

# **LHC Orbit-FB Bandwidth**

## **– Margins, Limits and Caveats –**

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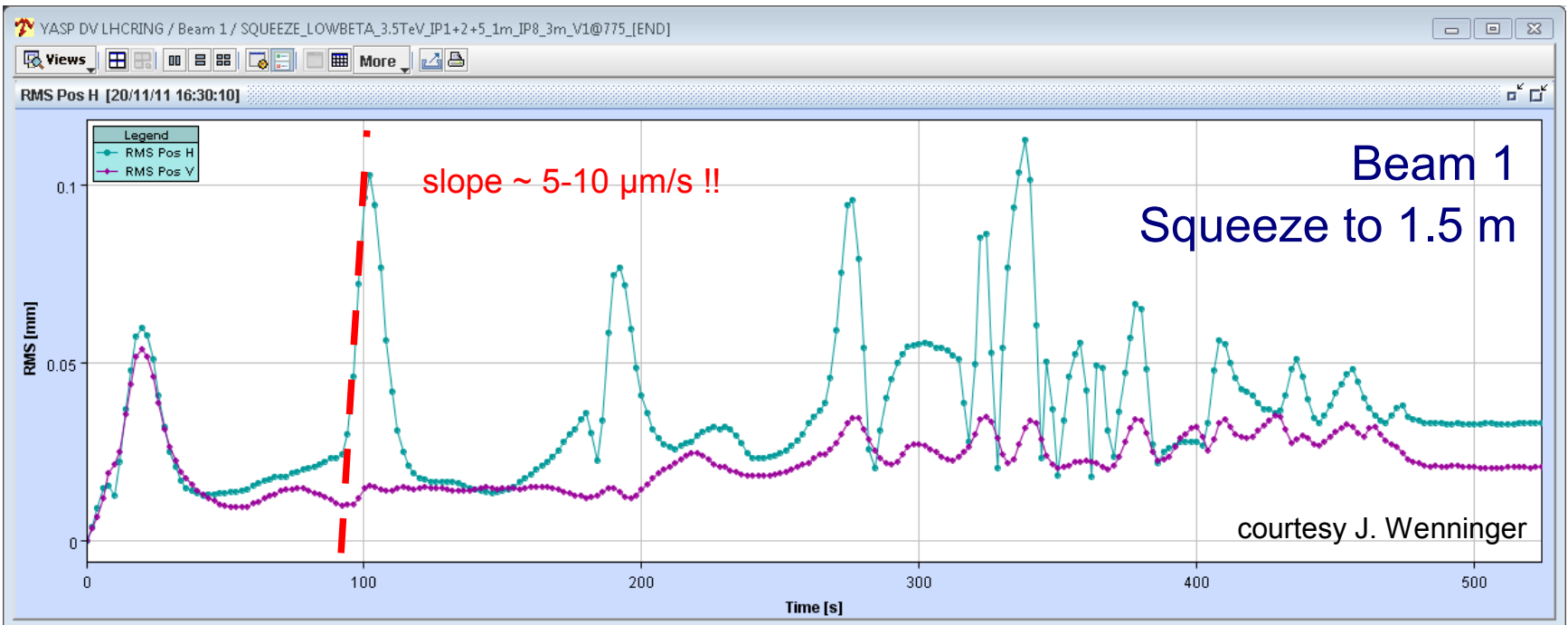
### **Some references:**

<http://cern.ch/AB-seminar/talks/AB.Seminar.rst.pdf> (CERN-AB-2007-049)

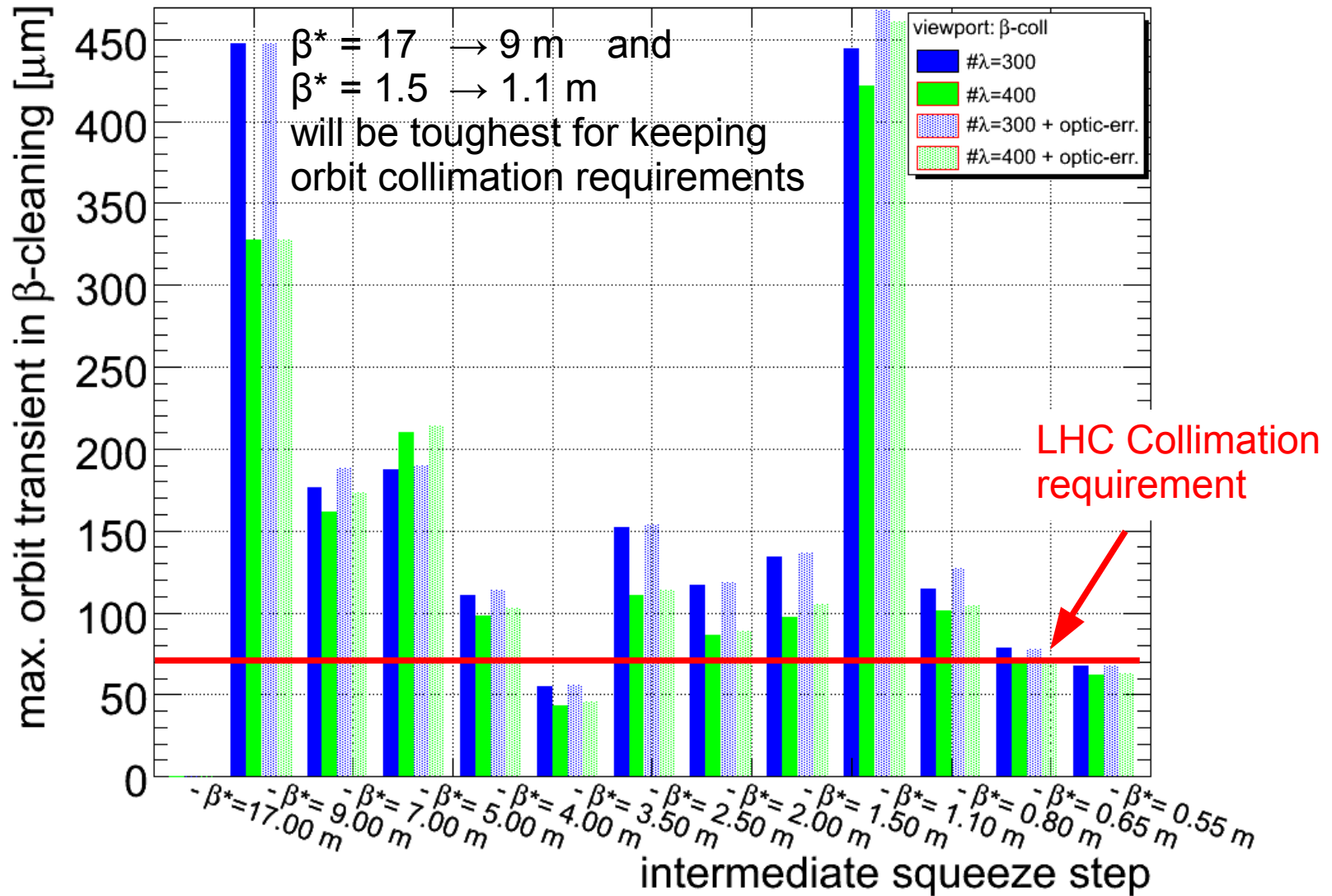
[http://lhccwg.web.cern.ch/lhccwg/Meetings/2007/2007.10.23/2007-10-23\\_LHCCWG-FAULTY\\_BPM.pdf](http://lhccwg.web.cern.ch/lhccwg/Meetings/2007/2007.10.23/2007-10-23_LHCCWG-FAULTY_BPM.pdf)

LHC-BPM-ES-0004 rev. 2.0, EDMS #327557, 2002,

- Orbit transients at the ‘matched points’ (where we pass a matched optics)
  - issue for tight collimators.
  - Transients are due to mismatch of the Xing and separation bumps due to optics interpolation between matched points.
  - LHCb Xing bump of 250 urad is driving source !
- Simplest cure: faster orbit FB for better damping...
  - ... but needed proof-of-feasibility prior to counting on this for 2012!



