

Fast Inter/Intra-Bunch Activities related to CLIC and LHC

Status and Update –

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- Motivation
- Present Limitations and LIU Upgrade Wishes/Plans
- Three complementary upgrade paths being investigated:
 - Synchrotron-Light based BPM \rightarrow dual use CTF3 & LHC

 - Optical regime may solve present power and bandwidth limitations
 - Fiber-based signal transmission
 - Collaboration effort with ACAS (Uni-Melbourne and ASLS)
 - (In-)direct EO-based BPM \rightarrow machine/beam type independent
 - Proof-of-principle by the end of this year, can re-use acq. system
 - Wide-band, electro-magnetic pick-up based
 - boost to 6 GHz by classic RF means \rightarrow BI-TB in January 2012 •



Head-Tail Instability



- First discovered at ACO and Adone (1969)¹
 - mixing of long. and transverse motion
 - Phase advance change:

$$\Delta Q := Q' \cdot \frac{\Delta p}{r}$$

• Synchrotron oscillation:

$$\frac{\Delta p}{p} = \frac{\hat{\Delta p}}{p} \cdot \sin(\omega_s \cdot n + \phi_i)$$

→ Head and Tail swap position after half a sync. period!

Damping time constant (1st order):

$$\frac{1}{\tau_{HT}} \sim N_{b} \cdot \frac{\hat{\tau} Q'}{Q^{2} \cdot \left[\alpha_{c} - 1/\gamma_{rel}\right]}$$

- Head-Tail motion becomes unstable if
 - above transition $(\eta > 0)$ & Q' < 0
 - \rightarrow usually keep Q' slightly positive!





1 M. Sands, "The Head-Tail Effect: An Instability Mechanism in Storage Rings", SLAC-TN-69-008, 1969





- Intra- and inter-bunch beam motion is one of the observables of this instabilities:
 - Lower-order modes: Inter-bunchBPM resolving bunch-by-bunch motion
 - High-order modes: Intra-bunch instabilities \rightarrow Head-Tail instabilities

e.g. J. Gareyte, "Head-Tail Type Instabilities in the PS and Booster", CERN, 1974









- a) mode m = 0, $\chi = 0$ b) m = 0, $\chi = 2.3$ radians b) m = 1, $\chi = 6.9$ radians d) m = 2, $\chi = 6.9$ radians
- PS: 120 ns bunch length \leftrightarrow less demanding in terms of bandwidth SPS/LHC: bunch length down to 1 ns \rightarrow requires GHz analog bandwidth



Head-Tail Motion Signal-Spectra Simulation



- Higher-order HT instabilities
- Can be detected e.g. via:
 - time-domain: counting number of zero-crossing, rising/falling-edges
 - freq.-domain: standard peak search, provides also indication for
 - mixed modes



time [ns]





difference signal [a.u.]





- Additional diagnostics: power spectrum is sensitive to bunch shape, bunch length, bunch intensity, longitudinal phases, …
 - But signal decreases significantly
 → amplification by 40 dB above 1-2 GHz







- Fast inter- and intra-bunch beam instability monitor used to detect and study e-could-, impedance- and TMCI-driven head-tail effects
- Diagnostics typically based on strip-line pick-ups, challenges:
 - very wide-band frequency response: 0 ... 2.5 (present)... 8 GHz (target)
 - − nom. bunch intensity: $n_{b} \approx 1.2 \cdot 10^{11} e$ +/bunch (20 nC) & $\sigma_{t} \approx 0.2 ns$
 - $\leftrightarrow V_{_{pp}} \sim n_{_{b}}^{^{2}} \sim 300 \text{ V}$ and frequency range of 0 1.7 3 GHz
 - \rightarrow LIU upgrade: $n_{b} \rightarrow 5.10^{11}$ e+/bunch

(x 16 power, not many HF & broad-band components that can handle this)

Standard HT approach: BPCL.421 – 60 cm long strip-line monitor:







Classical Head-Tail Instability Detection II/II



Implemented/tested at CERN-SPS, Tevatron, LHC:

