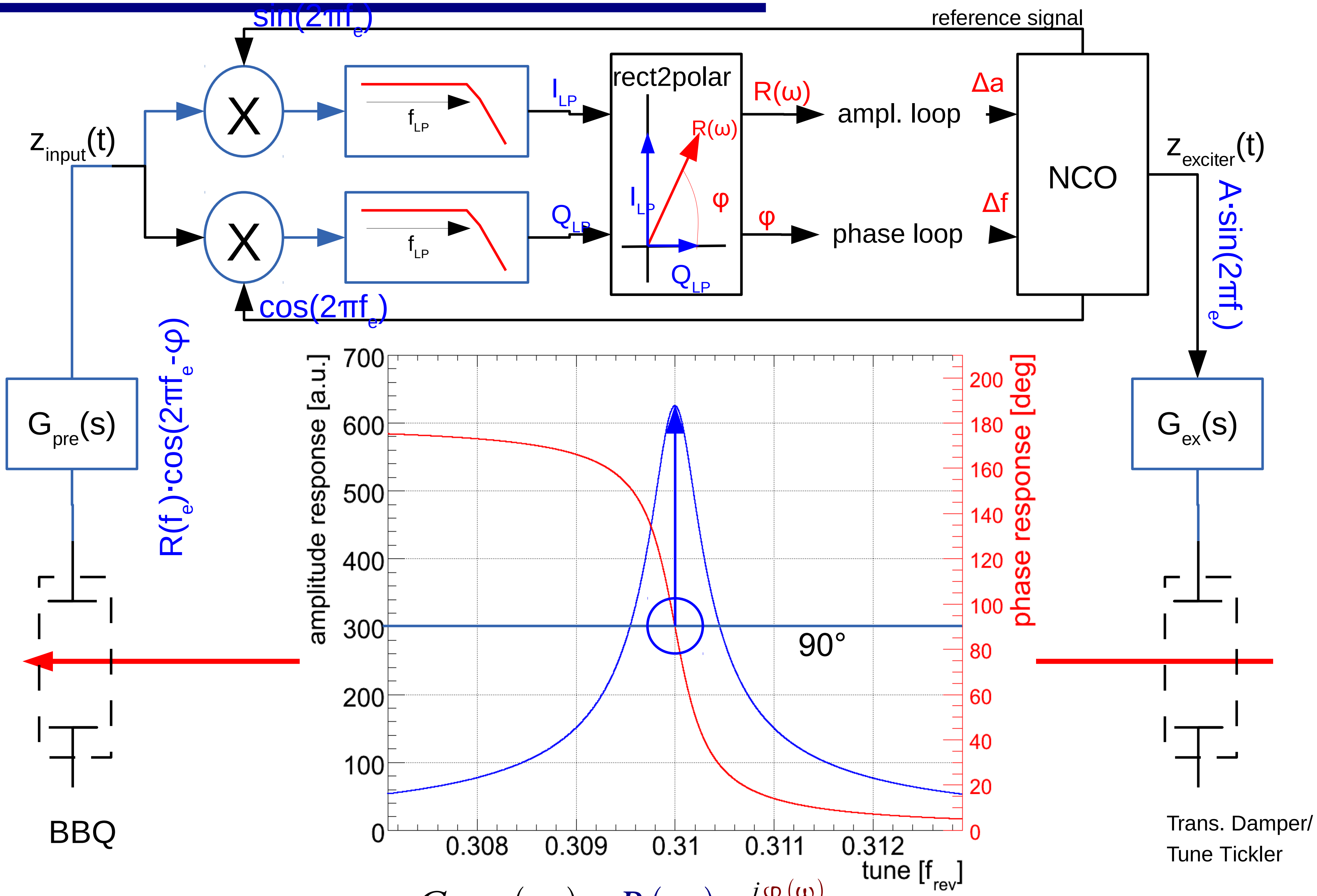
$$= \frac{AR}{2}\underbrace{\sin(\Delta\varphi(t))}_{\approx\Delta\varphi(t)} + \cancel{\frac{AR}{2}\sin(4\pi f_e - \Delta\varphi(t))}$$

for small phases

removed by low-pass filter

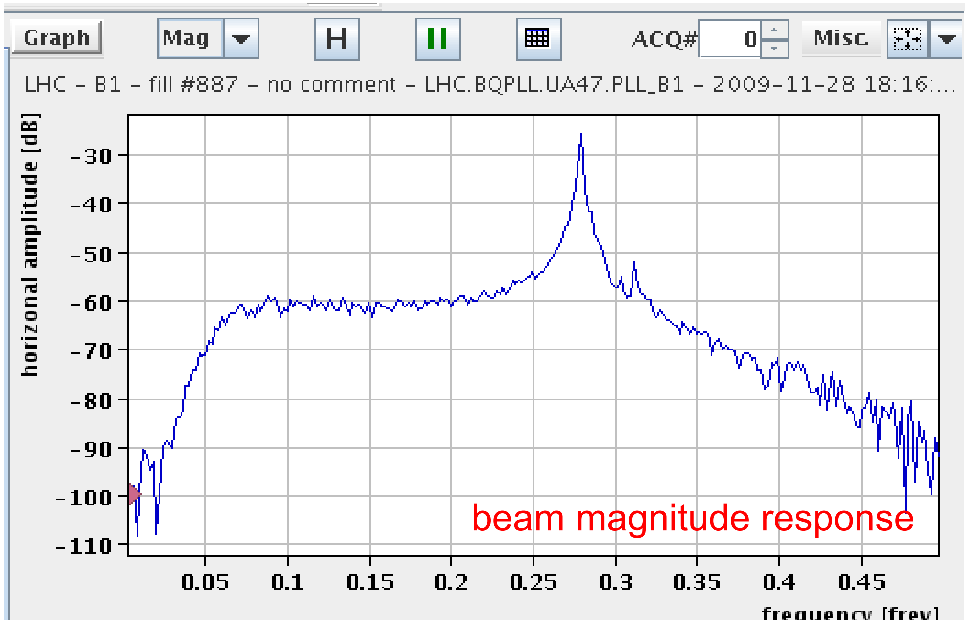
- Pro: robust analogue circuit implementation possible
- Con:
  - non-linear control signal for large phase difference Δφ
  - Control signal depends on beam response's amplitude $R(f_e)$