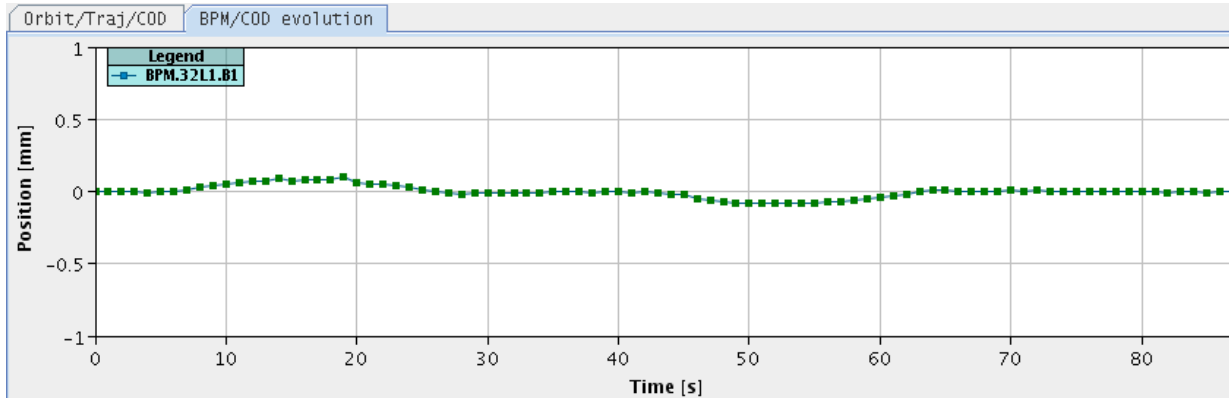
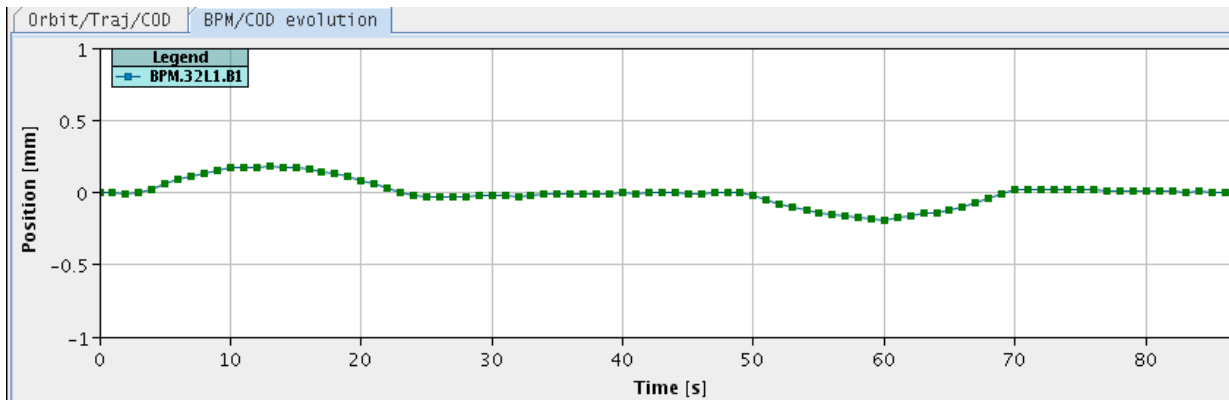
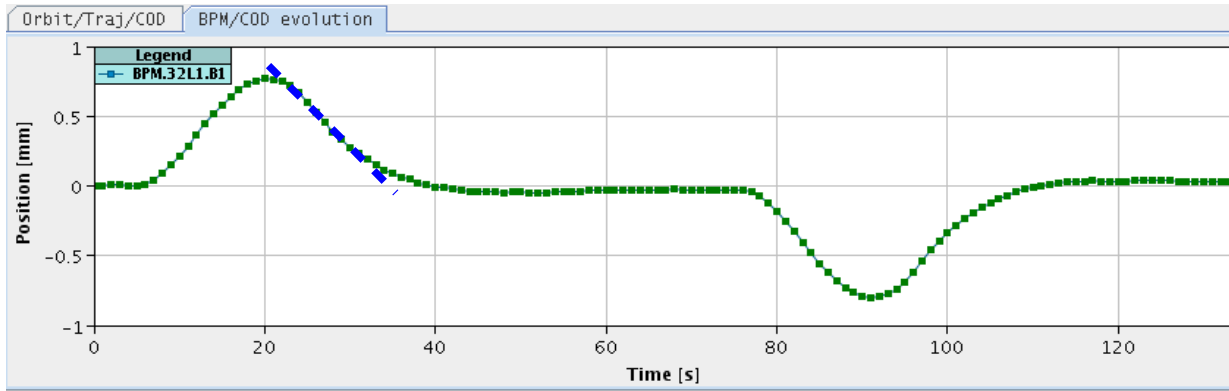
# Not a new Effect – has been studied before: Transient in Collimation Insertion vs. Squeeze Step



- Makes a fast orbit feedback practically mandatory during squeeze and nominal beam operation.



# OFB Bandwidth – Pre-Flight Checks @ 3.5 TeV Response to a kick



Default Bandwidth:

Setup: OL BW = 10 Hz @ 5μm/s

→ CL BW = 0.025 Hz@3.5 TeV

Measured:

$tr_{10-90\%} \approx 15s \leftrightarrow BW \approx 0.023 \text{ Hz}$

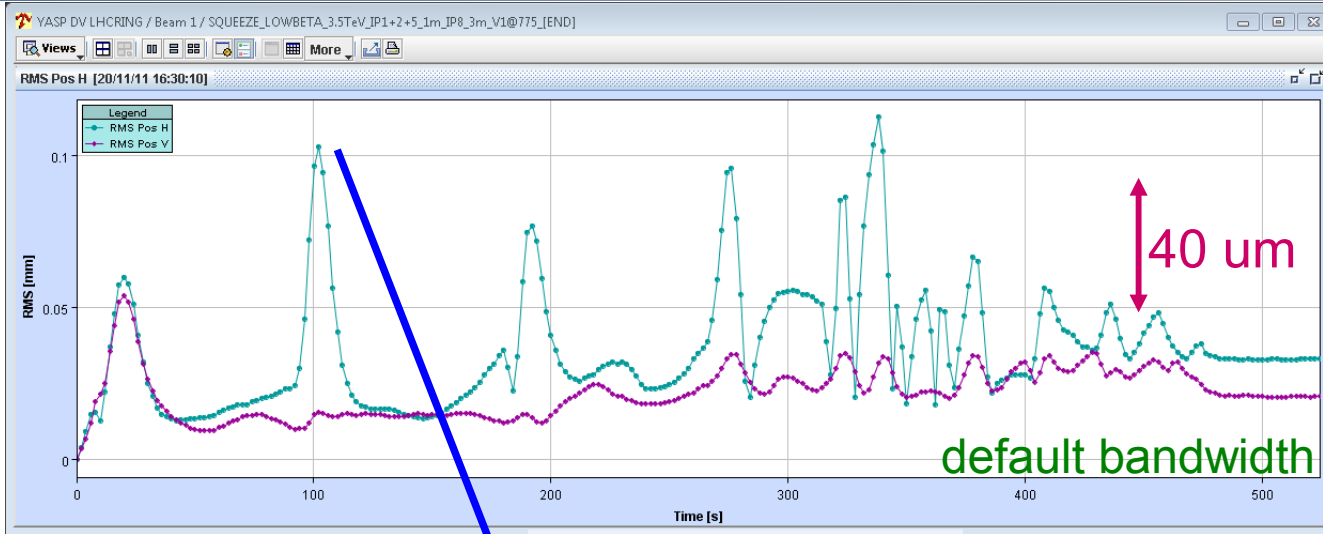
CL Bandwidth x 5

CL Bandwidth x 10

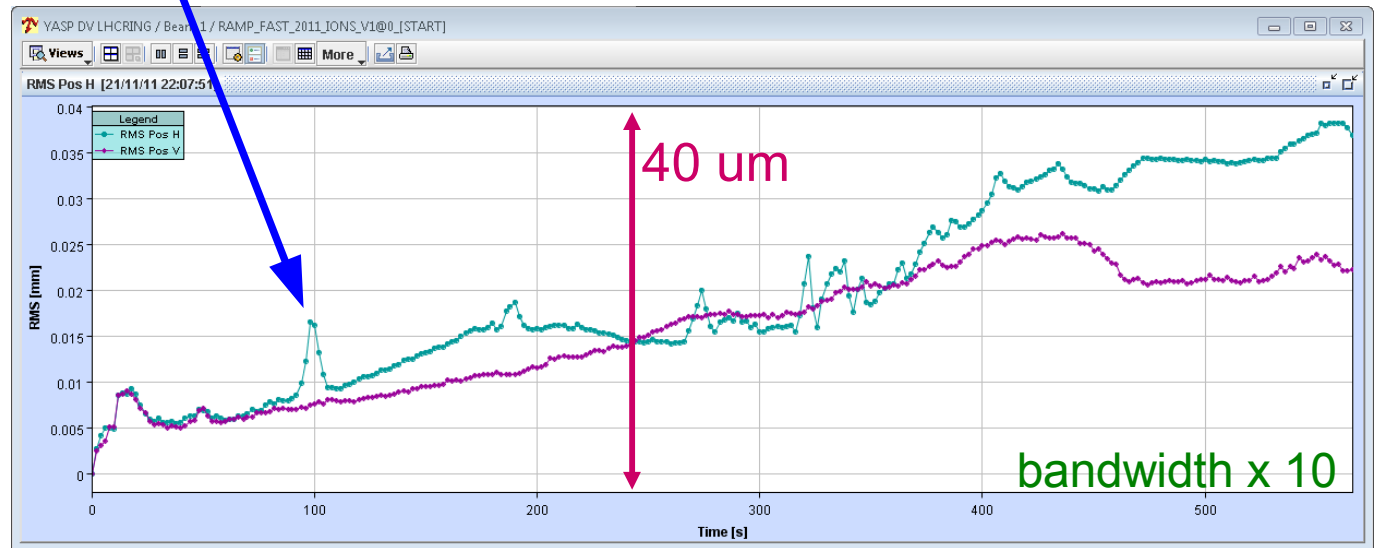
+ linear increase  
+ no sign of ringing  
→ there is some margin!!

N.B. IR 1+5 @1m, IR 8 @ 3 m

# Difference of Bandwidth – The Good...

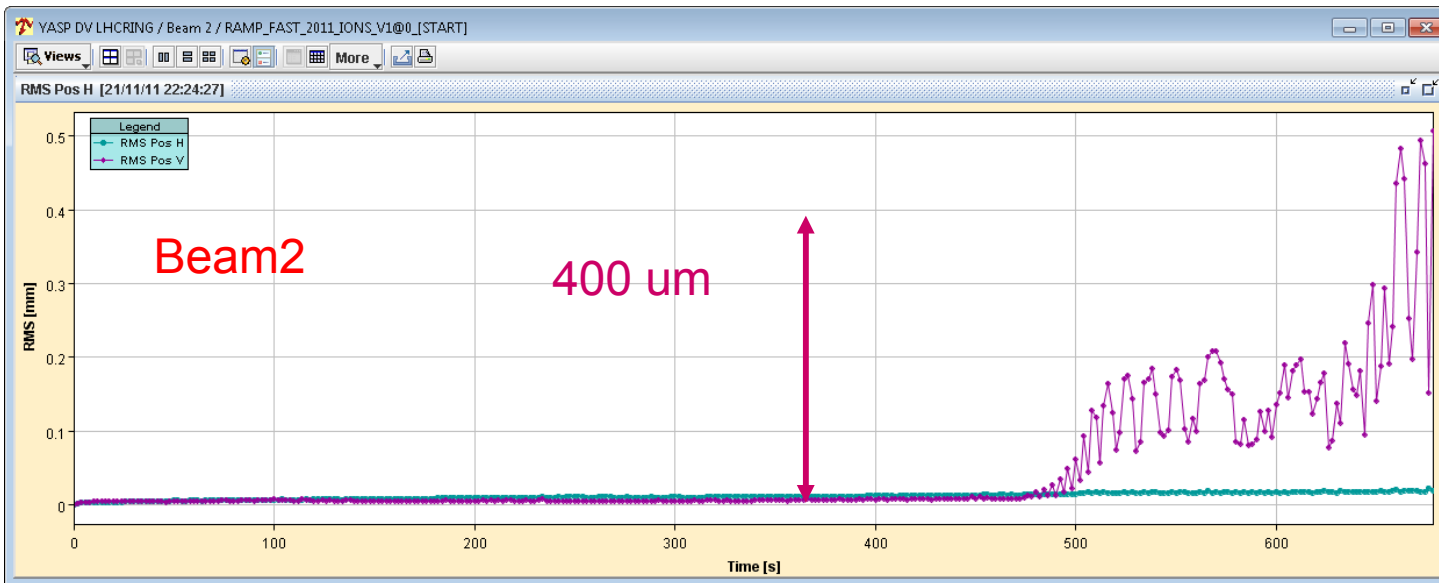
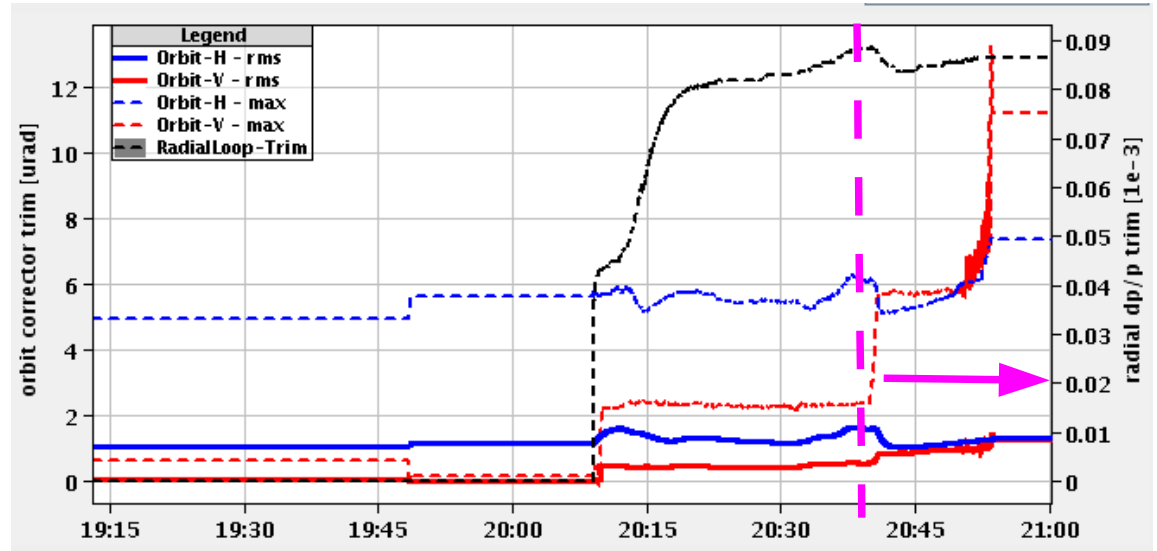


factor 10 reduction



- Great linear/design performance... *don't count chicken until they are hatched!*

# Difference of Bandwidth – The Bad ... Squeeze in IP2 $\beta^* = 3 \text{ m} \rightarrow 1 \text{ m}$



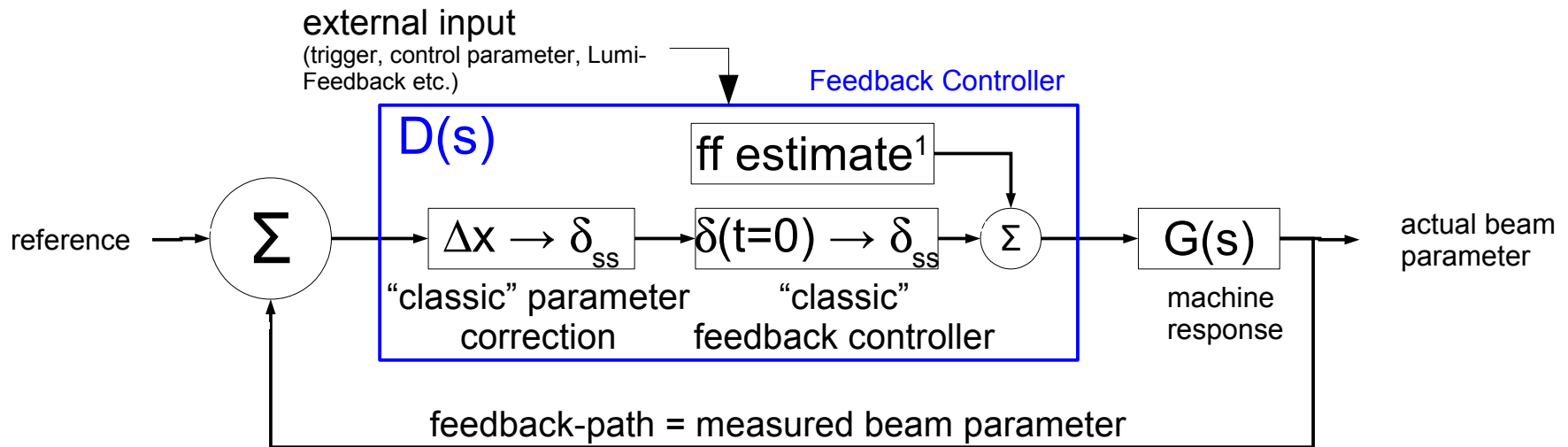
What happened?