- Long strip-line BPM (60 cm, to avoid signal-reflection mixing)
- Σ - Δ hybrid (removes common mode signal)
- Fast-sampling to resolve bunch structure
 - ~ ns bunch length \rightarrow GHz scope bandwidth
- Need to compensates for non-beam effects:
 - pickup- & hybrid response,
 - cable dispersion, ...
 - cable reflection, imperfect impedance matching
 - electrical offsets













What our ABP colleagues like to see (here: HT mode 'm=1'):





Limits of Classical Head-Tail Monitoring Approach 108° Hybrid Imperfections



- Batch variations of 22 wide-bandwidth hybrid couplers (H-9, 2-2000 MHz)
 - similar common-mode perf., gentle roll-off for HF (can be compensated)
- All Hybrids/RF components are equal...but some are more equal than others:
 - large variation of insulation between ports: (not a strong design constraint for manufactures)



- Best insulation level -40 dB @ high frequencies
 - \rightarrow limits minimum dynamic range of input to ~1% of BPM half-aperture
- electrical offset/non-centered orbit adds to this and is another dominant source of HT resolution reduction



Limits of Classical Head-Tail Monitoring Approach For Comparison: SPS/LHC HT System Response



Roll-off at ~ 3.5 GHz due to limited scope bandwidth Better performance for LHC HT \rightarrow SPS deserves something better





- Ringing in delta signal after main bunch....
 - Eventually traced back to upstream vacuum pipe discontinuity:
 BPV: elliptical ↔ rectangular ↔ round





Hybrid ferrite saturation \rightarrow addition common-mode signal in delta-signal

Additional bunch intensity induced ringing after signal \rightarrow hybrid saturation

- Can be mitigated by attenuation but may loose dyn. range without switches
 - variable gain attenuation at broad-band GHz frequency is not trivial

Head-Tail Upgrade Specification As prepared by B. Savant et al.

- Beam observables: transverse position resolved within the bunch
- Objective: qualitative observation of coherent modes of oscillations (intrabunch and interbunch).
- Acquisition:
 - Bunch by bunch,
 - Turn by turn for at least 1000 turns,
 - availability every cycle in both planes
- Dynamic range: 10¹⁰ protons to 5.10¹¹ protons per bunch
- Accuracy: accurate knowledge of absolute transverse position is not critical
 - Resolution: Changes of transverse position by 1% should be detected.
- Repeatability:
 - Within the same turn: same as resolution (1%).
 - From turn to turn: same as resolution (1%).
- Reproducibility from cycle to cycle: not critical.
- Logging: SDDS saving is sufficient for now (no need for logging).
- Required Bandwidth: aim at resolving 'm=6' head-tail modes (σ_t =1 ns \rightarrow ~6 GHz bw.)
- Miscellaneous: if possible, suppression of reflection signal would be appreciated

Australian Collaboration for Accelerator Science http://accelerators.org.au/

... a new network with the aim of providing:

university-based education, maintaining up-to-date au. accelerator science programme, direct funding into high priority research, capitalise on investment in accelerator based user facilities, foster collaborative research (CERN: CLIC-DR, BI), R&D for new facilities and innovative technology

- Idea: extend FPM functionality to measure dipole momentum of transverse beam distribution, e.g. via 2x2 photo- detector matrix
- Initial idea and infrastructure based on the ASLS's Fill Pattern Monitor (FPM)*

*D. Peak, M. Boland, R. Rassool, et. al., "Measurement of the real time fill-pattern at the Australian Synchrotron", NIMA, 2008¹⁷

Idea: measure dipolar momentum of the synchrotron light...

- Range limited to $\pm \sigma$ of beam spot size \rightarrow acceptable as an instability monitor
- Second order: scale is dependence on σ

- Hamamatsu's G4176-03 (TO5 package, SMA connector)
 - $t_r \approx 30 \text{ ps} \leftrightarrow \text{nom. 50\%}$ atten. @12GHz
 - 0.3 pF for active area of 0.2 x 0.2 mm²
 - typ. light input power ~5-10 mW (50% duty-cycle)
 - dark-current: 100 pA @23°C
 - max. est. S/N: ~150 dB (w/o cooling)

(very good value for money, prototyping!)