Advanced Phase-Locked-Loop Scheme

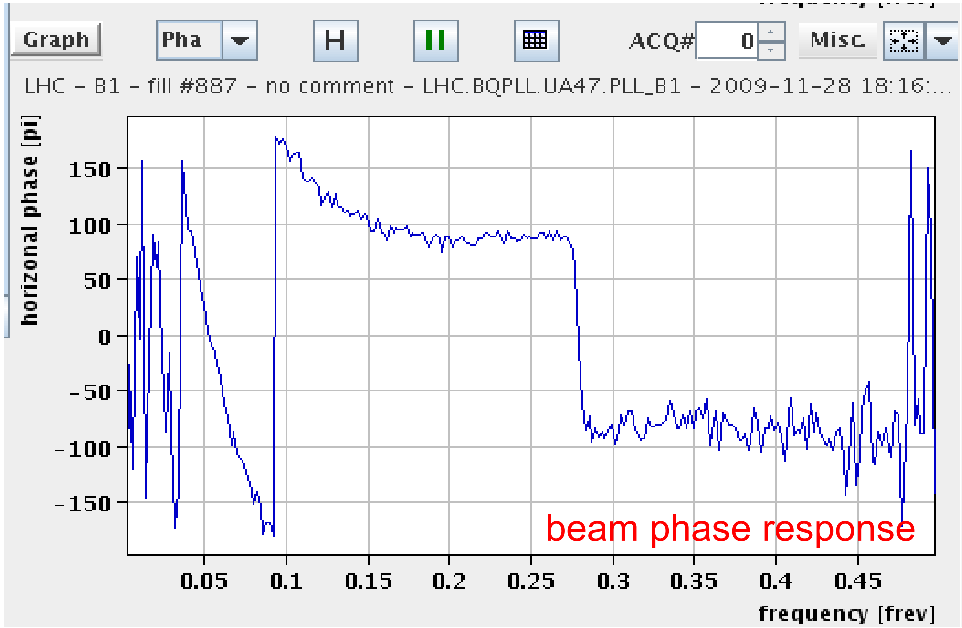


$$G_{Beam}(\omega)=R(\omega)\cdot e^{i\varphi(\omega)}$$

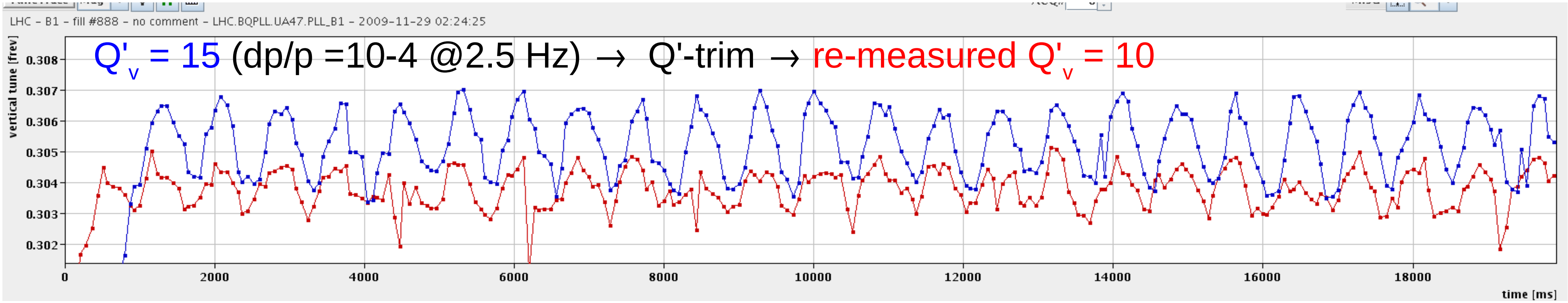
LHC Q/Q' Phase-Locked-Loop



beam magnitude response



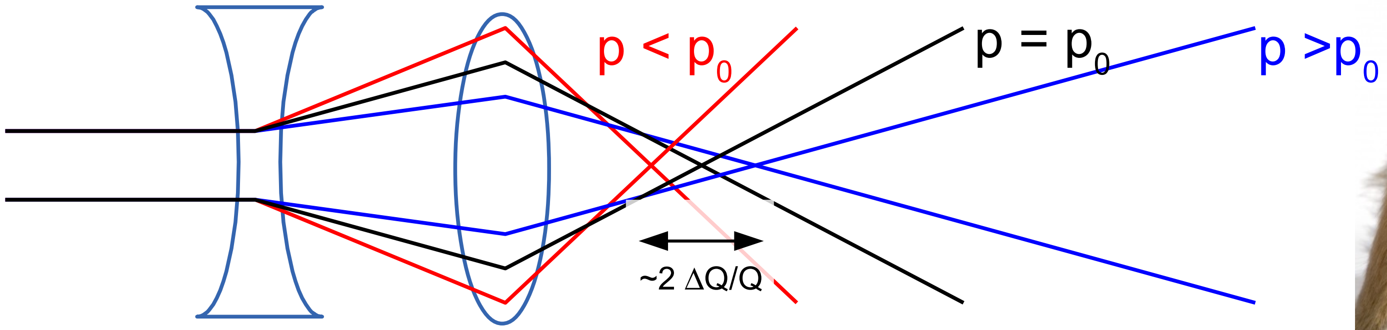
beam phase response

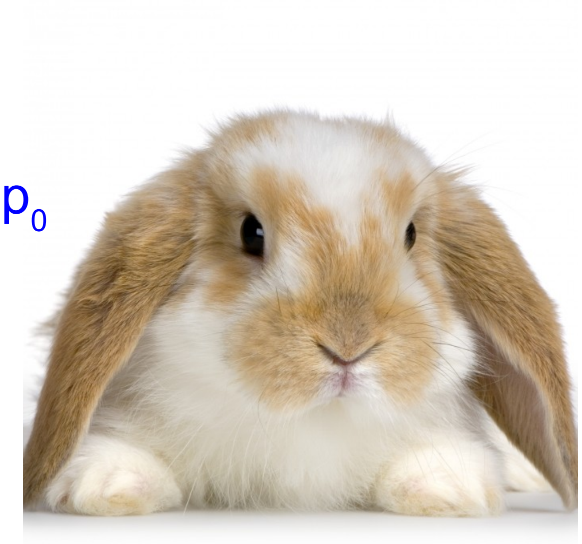


Q'_v = 15 (dp/p =10-4 @2.5 Hz) → Q'-trim → re-measured Q'_v = 10

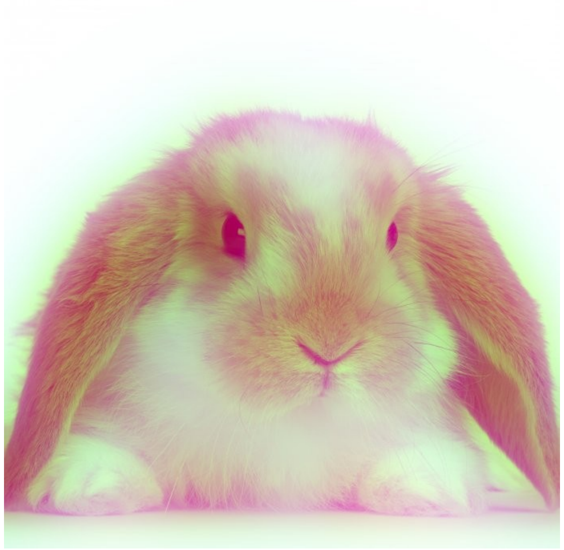
Beam Chromaticity - Primer

- Light optics analog: chromatic error





chromaticity



- Tune spread DQ/Q dependence on momentum spread Dp/p:

$$\Delta Q := Q' \cdot \frac{\Delta p}{p} \quad \text{or:} \quad \frac{\Delta Q}{Q} := \xi \cdot \frac{\Delta p}{p}$$

  - defines: (normalised) 'chromaticity' Q' (ξ)

  → also 1st order measurement principle

Recap: Tune & Quadrupole Gradient Errors

Hill's equation

$$\boxed{z'' + k(s)\cdot z = f(z)}$$

- Why do we need to measure the tune at all?
  - Quadrupole strength (hor. focusing): $k(s)=\frac{q}{\boxed{p}}\frac{\partial \boxed{B}}{\partial x}$

- Quadrupole gradient errors: $k(s)\rightarrow k_0(s)+\Delta k(s)$
  - saturation of iron yoke, magnet calibration errors, power converter ripple, etc.

$$\boxed{\Delta Q=\frac{1}{4\pi}\beta(s)\cdot\Delta k(s)}$$

→ watch out for quadrupole errors at large beta functions (e.g. final focus)!

Recap: Tune & Momentum Error I/II

$$\boxed{\Delta Q = \frac{1}{4\pi}\oint \beta(s)\cdot\Delta k(s)\,ds}$$

- Why do we need to measure the tune at all?
  - Quadrupole strength (hor. focusing): $k(s) = \frac{q}{\boxed{p}}\frac{\partial \boxed{B}}{\partial x}$

- Beam momentum error: $p \to p_0 + \frac{\Delta p}{p_0}$

$$f_x(s) = k(s)\cdot x \approx k_0(s)\cdot x - \boxed{k_0(s)\cdot\frac{\Delta p}{p}}\cdot x + \underbrace{k_0(s)\left(\frac{\Delta p}{p}\right)^2}_{\sim Q''}\cdot x \qquad \left(\frac{1}{1+x} = 1 - x + x^2 + h.o.\right)$$

→ Δk(s)

  - Inserting 'Δk' into tune shift formula yields 'natural chromaticity' Q'_{nat} definition:

$$\boxed{\Delta Q = -\frac{1}{4\pi}\left[\oint \beta(s)\cdot k(s)\,ds\right]\cdot\frac{\Delta p}{p_0} := Q'_{nat.}\cdot\frac{\Delta p}{p_0}}$$

  - ~ number of quadrupoles (~ accelerator circumference)
    - always negative (since β(s) > 0) → drives head-tail instability

→ needs to be compensated for nearly all (big/high intensity) machines

Recap: Tune & Momentum Error II/II

Hill's equation

$$\boxed{z'' + k(s)\cdot z = f(z)}$$

- Sextupolar field:

$$f_{x/y}(s) = \left\{ \begin{matrix} +\frac{1}{2} m(s)\cdot\left(x^2 - y^2\right) \\ -m(s)\cdot x\cdot y \end{matrix} \right\} \quad with \quad m(s) = \frac{q}{p}\frac{\partial^2 B}{\partial x^2}$$

- Off-Momentum particle passage through sextupole (assume y=0): $x \rightarrow D\cdot\frac{\Delta p}{p} + x_\beta$
  - keep only relevant order (estimate: D ~ m, Δp/p ~10^{-4} & x_β ~ 10^{-4} m)

$$\begin{aligned} f_x(s) &= +\frac{1}{2} m(s)\cdot\left[\left(D\cdot\frac{\Delta p}{p} + x_{beta}\right)^2\right] \\ &= +\frac{1}{2} m(s)\cdot\left[\left(D\cdot\frac{\Delta p}{p}\right)^2 + 2\left(D\cdot\frac{\Delta p}{p}\right)\cdot x_{beta} + x_{beta}^2\right] \\ &= \underbrace{+m(s)\left(D\cdot\frac{\Delta p}{p}\right)\cdot x_{beta}}_{\sim Q'} + \underbrace{\frac{1}{2} m(s)\cdot\left(D\cdot\frac{\Delta p}{p}\right)^2}_{\sim Q''} + \underbrace{\frac{1}{2} m(s)\cdot x_{beta}^2}_{\rightarrow \text{Landau damping}} \end{aligned}$$

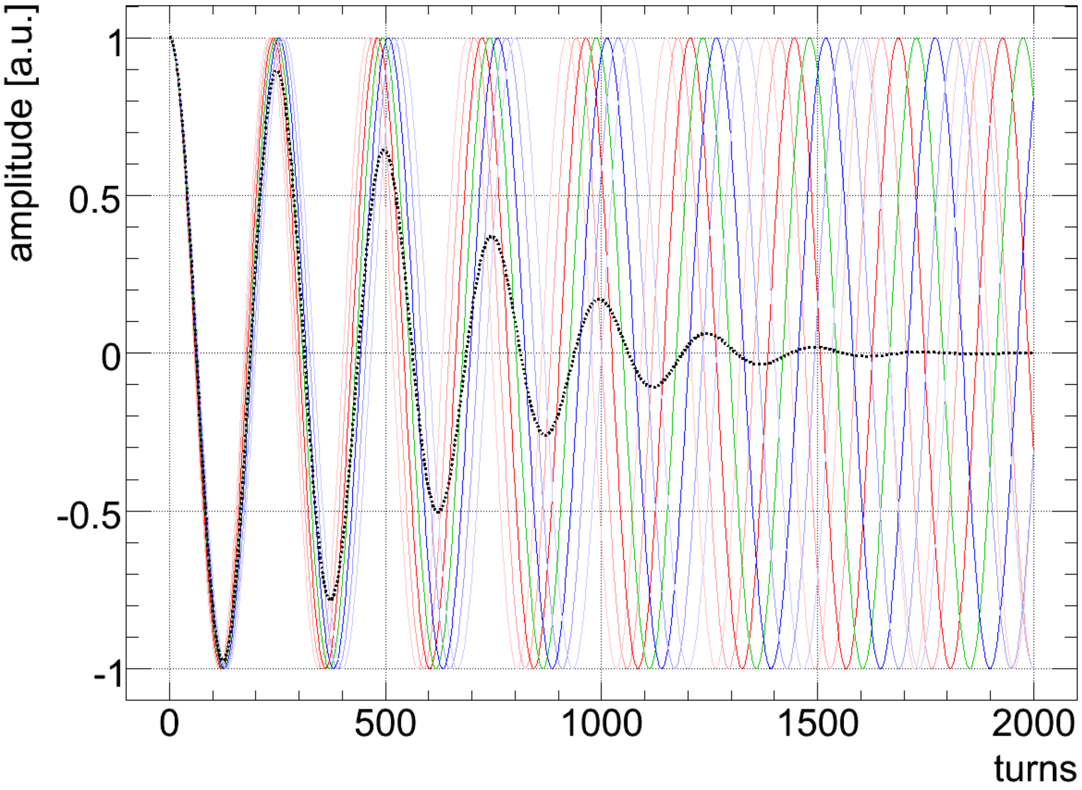
  - linear natural chromaticity compensated if $m(s)\cdot D(s) = k_0(s)$

- General linear chromaticity compensation relation:

$$\boxed{Q' = \frac{1}{4\pi}\oint \left[D(s) m(s) - k(s)\right] \beta(s)\, ds}$$

Recap: "Landau Damping"

- Individual bunch particles usually differ slightly w.r.t. their individual tune → Literature: "Landau Damping" (Historic misnomer: particle energy is preserved!)