- 'Divide and Conquer' feedback controller design approach:

- 1 Compute steady-state corrector settings  $\vec{\delta}_{ss} = (\delta_1, \dots, \delta_n)$  based on measured parameter shift  $\Delta x = (x_1, \dots, x_n)$  that will move the beam to its reference position for  $t \rightarrow \infty$ .
- 2 Compute a  $\vec{\delta}(t)$  that will enhance the transition  $\vec{\delta}(t=0) \rightarrow \vec{\delta}_{ss}$
- 3 Feed-forward: anticipate and add deflections  $\vec{\delta}_{ff}$  to compensate changes of well known and properly described sources

space domain

time domain



- (N.B. here  $G(s)$  contains the process and monitor response function)

- Effects on orbit, Energy, Tune, Q' and C<sup>-</sup> can essentially cast into matrices:

$$\Delta \vec{x}(t) = \underline{R} \cdot \vec{\delta}(t) \quad \text{with} \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi Q)} \cdot \cos(\Delta \mu_{ij} - \pi Q) + \frac{D_i D_j}{C(\alpha_c - 1/\gamma^2)}$$

matrix multiplication

- LHC matrices' dimensions:

$$\underline{R}_{orbit} \in \mathbb{R}^{1070 \times 530} \quad \underline{R}_Q \in \mathbb{R}^{2 \times 16} \quad \underline{R}_{Q'} \in \mathbb{R}^{2 \times 32} \quad \underline{R}_{C^-} \in \mathbb{R}^{2 \times 10/12}$$

- control consists essentially in inverting these matrices:

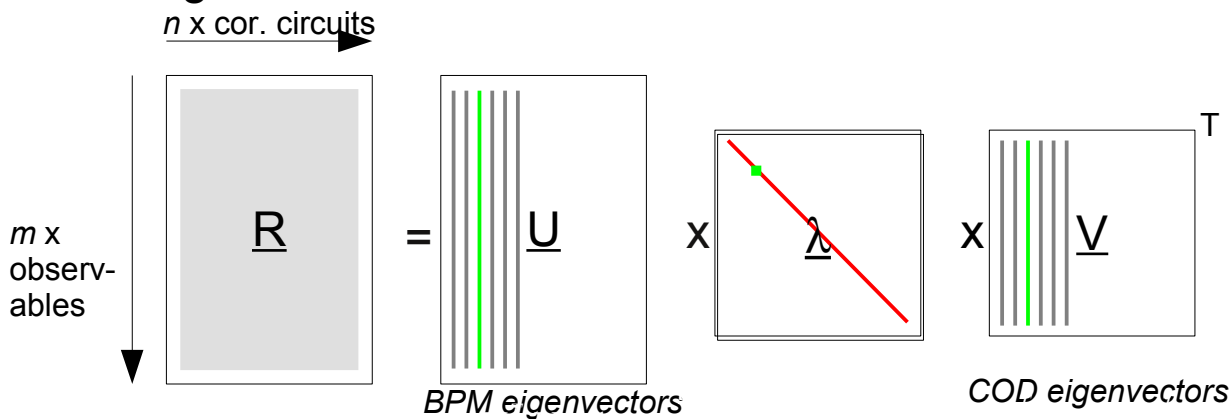
$$\|\vec{x}_{ref} - \vec{x}_{actual}\|_2 = \|\underline{R} \cdot \vec{\delta}_{ss}\|_2 < \epsilon \rightarrow \vec{\delta}_{ss} = \tilde{R}^{-1} \Delta \vec{x}$$

- Some potential complications:

- Singularities = over/under-constraint matrices, noise, element failures, spurious BPM offsets, calibrations, ...
- Time dependence of total control loop → “The world goes SVD....”



Linear algebra theorem\*:



eigen-vector relation:

$$\lambda_i \vec{u}_i = \underline{R} \cdot \vec{v}_i$$

$$\lambda_i \vec{v}_i = \underline{R}^T \cdot \vec{u}_i$$

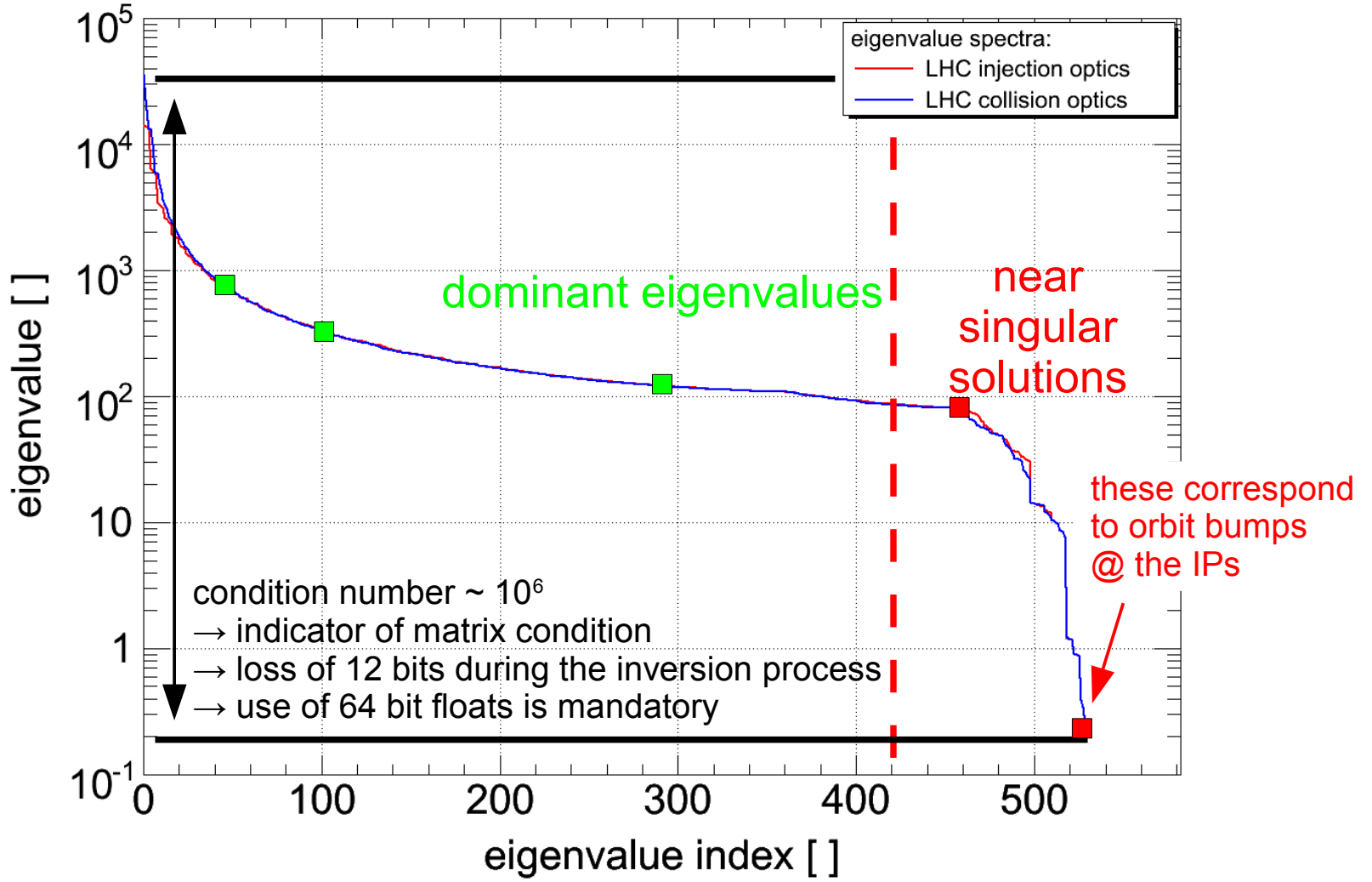
- though decomposition is numerically more complex final correction is a simple vector-matrix multiplication:

$$\delta_{ss} = \tilde{R}^{-1} \cdot \Delta \vec{x} \quad \text{with} \quad \tilde{R}^{-1} = \underline{V} \cdot \underline{\Lambda}^{-1} \cdot \underline{U}^T \quad \Leftrightarrow \quad \delta_{ss} = \sum_{i=0}^n \frac{a_i}{\lambda_i} \vec{v}_i \quad \text{with} \quad a_i = \vec{u}_i^T \Delta \vec{x}$$