N.B. alternative variant for infra-red: G7096-03

Metal-Semiconductor-Metal (MSM) Photodetector II/II

... not quite a P(i)N junction (diode)!
 no polarity, requires bias-voltage (typ. 10 V)

- Speed determined by doping of (In)GaAs SC material and PD geometry (reflection, C, ...)
 - Not quite a MS Schottky Diode
- Variants available exceeding 100 GHz bw.
 but makes fiber-coupling mandatory
- \rightarrow KISS: initial prototyping with in-air design

Advantages:

- even lower noise than pure MSM
- Simple phase compensation
- Simple adaptive orbit offset comp.
- 50Ω vs. high-impedance (glued to ADC)
- Can keeps sensitive (== expensive) equipment/control outside the tunnel

Advantages:

- incoherent sum → indep. on phase of laser wave-front (no expensive PANDA fibres!)
- Can be re-used for other EO-options
 → see second part of summary
- Future: dependence on beam size \rightarrow extend scheme to measure σ

- Specific advantage of MSM photo detect. vs. diodes: no specific bias polarity \rightarrow can be exploited to dynamically flip signal between delta and sum mode
- Can detect DC changes
- Bias voltage and polarity can be used to:
 - high-frequency difference controlled via regulated DC voltage
 - Noise is an issue → presently batteries (proven* lowest noise impact)
 - simple calibrate for phase difference:
 - Put one PD bias voltage to zero measure signal with the other and vice versa and adjust for the phase difference (externally e.g. mechanically stressing optical fiber)

- MSM-Photodetector scheme well established
- Small spot size makes 2x2 matrix difficult to tune/operate remotely
 → decided early to focus on robust delivery system studies
- Proposal of using imaging-fiber optics type synchrotron-light delivery system:
 - Can use spatially distributed, discrete MSM photodetectors
 - Can keep sensitive, less radiation tolerant, expensive equip. in alcoves
 - Some operational experience...

FPM² Laboratory Test Setup II/III

Lab test setup with pulsed Laser source:

- ASLS 3GeV Main Ring CTF3 – Combiner Ring "La Favella" Streak Camera Lab **Optical Diagnostic Beam-Line** ODB Spectroscopy x 10 1400 1.8 1200 1.6 1000 1.4 rel. intensity [a.u.] 1.2 Intensity 800 1 0.8 600 0.6 400 0.4 0.2 200 200 300 600 700 800 400 500 900 1000 300 400 500 600 700 200 800 Wavelength (nm) wavelength [nm]
 - mm-size beam spot size >> PD
 - re-focus with x40 micros. optic
 - 13 GHz amplifier
 - \rightarrow mW-level integrated light power

- compatible power (spectrum)
- stage I: prototyping at ASLS (S/N ratio, resolution, systematics, ...)
- stage II: @ CTF3
 (tbd: 12 GHz BW, systematics, ...)

- Photodetector sensitivity → about 5-7 mW light impinging on the the detector
 However: beam spot size >> active detector size (few mm vs. 200 um)
 - needs re-focusing and/or amplification (both available now)
 - delta signal is expected to be on the percent level of this carrier signal
 - \rightarrow aim at 200 (500) mV (amplified) peak signal

Some slides on EO Optical-BPM and classical RF-based HT Upgrade Activities

Refraction in birefringent crystals depends on ex. electrical field:

• Typically the *'half-wavelength voltage* V_{π} ' is used to describe electro-optical modulators, i.e. the voltage required to achieve destructive interference:

$$\Delta \phi := \pi \rightarrow V_{\pi} = \frac{\lambda}{r_{33}n_e^3 - r_{13}n_o^2} \cdot \frac{d}{L}$$

- wavelength λ , crystal height *d* and length *L* are basically free parameter
- Large variety of crystals (KTP, GaAs, ...), I chose:
 - Lithium Niobiate (LiNbO₃) 5x5x15 mm³
 - · common and the 'standard' in telecommunication
 - typ. (only) low V_{π} ~6-10 V available
 - Lithium Tantalate (LiTaO₃) 3x3x15 mm³
 - more robust but similar to LiNbO3 or Al₂O₃