- E.g. if f(ΔQ) is a narrow Gaussian distribution with with σQ << Q:

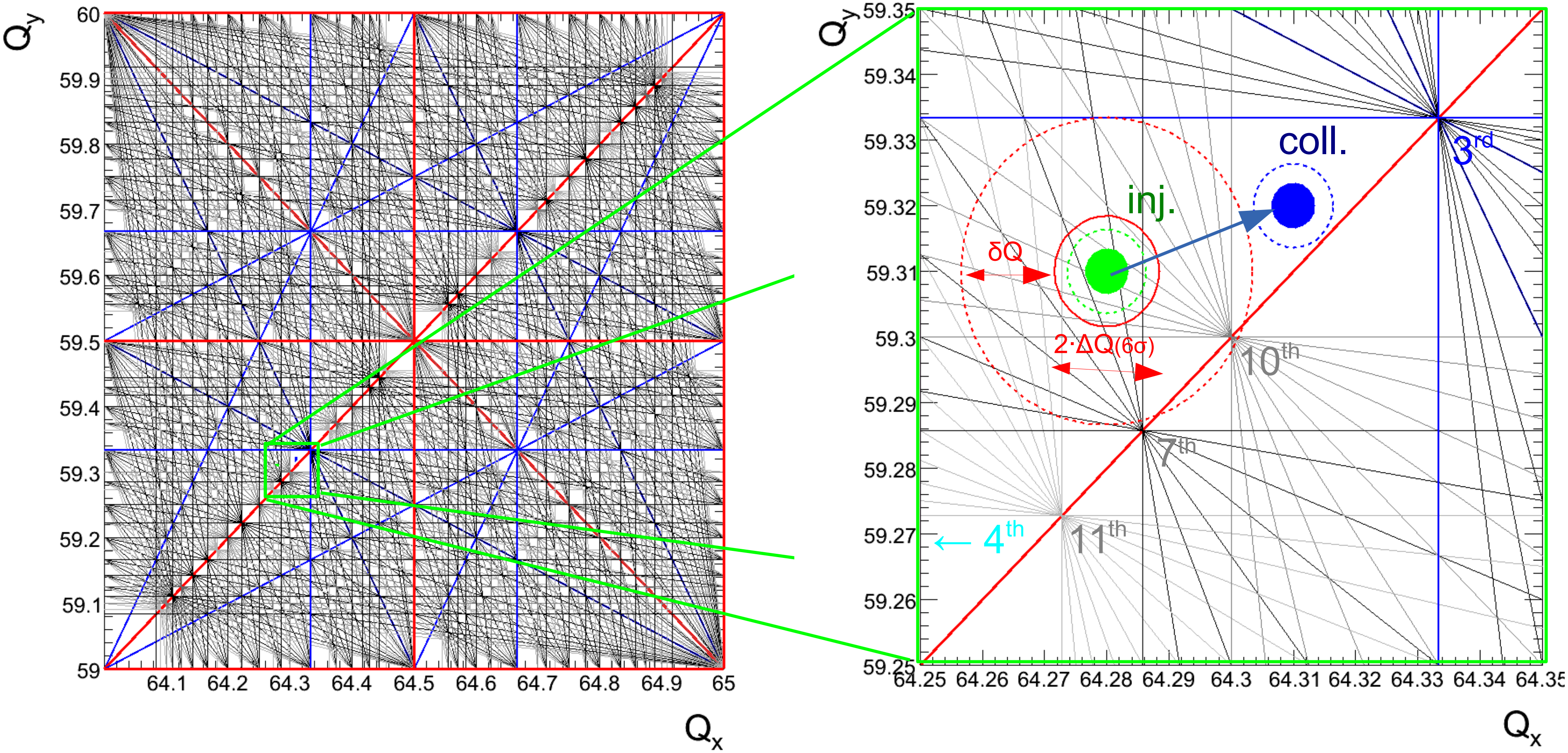
$$\boxed{\bar{z}(t)=\bar{z}_0\cdot \underbrace{e^{-\frac{1}{2}\cdot\sigma_Q^2 n^2}}_{\text{dampening}}\cdot\underbrace{\cos(2\pi Q\cdot n)}_{\text{tune oscillations}}}$$

→ large tune spread ↔ fast damping of e.g. head-tail instabilities

→ Tune oscillations are usually damped

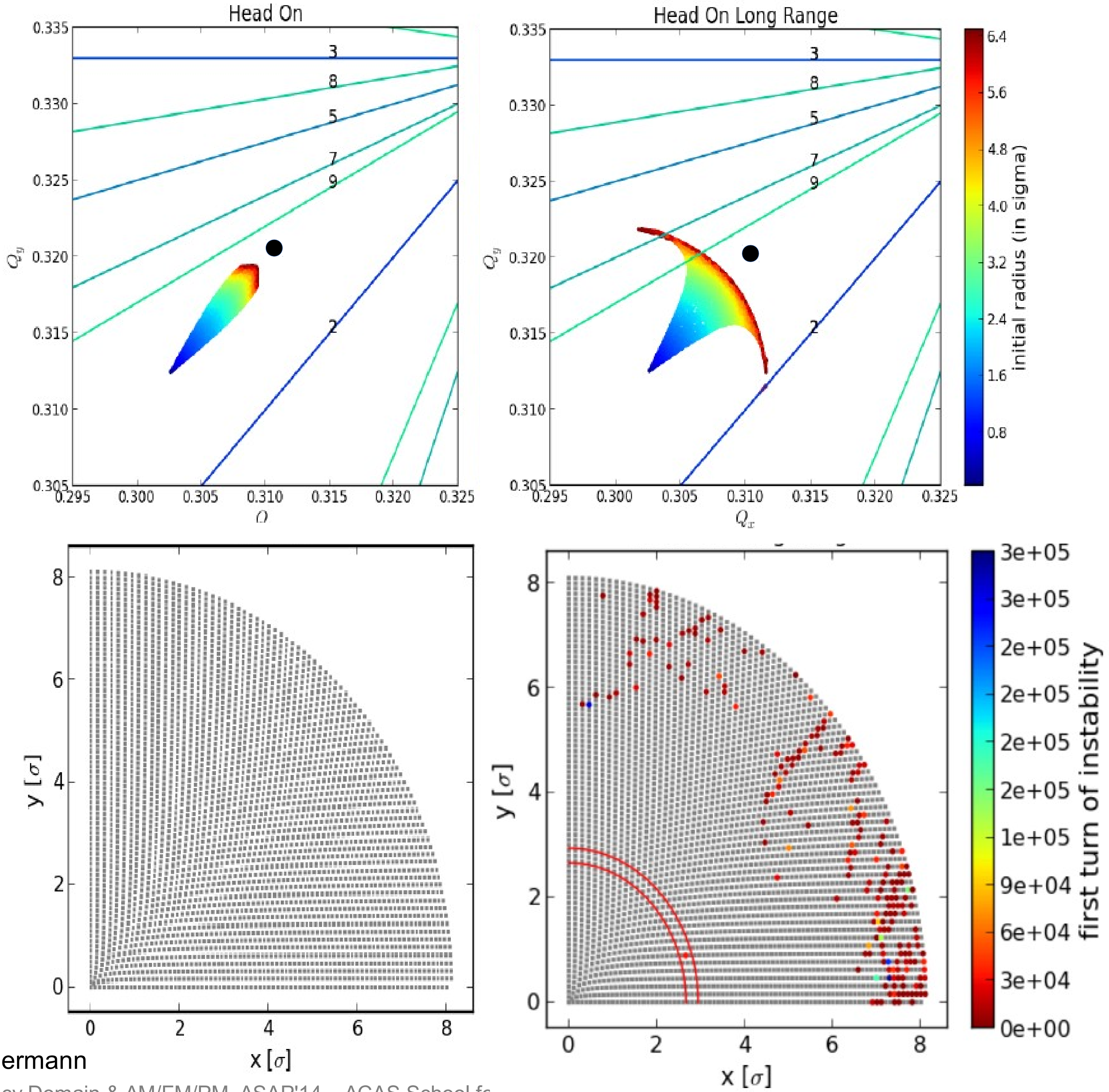
Why bother about measurement, stability & control of Q', Q'', ...?

- Increases footprint in Q diagram and causes resonances for off-momentum particles
  Example LHC (RF cavities 'off'):



  - need to obey this if we want to have more than one particle in the machine.
- Head-Tail instability → requires positive chromaticity for machines above transition
  - practically all lepton accelerators (e^+e^- collider, light sources, …)
  - high-energy proton accelerator (Tevatron, RHIC, SPS, LHC, ...)