- numerical robust, minimises parameter deviations  $\Delta x$  and circuit strengths  $\delta$
- Easy removal of singularities, (nearly) singular eigen-solutions have  $\lambda_i \sim 0$ 
  - to remove those solution: if  $\lambda_i \approx 0 \rightarrow '1/\lambda_i := 0'$
  - discarded eigenvalues corresponds to solution pattern unaffected by the FB**

\*G. Golub and C. Reinsch, "Handbook for automatic computation II, Linear Algebra", Springer, NY, 1971

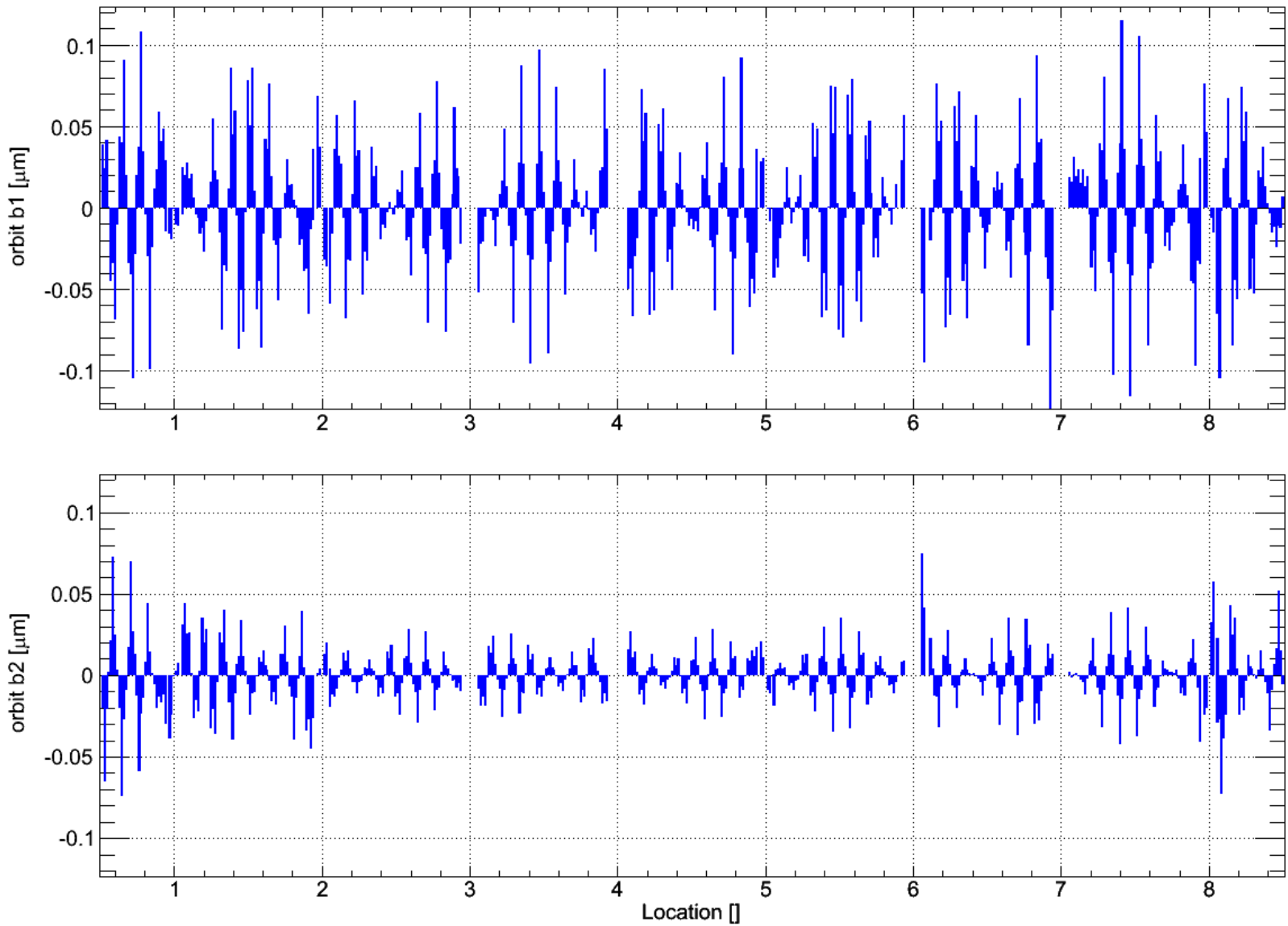
Eigenvalue spectra for vertical LHC response matrix using all BPMs and CODs:





# Space Domain:

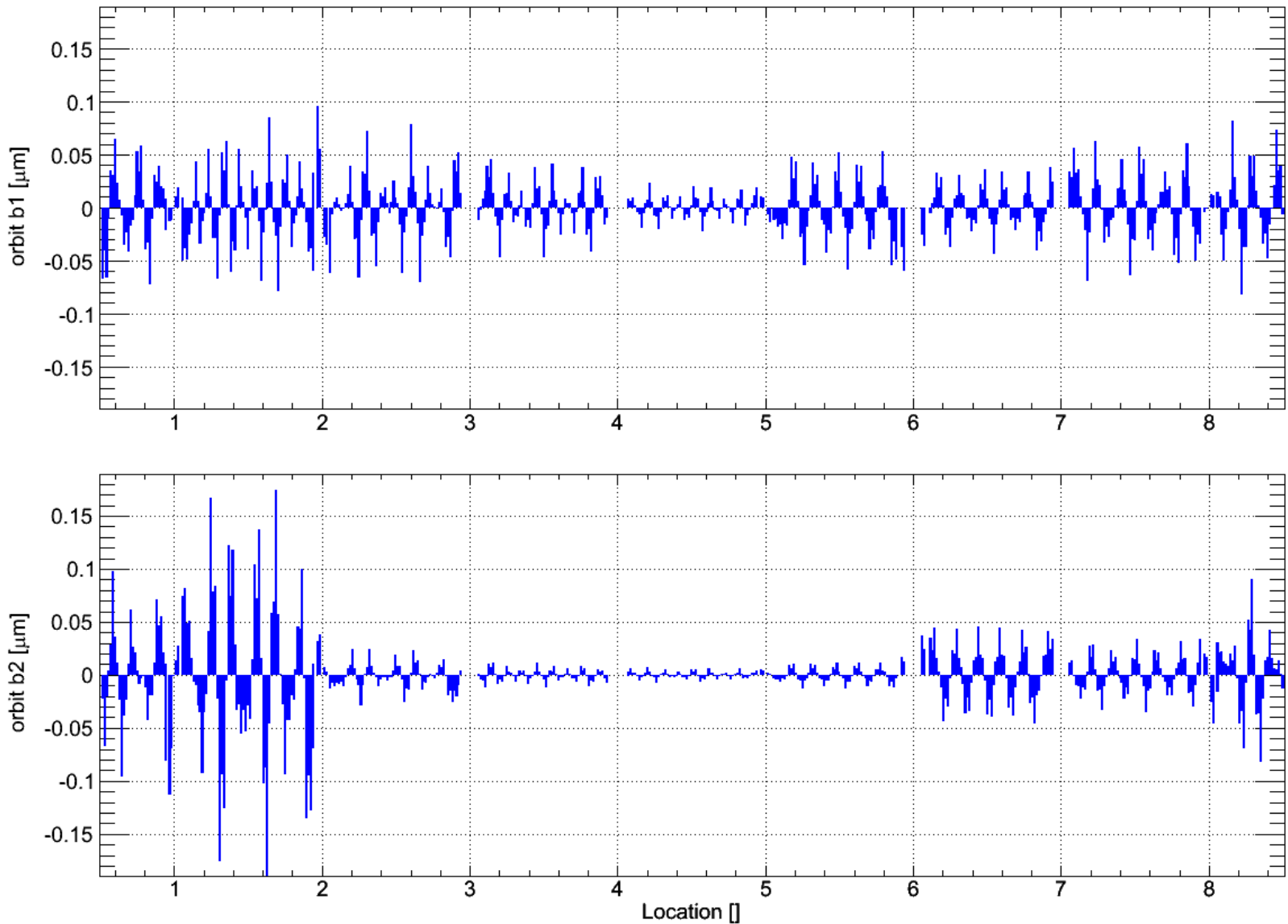
## LHC BPM eigenvector #50 $\lambda_{50} = 6.69 \cdot 10^2$