	Lithium Niobate	Lithium Tantalate
	LiNbO ₃	LiTaO ₃
Density:	4.65 g/cm ³	7.46 g/cm ³
Melting point:	1257 °C	1650 °C
Thermal expan. [10 ⁻⁶ K ⁻¹]	15, 5	16, 4
Thermal cond. [W/mK ⁻¹]	5.6	4.6
Damage threshold	250 MW/cm ²	500 MW/cm ²
ε _r @ 100kHz	ε _⊥ 85, ε _∥ 29	ϵ_{\perp} 54, ϵ_{\parallel} 43
transmission range [nm]	350-5500	400 - 5500
refractive index (@589 nm, 25°C & @633 nm, 25°C)	n _o 2.30, n _e 2.21	n _o 2.19, n _e 2.18
EO-coefficient* [pm/V]	$r_{13} = 9.6, r_{33} = 30.9,$ $r_{22} = 6.8, r_{51} = 32.6$	$r_{13} = 8.4, r_{33} = 30.5,$ $r_{22} = 20$
Non-linear EO coeff. [p/m/V] @ 1064 nm	d ₃₁ = -4.5, d ₃₃ = -0.27, d ₂₂ = 2.1	d ₂₂ = 2.0, d ₃₁ = - 1, d ₃₃ = -21

Refractive Index Dependence on Wavelength

• LiNbO₃ – gain control possible but limited to factor ~ 10

... thus acquired 530 nm (green) and 1550 um (infra-red) laser for testing this.

Robustness w.r.t. Radiation Damage

- LiNbO₃ and LiTaO₃ are related to Al_2O_3 , known to be fairly radiation hard
- Nevertheless, should get more precise numbers to assess long-term damage
 - Radiation damage level on LiTaO₃ according to [1,2]:

Conversion factor tbc. but '10¹⁷ Ar⁺⁺' is probably much more than 100 kGy

¹: C. J. Wetteland et al., "Radiation Damage Effects in [..] LiTaO3 Single Crystals", Mat. Res. Soc. Symp. Proc. Vol. 504, 1998 ²: R. H. West, S. Dowling, "Effects in [..LiTaO3..] Exposed to Radiation from a Flash X-Ray Source", Royal Military College of Science, IEEE TRANSACTIONS ON NUCLEAR SCIENCE vold. 41, #3, 1994

- Two stage demonstration:
 - Re-use existing MSM-PD-based light-electrical conversion scheme
 - two options: optical (I) or electrical (II, favoured) hybrid
 - Sensitivity: 1% beam movement \leftrightarrow 3V signal, resolve a fraction of this
 - Michelson interferometer with EO-crystal as trans. Modulator
 - EO-crystal as amplitude modulator per pick-up
 → insensitive/lose laser phase information → turns out to be more robust...
 - Setup ready, need to redo scans (with low-intensity meter, DSO constraints)
 - Bandwidth: commercial LiNbO3 20 GHz EO-Modulator
 - to be tested: S/N ratio, reflections (limited by coupler)
 - Mini-Laserlab nearly finished \rightarrow test before Christmas!
 - After that: design of purely-optical BPM pick-up (2013+?)

Test & Evaluation Programme Sensitivity Setup – Phase Modulation-based

Mach-Zehnder or Michelson Topology

- Utilises wave-front phase interference to suppress common mode signal However:
 - Need to maintain polarisation within (larger) structure
 - More delicate/less robust w.r.t. alignment, stability of mirrors and split ratio
 - would need to be done locally close to the pick-up for re-tuning (remote motorisation, local instrumentation, ...yikes)

LiNbO3 Sensitivity Setup – Phase Modulation-based here: < 1mW, 630 nm Laser

- E.g. polarisation (\rightarrow pockels cell) or phase retardation (Fabry-Perot) +U $\mathsf{U}_{_{\text{bias}1}}$ In the tunnel ... EO crystal 1 ectric/mechani few m to km of -40 mm RF out single-mode fiber S p-beam σ õ EO crystal 2 Ρ l_{bias2} e.g. ±10∨ Simple \rightarrow robust design: somewhere else next to DAQ
 - no setup or retuning of electrical/mechanical parts in tunnel
 - complexity kept at DAQ
 - Leverage same MSM-detector desgin as for synch-light based BPM
 - Phase and amplitude matching possible via $\rm U_{_{o}}$ and $\rm U_{_{bias}}$
 - Less radiation issues, could consider cryo-cooling MSM detectors
 - Could daisy-chain/mix multiple pick-ups on the same two optical fibres

Test-setup:

- Aim: confirm bandwidth and achievable S/N ratio
- Basically this is a standard telecommunication setup (modulo fiber length)
 - Reproduce bandwidth
 - Explore limits of link, noise sources etc.