Beam-Beam Interactions – Simulations



analysis: T.Rijoff & F. Zimmermann

LHC Base-Line Q/Q' Diagnostics Overview

- RF momentum modulation

$$Q' = \frac{\Delta Q}{\Delta p/p}$$

measured tune change

RF induced momentum change (known)

  - Measurement procedure (manual – human driven):
    1. Step: measure tune Q_1
    2. Step: change Δp/p (RF cavities), measure tune Q_2 → $\Delta Q = Q_2 - Q_1$
    3. Step: enter ΔQ & Δp/p into above definition → Q'

- Kicked Head-Tail Phase-Shift
  - Q' driven phase shift of bunch head- versus tail-oscillation

- Tune-width and de-coherence based methods
  - PLL Side-exciter & higher order fits

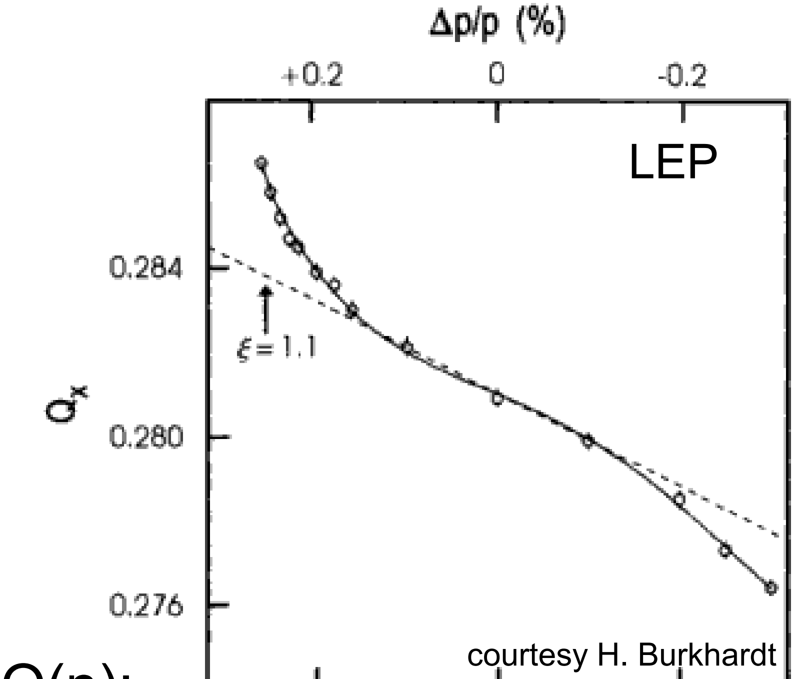
collective effects
- handle with care

Non-Linear Chromaticity

- Tune-shifts may depends not only linearly but also quadratically on Δp/p

  → Second order Chromaticity Q''

$$\Delta Q = Q'' \cdot \left(\frac{\Delta p}{p}\right)^2$$

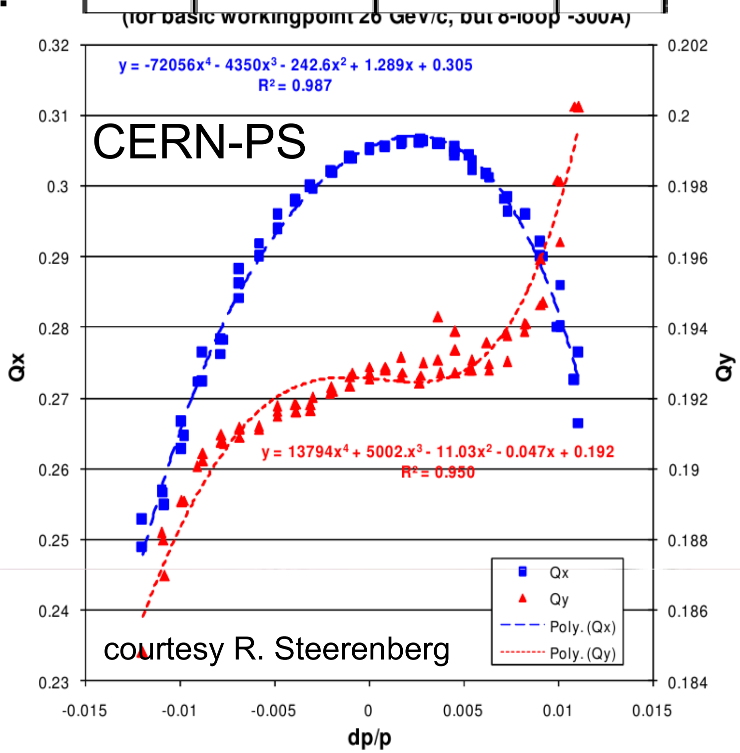


courtesy H. Burkhardt

- Can be generalised to higher orders Q'''... Q(n)

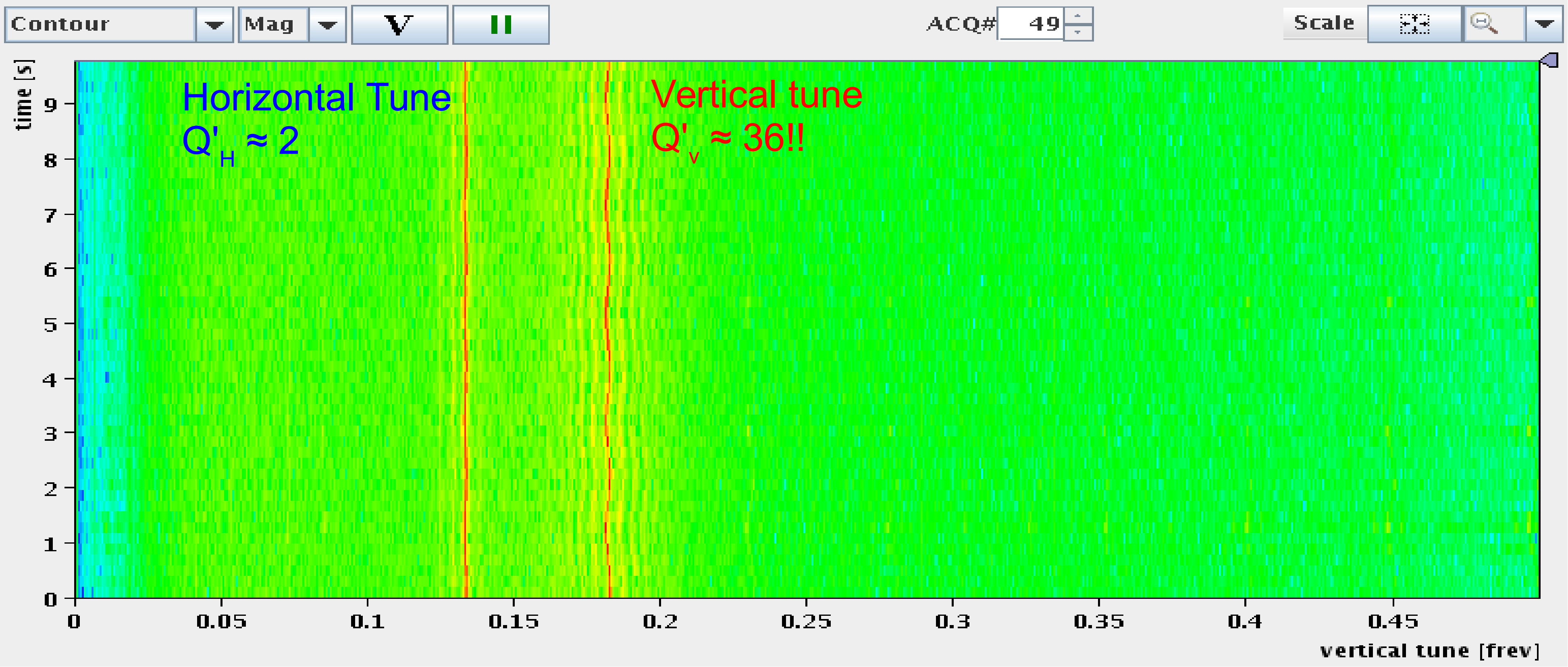
$$\boxed{Q^{(n)} = \frac{\partial^{(n)} Q}{\partial \delta^{(n)}} \quad with \;\; \delta := \frac{\Delta p}{p}}$$

- Principle stays the same:
  - Measure Q as a function of Δp/p
  - Fit n-th order polynomial to the tune shift
  - returns: Q, Q', Q'', Q''', ...
- However: correction is highly non-trivial!!



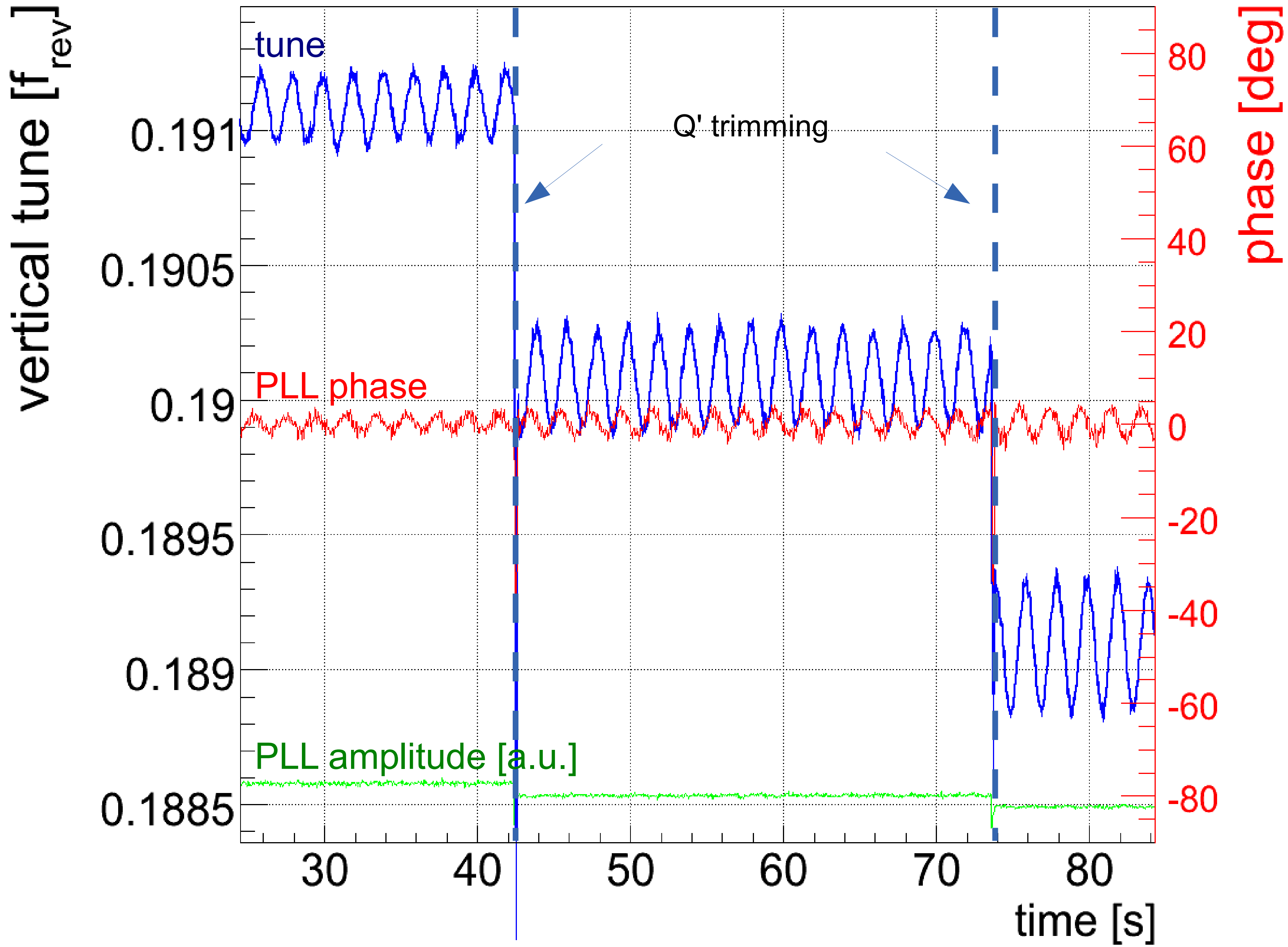
courtesy R. Steerenberg

Example: CERN-SPS – Δp/p ~ 2·10^-5 Modulation

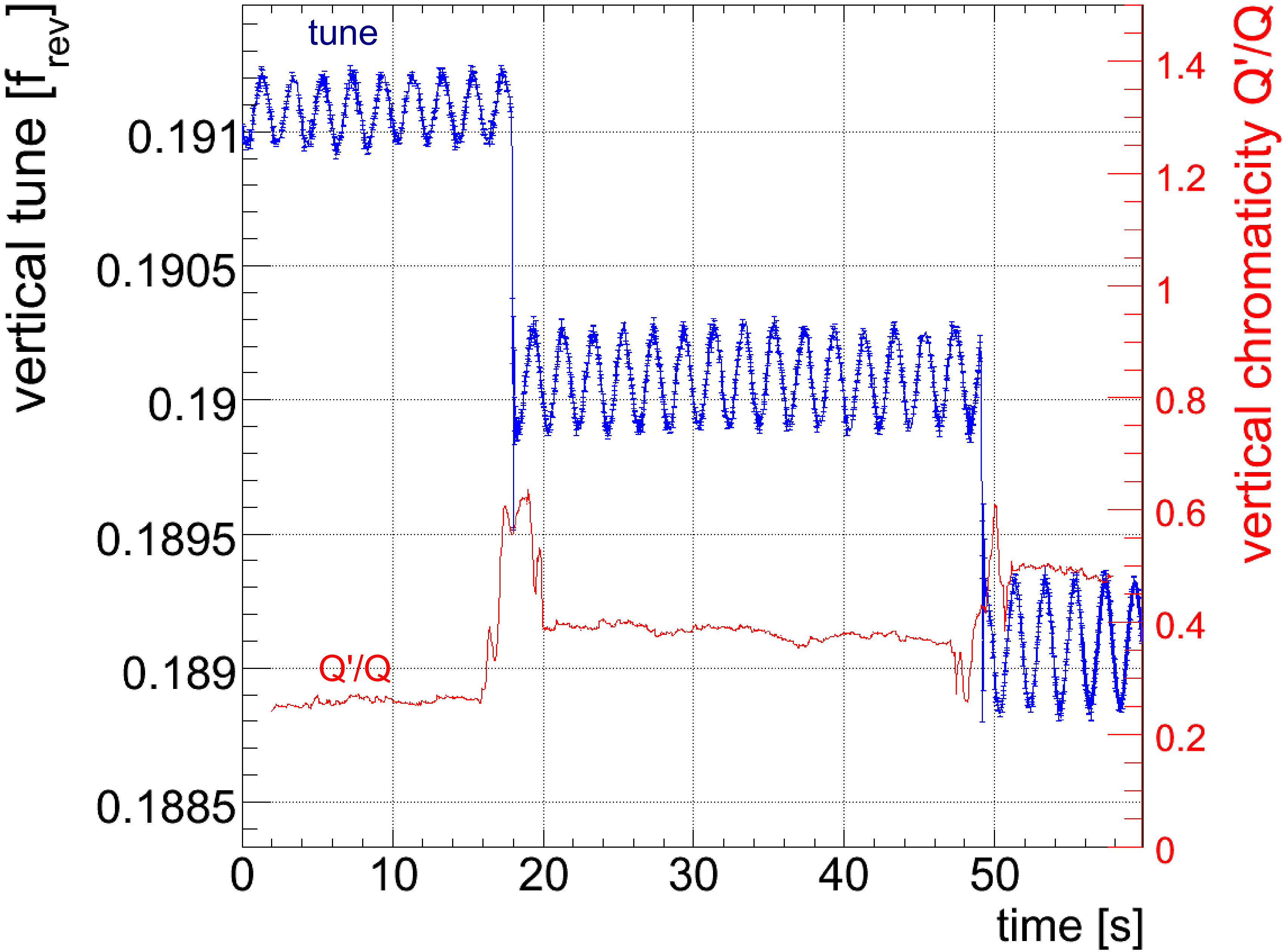


- Q' resolution is limited by tune resolution
  - large Q' increases frequency spread → Landau damping
  - can be improved by e.g. using a PLL
    - achievable frequency resolution $\Delta Q_{res} \sim 10^{-5} \ldots 10^{-6}$

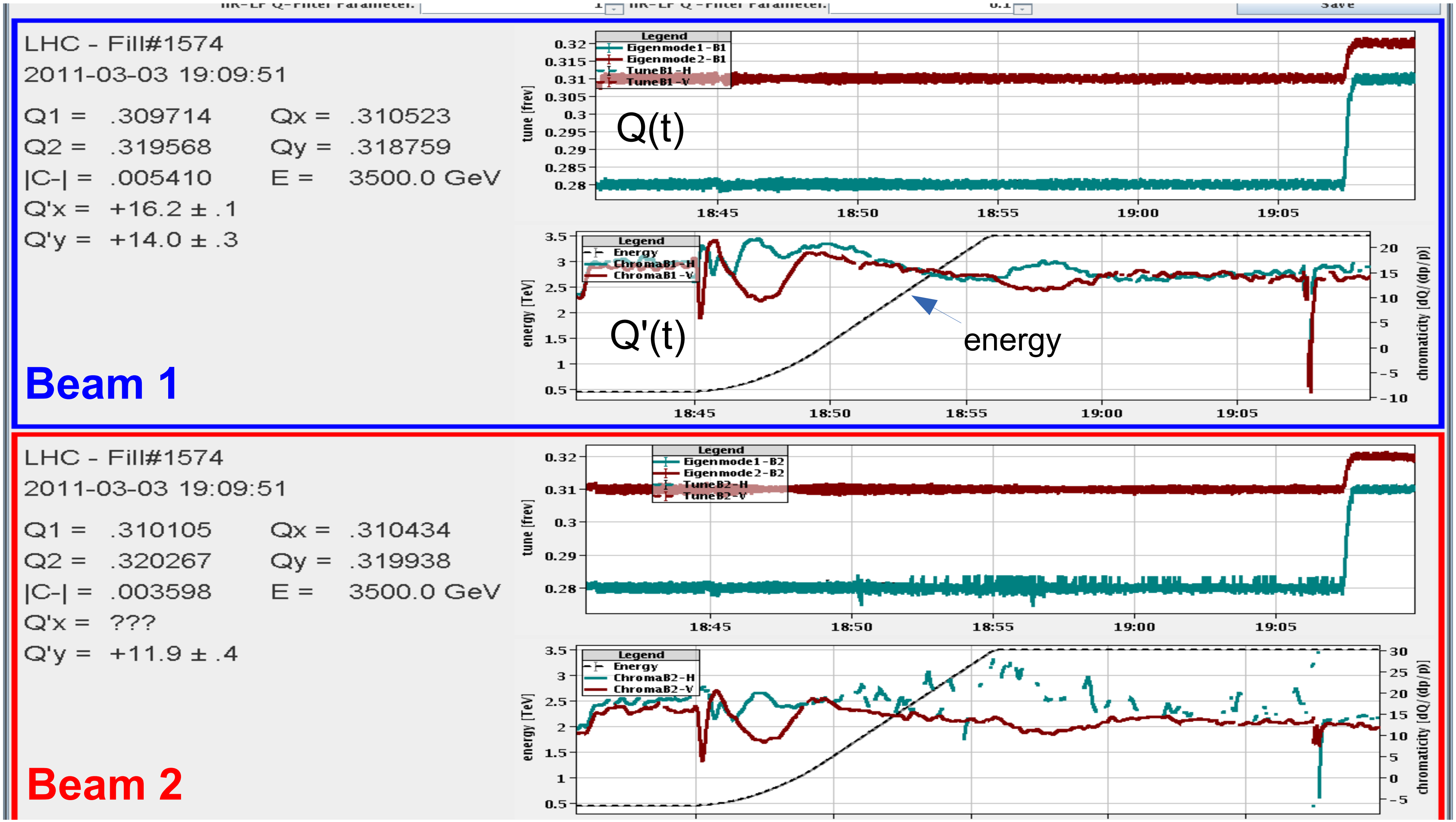
Example: SPS-PLL based Q/Q' tracking Δp/p ≈ 1.85·10^{-5}