- DSO: > 1 mW \rightarrow operation in dedicated lab and armoured fiber mandatory
 - Don't have one on the Prevessin site ...

LiNbO3 Bandwidth Setup Behold: BI-QP's New Laser-Lab

 Requirement from our DSO: light-tight confinement, only tool-based access allowed, laser cut if lid is opened

Courtesy Philippe Lavanchy; awaits DSO approval

- Creating an optical hybrid translates to the same classical RF hybrid issues
- EO-modulator basically are (un-)matched micro-/strip-line structure
- Critical aspects:
 - Impedance matching
 - Geometry \rightarrow tricky but similar to our other RF pick-ups
 - Larger dielectric loss-tangent due to $\epsilon_r(t) \sim 80$ (BPMs strip-lines are in vacuum)
 - Insulation
 - hasn't been demonstrated (yet) that we can achieve -40dB or better

CERN

Alternatives... Electrical → Optical BPM Signal conversion II/II

- Directly connect em-BPM \rightarrow eo-modulator \rightarrow fiber \rightarrow MSM detector
- Compared to BPMs, Eo-mod. typ. have badly matched strip-lines

- Not a design criteria for telecommunication (digital signals)
- Reflections may possibly perturb measurements of consecutive bunches
- If not done properly probe laser noise (typically 1%) may propagate and superimpose onto the beam signal
 - \rightarrow balanced detector scheme may mitigate this to some extend.

Independent of the electro-optical detection scheme:

- Sensitivity and signal levels are given by crystal geometry which can be adjusted to the expected maximum/minimum bunch intensity, however:
 - A priori static sensitivity, i.e. easy no optical gain switching
 - with added complexity: add more than one crystal per pick-up

 - gain adjustment would need to be addressed on the analog front-end
 - Radiation damage effects of fibers vs. cables need to be further assessed

Amplitude Modulation-based Scheme Optical-BPM

- All-Optical-BPM layout scheme, re-use conceptually LHC BPM design:
 - Keep the same body, keep external button form-factor

 Impact of EO-crystal (dielectric stup) on machine impedance probably small but should be re-checked by FE-EM simulation

- Mechanical design & construction in 2012/13
 - Need to investigate crystal clamping and fiber-to-feed-through alignment
 - Possible prototype installation in LS1?

- Split signal into manageable bandwidths and treat them separately and recombine them in the end
 - Attenuate/amplify bands with expected strong/weak power contributions
 - Post-processing (de-convolution) probably mandatory (difficult to passively match each part)
- For the time being: aim at two hybrids
 - H-9 or H-183 working from 0.002 (0.003) \rightarrow 2 (3) GHz, no amplification
 - New broad-band hybrid \rightarrow 8 GHz, ~2GHz HP and 40 dB amplification

- Dynamic range of oscilloscope <40 dB vs. bunch peak n_b/σ_t variation > 60 dB
 - adaptive signal attenuation mandatory
 - Requires tlc at n-GHz range and given power levels
 - matching, particularly insulation and gain calibration

- Some high-frequency SP67 and DPDT switches being evaluated
 - Have <20 ps pulse generators (PSPL¹) but looking into robust solutions that can be placed permanently in tunnel (radiation issues):
 - Initially avalanche generator (<1 ns), NLTL
 - 50 ps generator based on fast comparator ADCMP572BCPZ, ADCMP580BCPZ³

1: James R. Andrew, "Picosecond Pulse Generation Techniques and Pulser Capabilities ", Application Note AN-19, PSPL, 2008 2: Linear Technologies (p.93 & 94): http://www.linear.com/pc/downloadDocument.do?navId=H0,C1,C1154,C1009,C1028,P1219,D4138 3. Analog Devices: specs at www.analog.com