Example: SPS-PLL based Q/Q' tracking $\Delta p/p \approx 1.85 \cdot 10^{-5}$



Typical Q/Q'(t) Control Room View



Beam 1

LHC - Fill#1574
2011-03-03 19:09:51

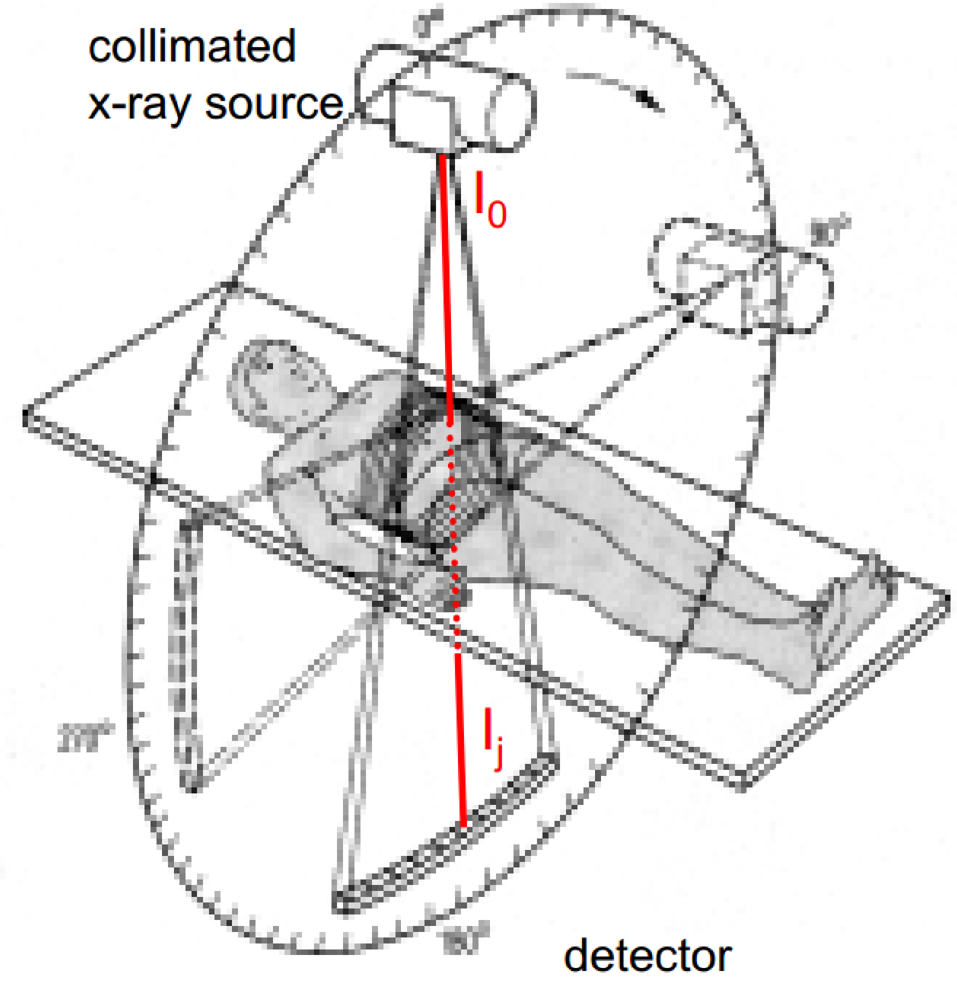
Q1 = .309714 Qx = .310523
Q2 = .319568 Qy = .318759
|C-| = .005410 E = 3500.0 GeV
Q'x = +16.2 ± .1
Q'y = +14.0 ± .3

Beam 2

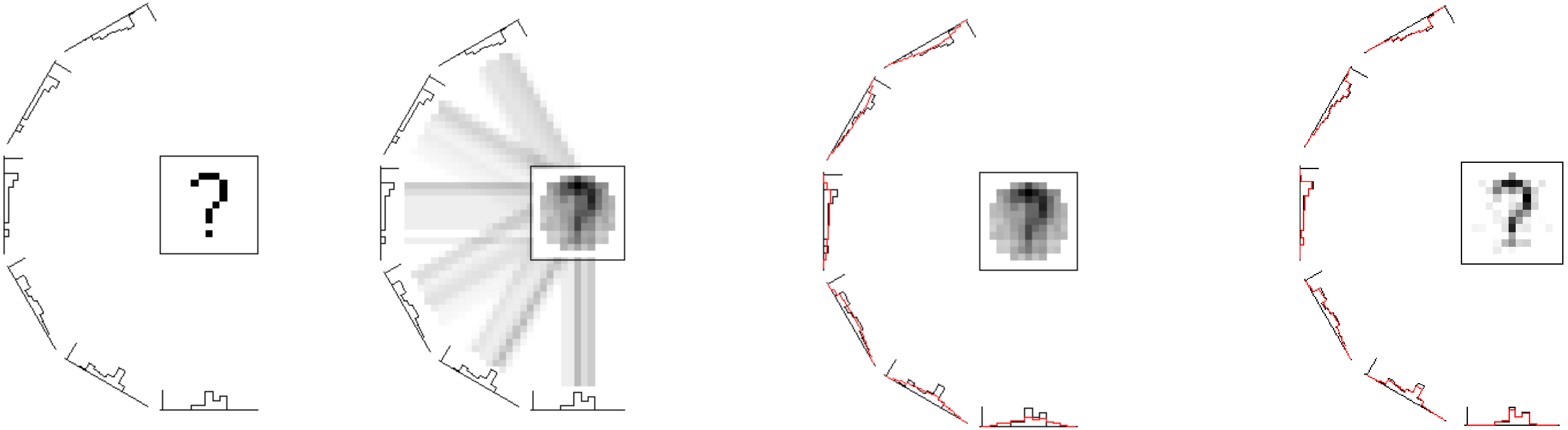
LHC - Fill#1574
2011-03-03 19:09:51

Q1 = .310105 Qx = .310434
Q2 = .320267 Qy = .319938
|C-| = .003598 E = 3500.0 GeV
Q'x = ???
Q'y = +11.9 ± .4

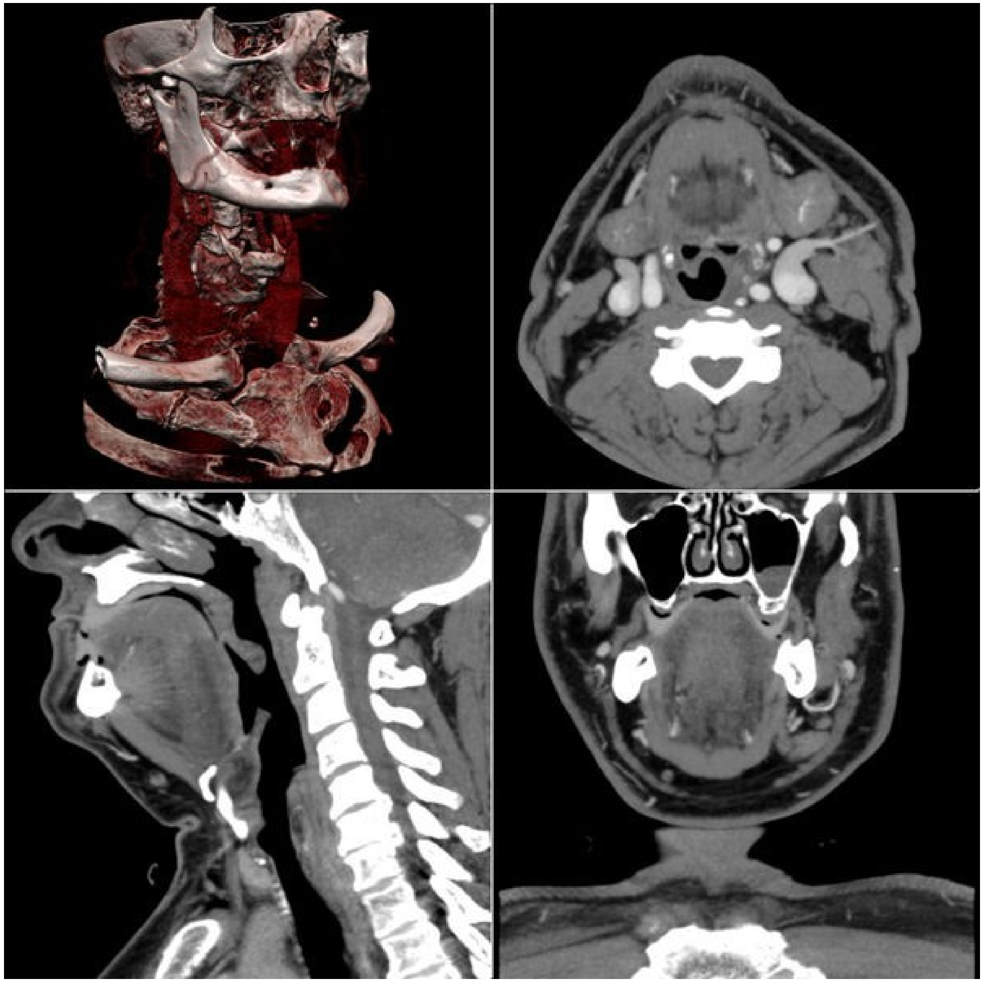
Computed Tomography (CT)

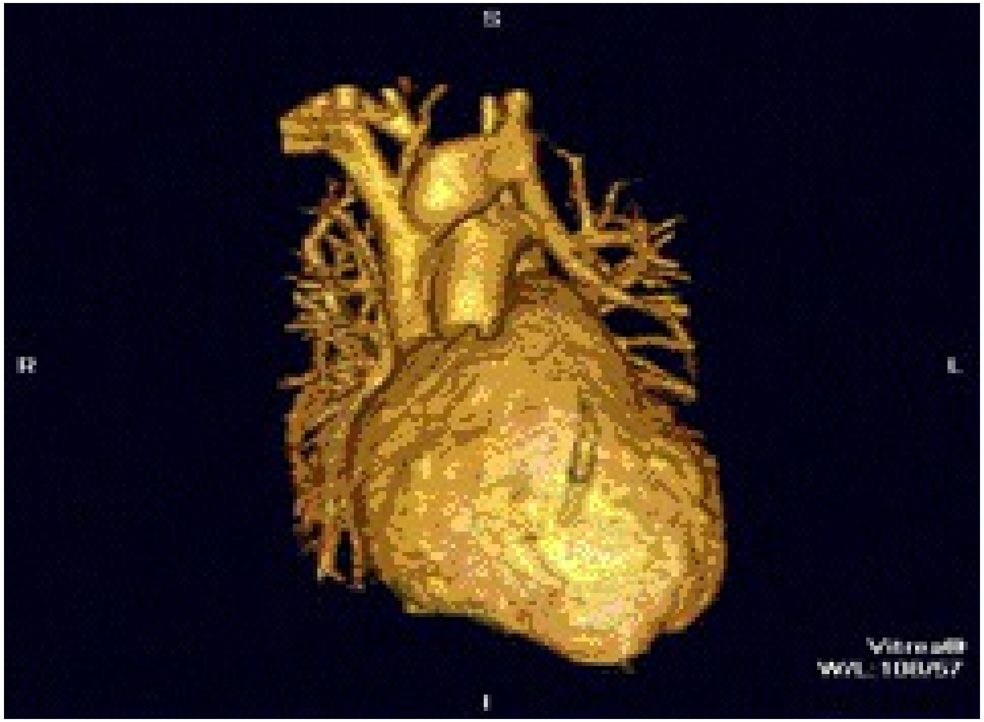


CT Reconstruction

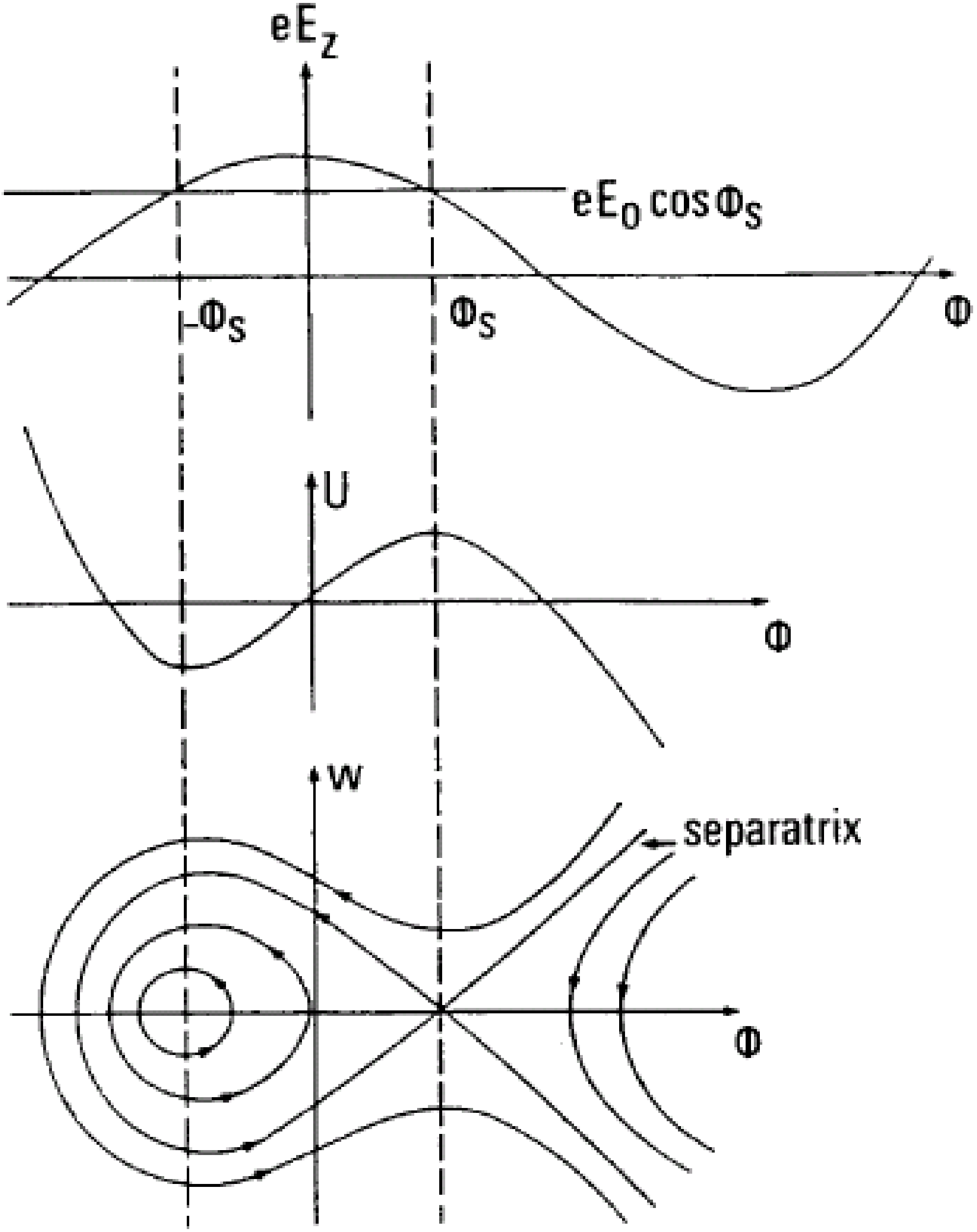


Some CT Results

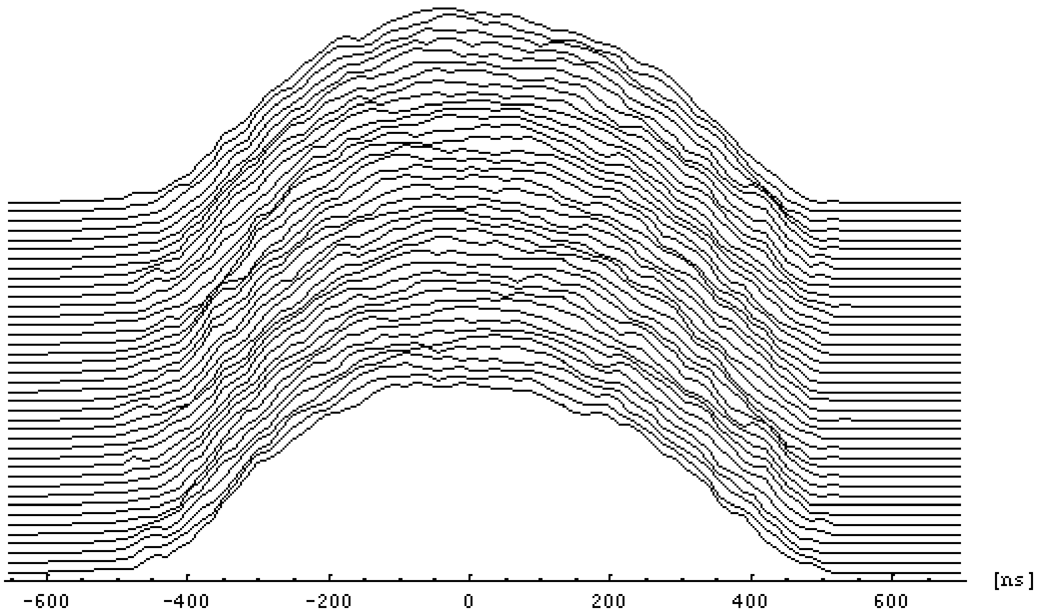


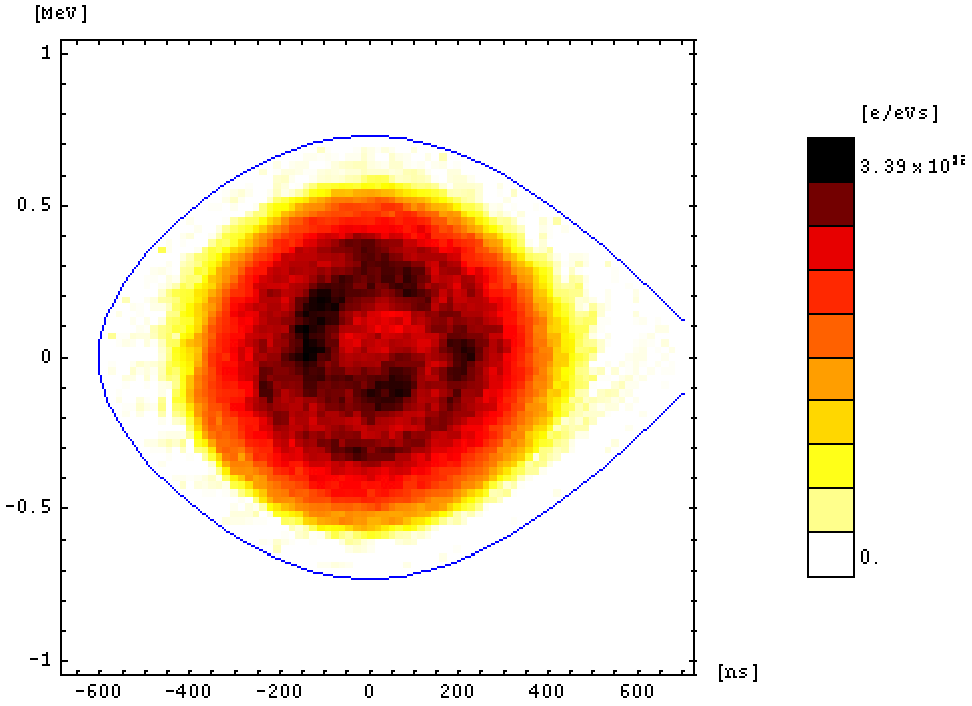


Computed Tomography and Accelerators

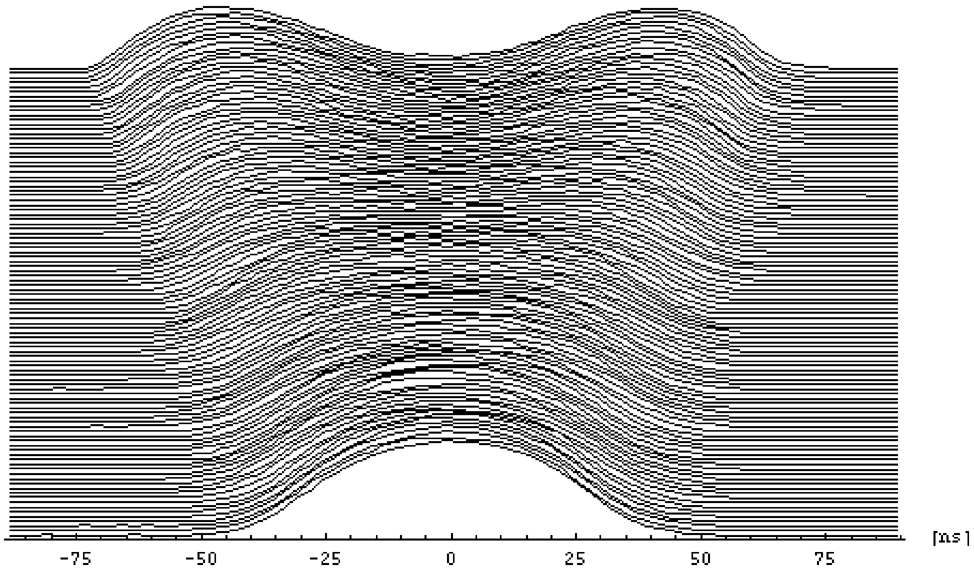


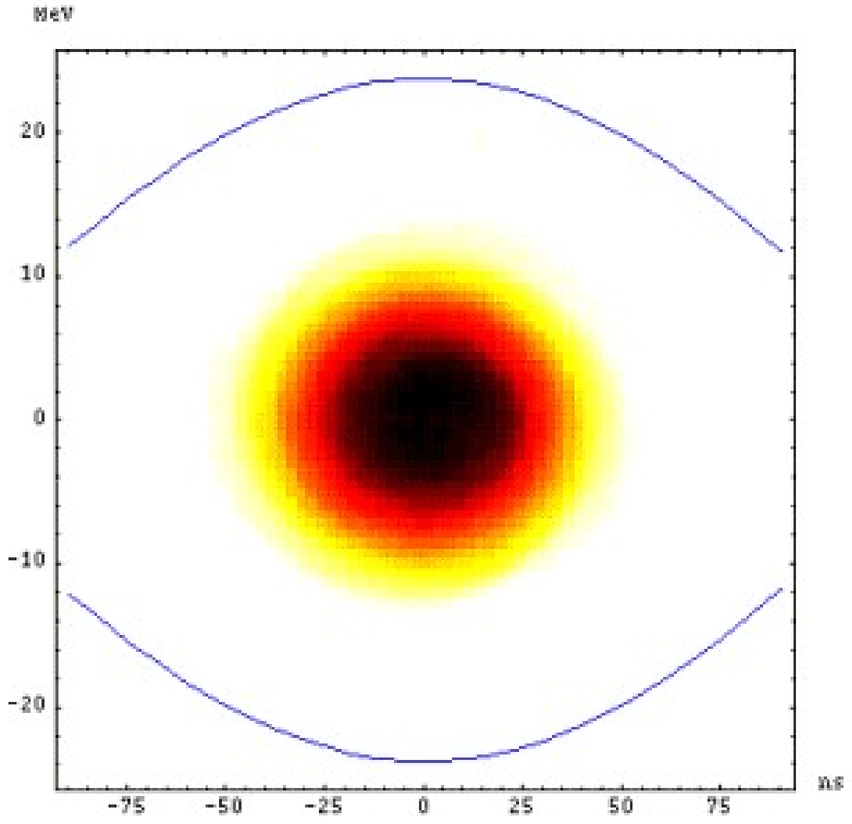
Reconstructed Longitudinal Phase Space





Bunch Splitting

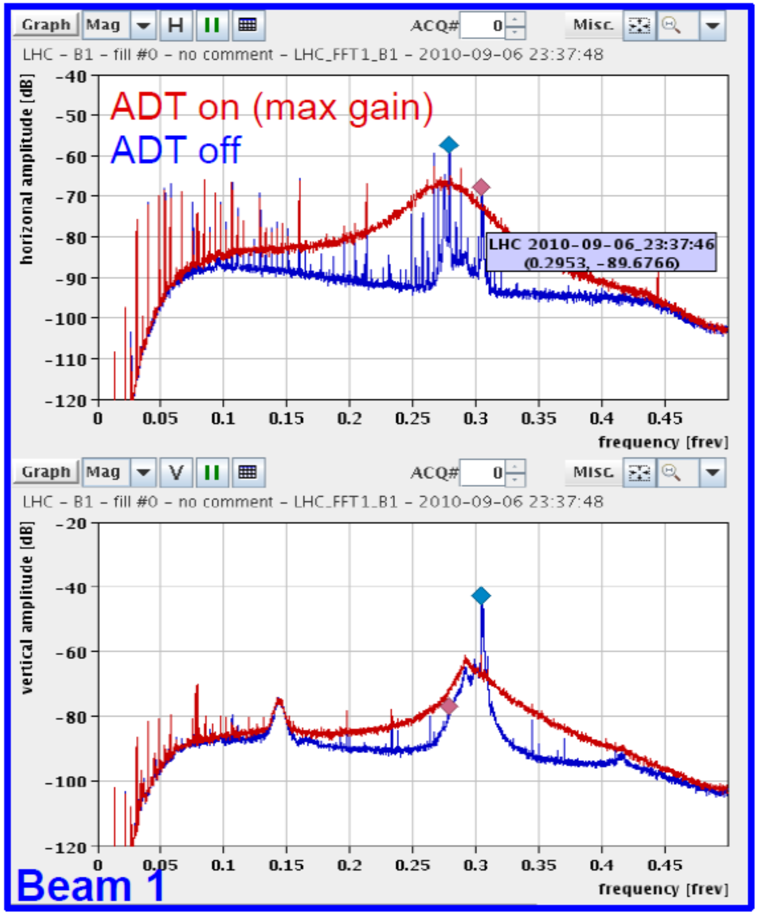




Tune Feedback & Active Damping

- BBQ noise-floor raised by 30 dB
  - wide tune peak → reduces tune resolution from 10^{-4} → $\sim 10^{-2}$