- track 1 bunch over 1000 turns
 - \rightarrow now need to track batches of bunches over 1000 turns
 - Requires more memory \rightarrow more post-processing to reduce data rates
- measure 'm=1' HT modes (~1 GHz) → need to go higher for 'm>1' modes
- 'Usual suspects' for suppliers for systems with f_{bw}>2 GHz bandwidth, all providing systems with >6 GHz bandwidth but with difference w.r.t. RAM:
 - Agilent (DAQs and Scopes): up to 1 GB/channel
 - LeCroy (Scopes only): up to 256 MB/channel
 - Tektronix (Scopes only): up to 128 MB/channel
 - Guzik (DAQ): up to 32 GB/channel
 - Huge buffers simplify triggering and data selection but also make smart memory management, online and automatic post-processing mandatory, i.e.
 "1 GB of data requires >10 seconds for read-out with standard Gigabit Ethernet link no post processing yet" we ideally desire an answer/pre-analysis every ~30 seconds"
 - technically feasible but should be kept in mind during design stage

LHC Injectors Upgrade

- Gigabyte Sampling Buffers: The aim is not to systematically process, analyse and store the whole buffer but to allow data-mining in case of instabilities:
 - Use-case example 1: Unstable bunches or batches are not known in advance. However, the guilty ones and timing can be identified with e.g. FastBCT and other instruments after the instability occurred
 → can be used to narrow the range of the data to be retrieved
 - Use-case example 2: instrument could be used by several users with different diagnostics indicators at the same, e.g. 'user 1' monitors few bunches over maximum number of turns while 'user 2' acquires the first 500 full turns looking for bunch-by-bunch oscillations but at a reduced sampling frequency.
 - Use-case example 3: do not just fit the minimum required but keep specification open to allow future upgrades and R&D, i.e. present HT monitor was designed to measure Q' but not as instability monitor, larger buffer and bandwidth allowed the exploitation for other beam studies outside the scope during the design.

(N.B. small difference in price for going from 2.5 \rightarrow 6 GHz, ~ 25% of total costs)

- Exploitation using scopes from Agilent, LeCroy and Agilent are time-proven but ultimately limited in available sampling memory, post-processing and integration (M\$ Windows oscilloscopes, Gigabit Ethernet data transfer, ...)
- Interesting new candidate: Guzik's GSA & ADC 6000 Series
 - 4 (2, 1) channels
 @ 4 (6.5/8, 13) GHz

- 16 (32) GB/ch sampling buffer \leftrightarrow monitoring of 1.6 s of beam data!
- Various on-line processing that could be exploited to pre-select data:
 - FFT & DFT (per bunch, mag. Average, sub-frequency ranges), rise-time, bunch-width (FWHM), bunch statistics (average, r.m.s., min., max. signal), number of rising/falling edge → possible HT mode detection already in HW?
 - Functionality to be explored → requested demo system for evaluation (closed FPGA firmware for the time being but user-specifc space under investigation)
- Some ongoing negotiations with the manufacturer:
 - Price is important but similar to others if normalised to sampling buffer
 - PCIe bridge integration aspects: what would need to be done by us?

Summary and Outlook

- Present limitation hierarchy:
 DAO hybrid transmission (cable/ont)
 - $DAQ \rightarrow hybrid \rightarrow transmission (cable/optic) \rightarrow new/better pick-up (response)$
- Several options addressing immediate and future performance aspects
 - Synchrotron-Light higher-bandwidth, better adapted to ultimate bunch charge
 - Electro-Optical Pick-up higher-bandwidth, easier signal transmission
 - Improve existing system by classical RF means (pushing the limit)
 - Upgrade plans/wishes/target:
 - 2012/13: upgrade of the DAQ system for HT (LIU budget confirmed)
 - aim: higher bandwidth, more memory, more in-built post-processing
 - Can be re-used whatever the other upgrade decisions
 - Mid-end 2012:
 - install and test hybrid RF-hybrid scheme
 - Synch-Light BPM Prototype at ASLS \rightarrow translate this to CTF3
 - EO BPM design?
 - LS1?!? New Pick-up Prototype Installation:
 - Synch-Light based HT monitor (CTF3/LHC more suitable)
 - EO BPM prototype installation in SPS

Reserve slides

Head-Tail based Q' Diagnostic I/II

- Instability can be exploited to give in estimate on Q' in the first place:
 - track two slices, 'head' and 'tail' in the bunch distribution $A_{HT}(n) \propto \sin(\psi_{HT}(n))$ (tune: Q, long slice position: τ, synchrotron frequency: ω_s , turn: n)

bunch position

long.