  - Impacts reliable tune (and coupling) measurement & feedback
  - Incompatible with chromaticity measurements using small Δp/p-modulation

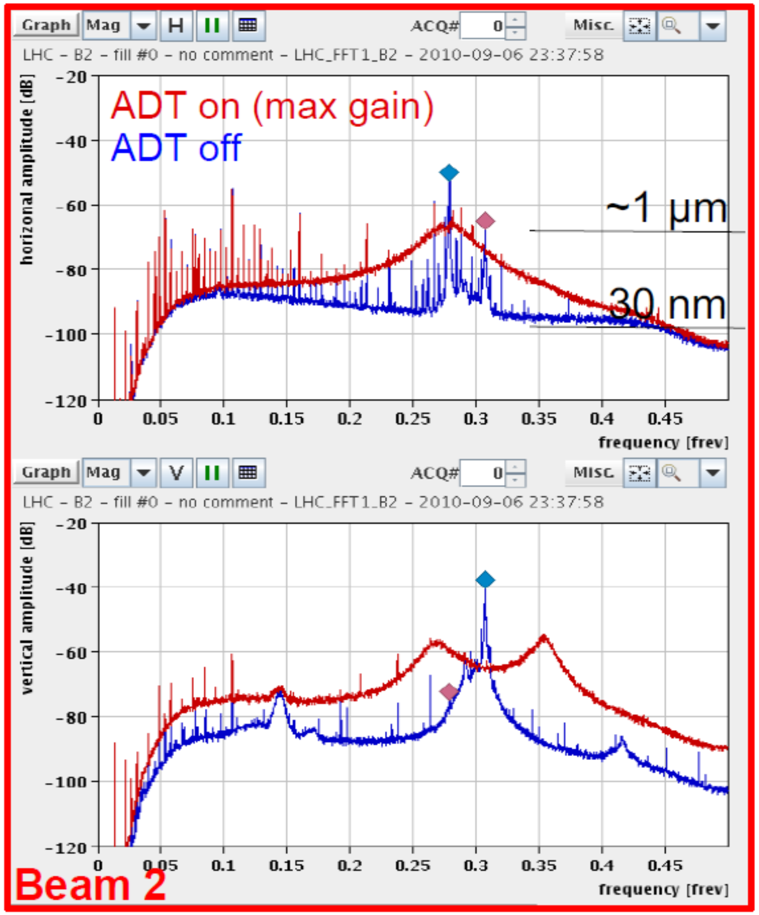
→ track tune on subsets of bunches with reduced damper gain



LHC – B1 – fill #0 – no comment – LHC_FFT1_B1 – 2010-09-06 23:37:48

LHC 2010-09-06_23:37:46 (0.2953, -89.6766)

LHC – B1 – fill #0 – no comment – LHC_FFT1_B1 – 2010-09-06 23:37:48



LHC – B2 – fill #0 – no comment – LHC_FFT1_B2 – 2010-09-06 23:37:58

LHC – B2 – fill #0 – no comment – LHC_FFT1_B2 – 2010-09-06 23:37:58