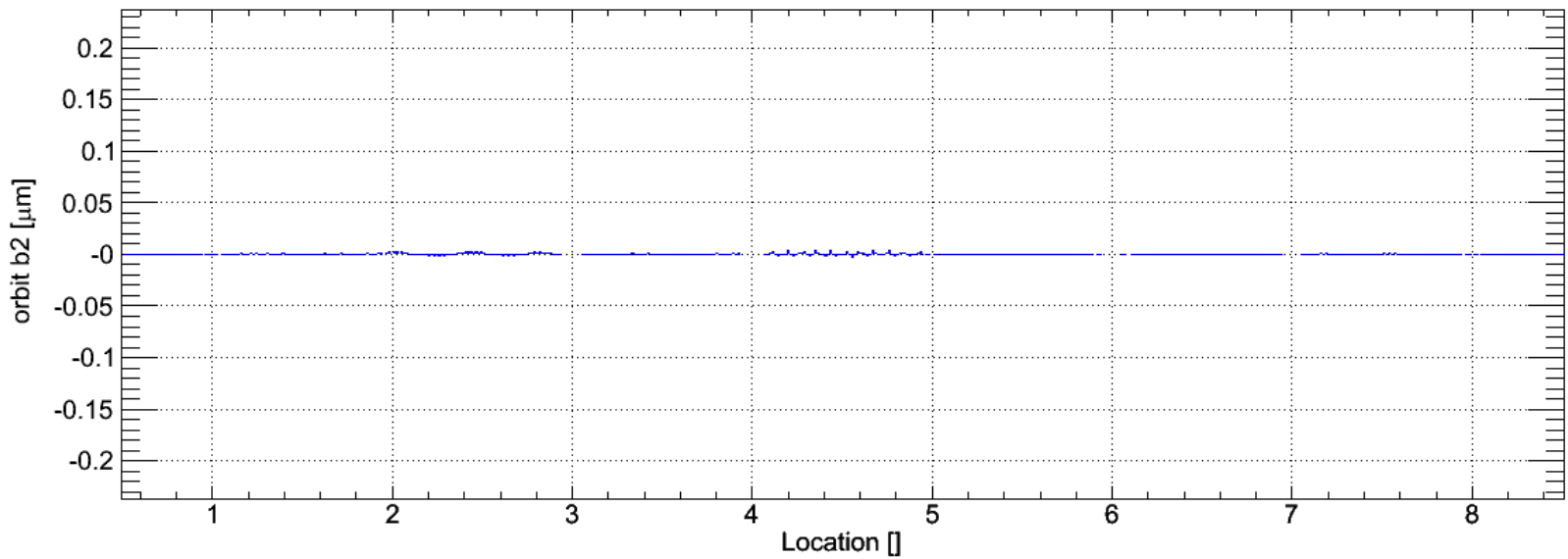
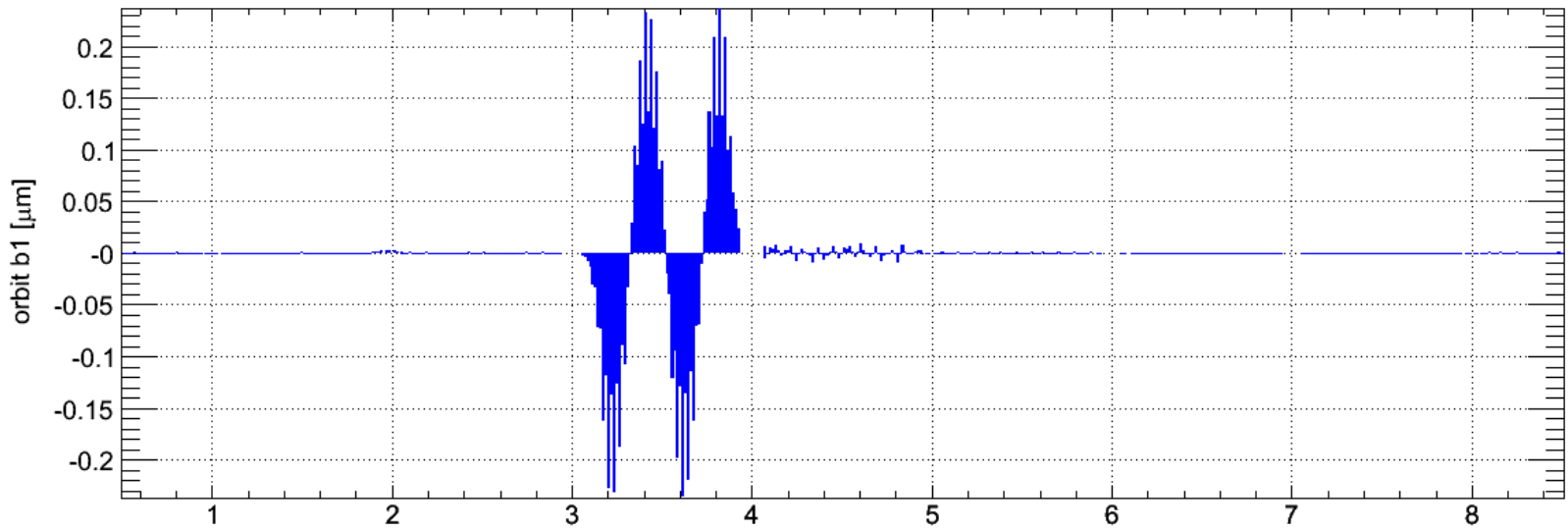
# Space Domain:

## LHC BPM eigenvector #100 $\lambda_{100} = 3.38 \cdot 10^2$





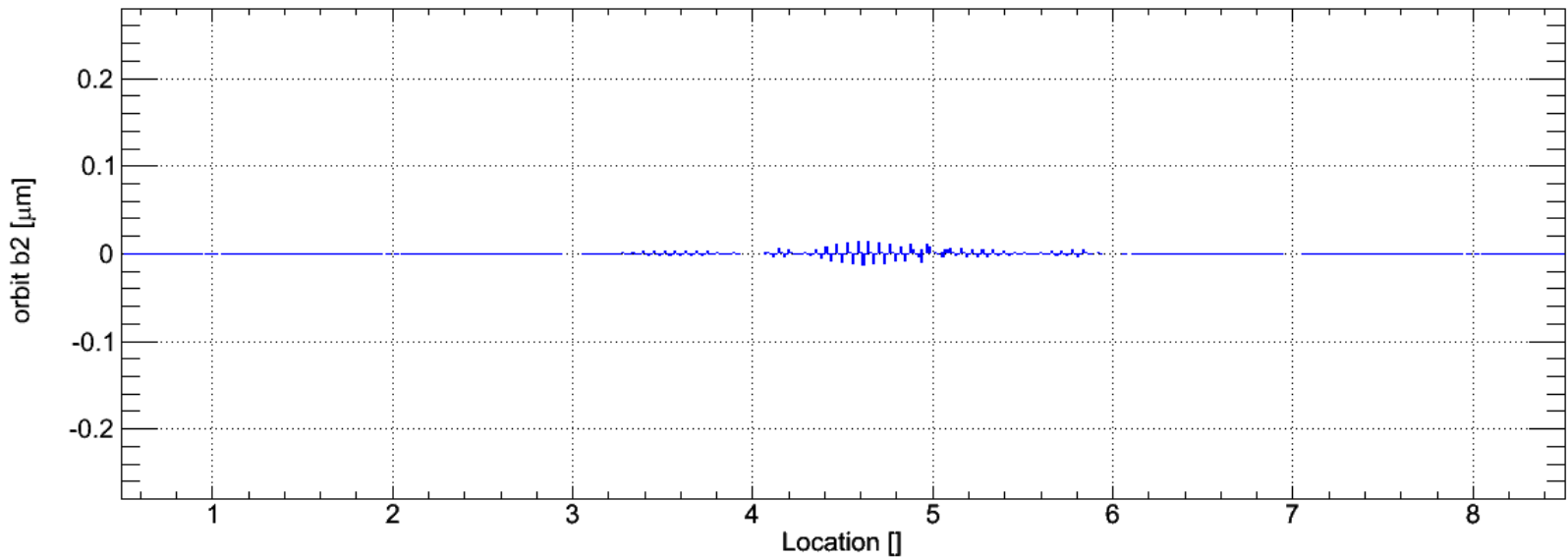
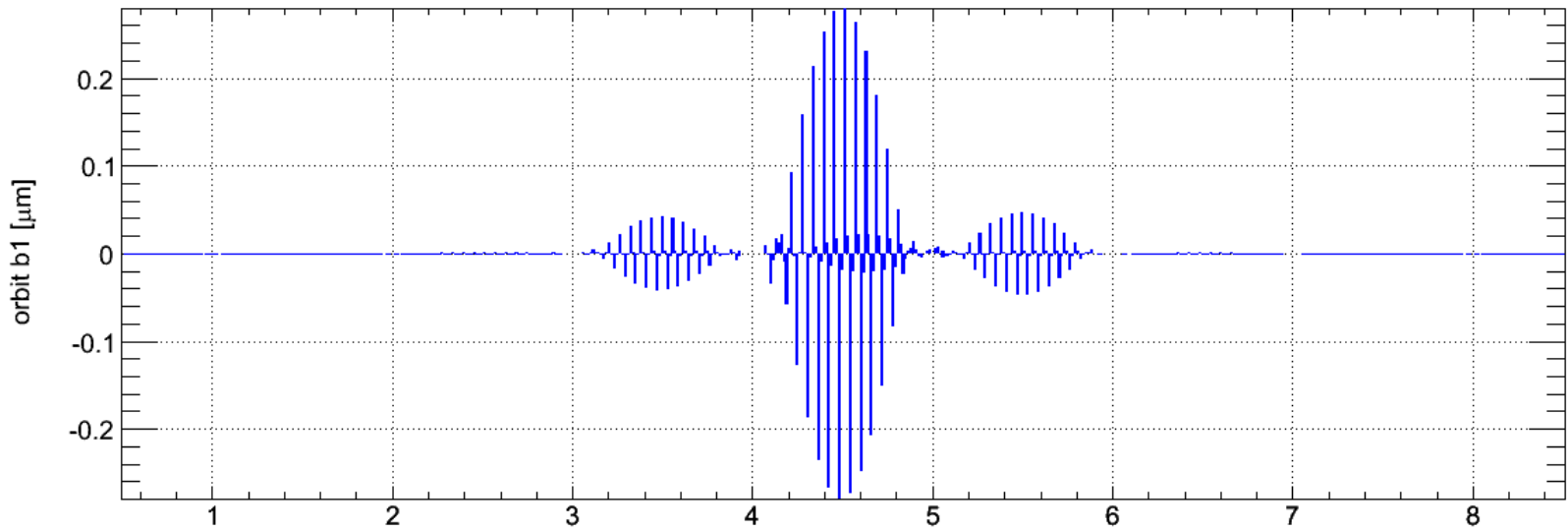
# Space Domain: LHC BPM eigenvector #291 $\lambda_{291} = 2.13 \cdot 10^2$





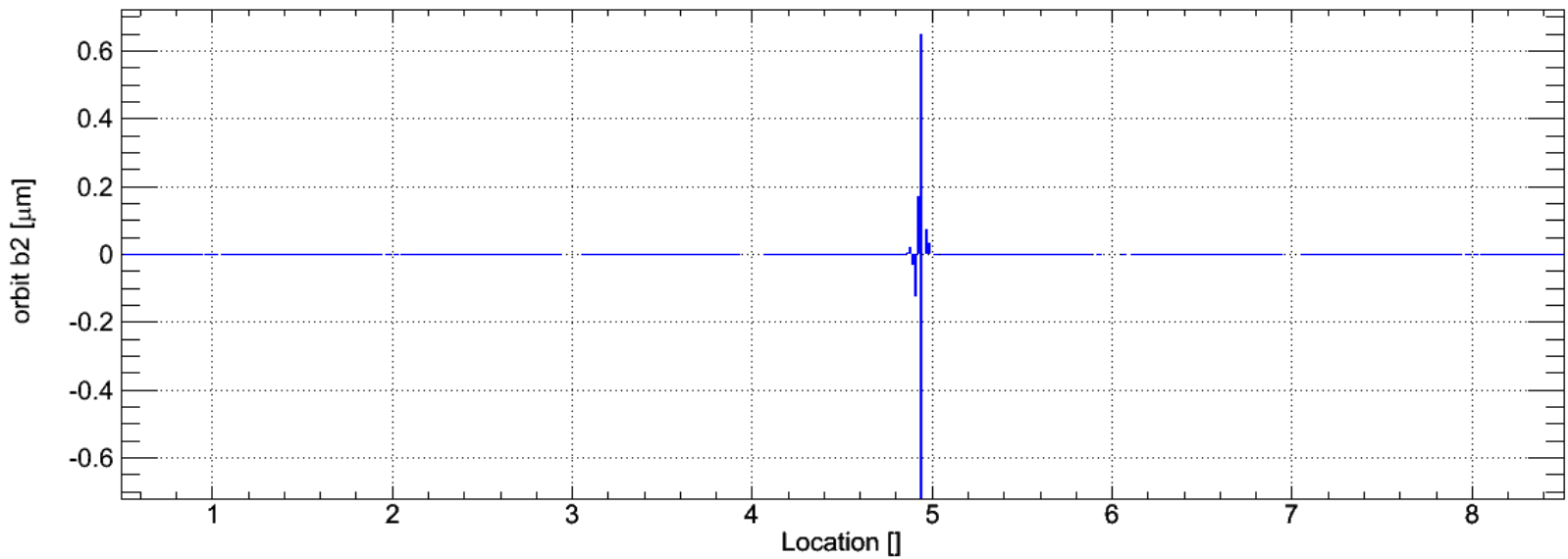
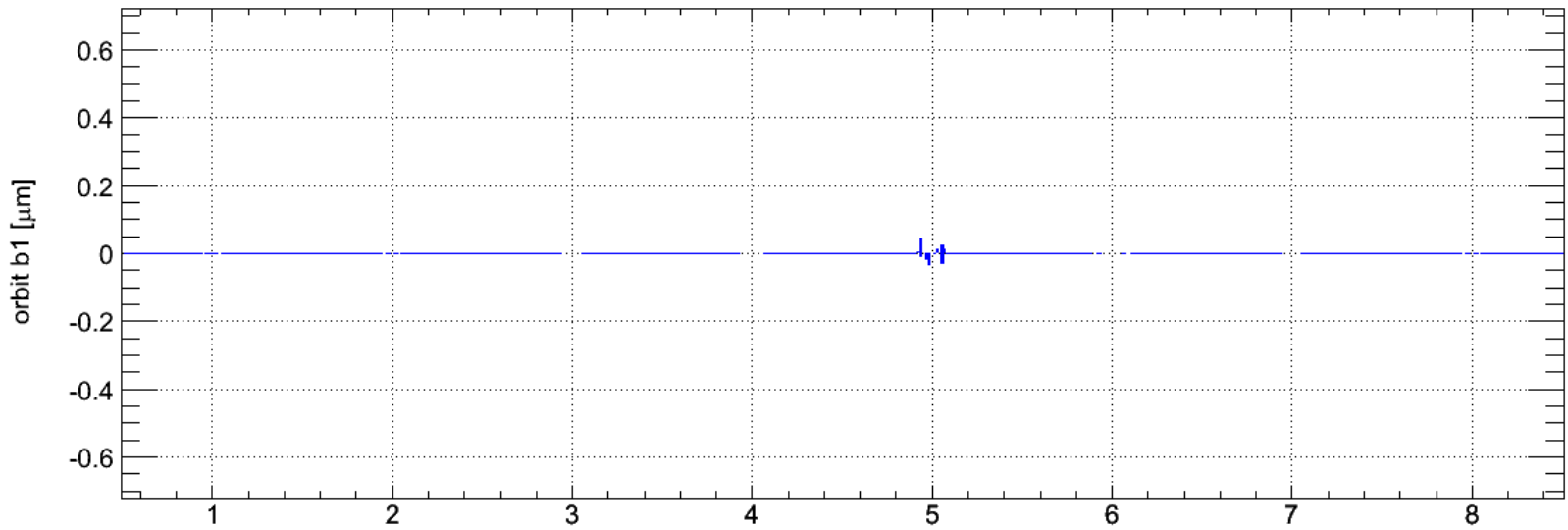
# Space Domain:

## LHC BPM eigenvector #449 $\lambda_{449} = 8.17 \cdot 10^1$

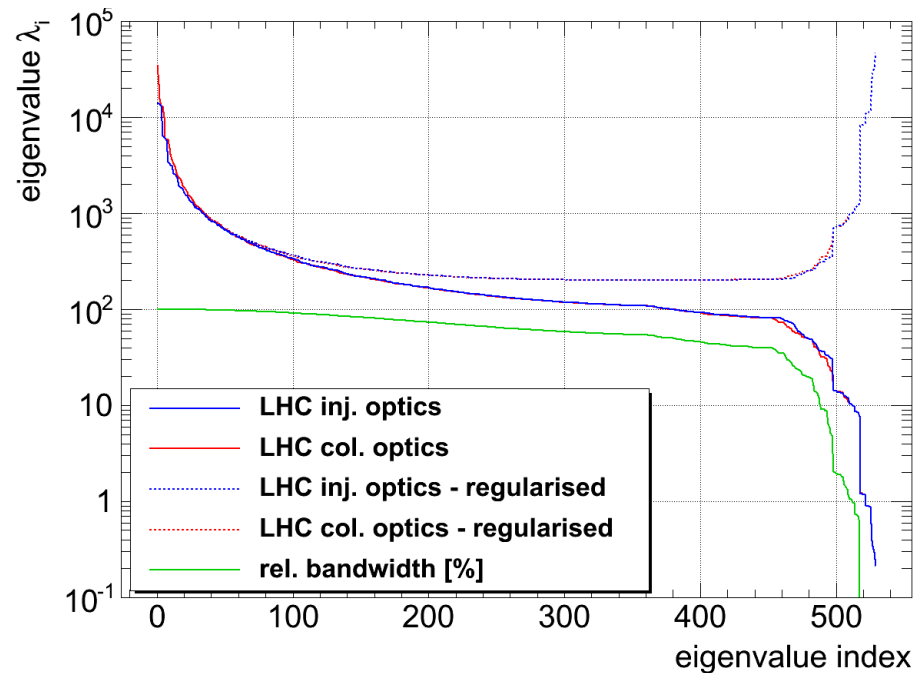
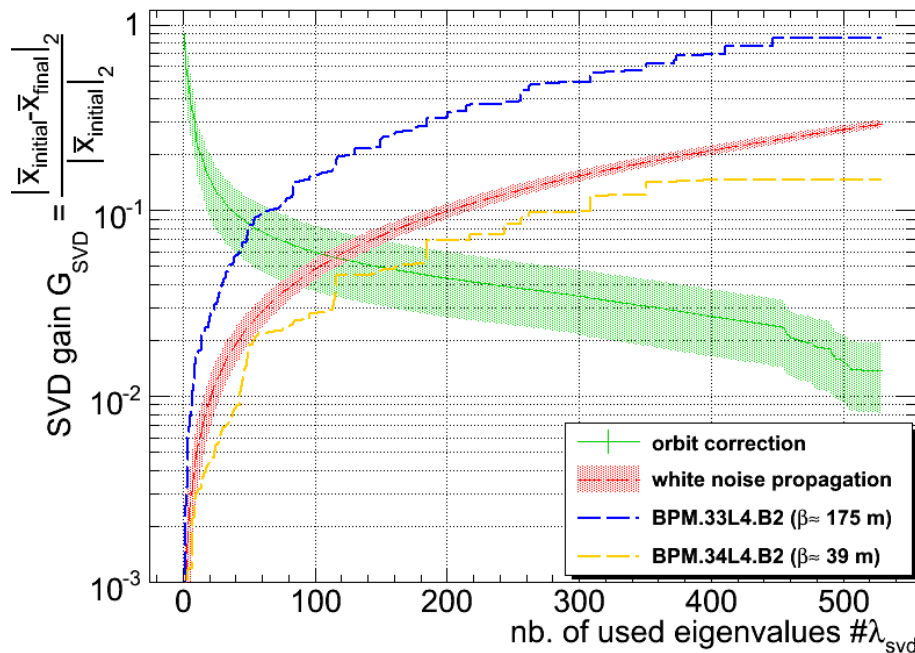




# Space Domain: LHC BPM eigenvector #521 $\lambda_{521} = 1.18 \cdot 10^0$

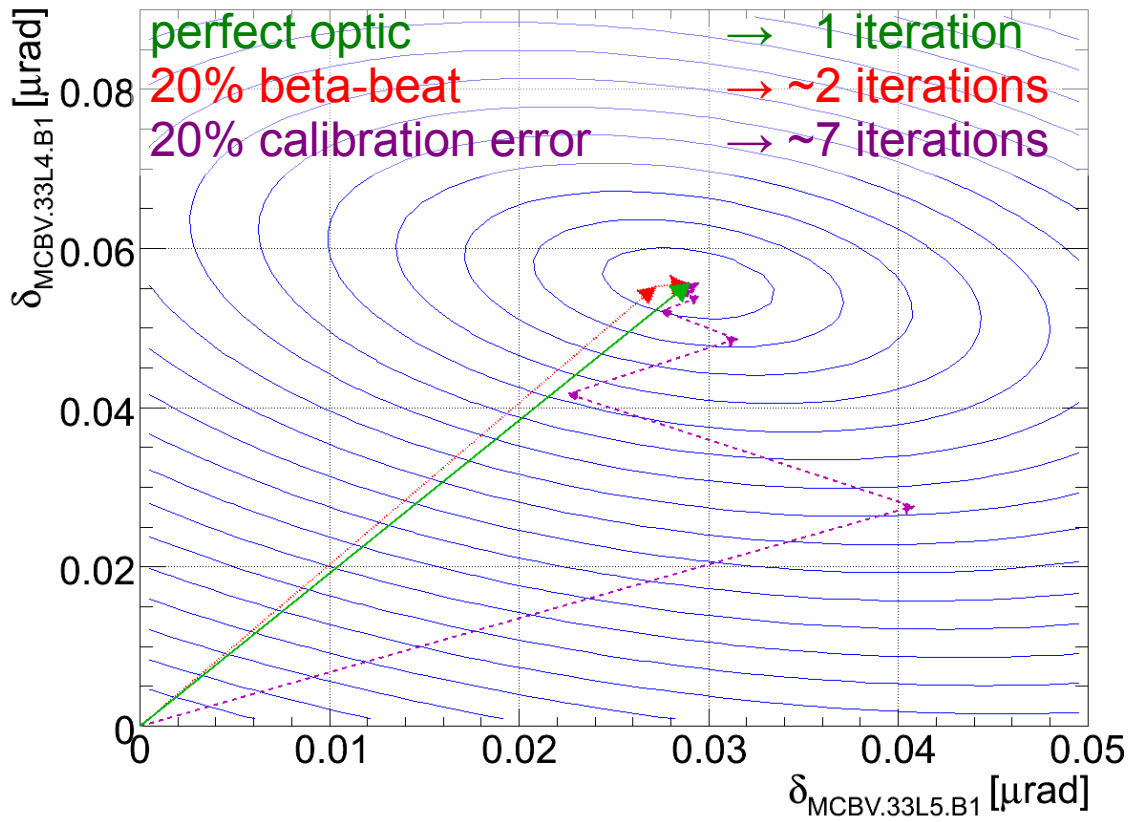


- Initially: Truncated-SVD (set  $\lambda_i^{-1} := 0$ , for  $i > N$ )
  - not without issues: removed  $\lambda_i$  allowed local bumps creeping in (e.g. collimation)
  
- Regularised-SVD (Tikhonov/opt. Wiener filter with  $\lambda_i^{-1} := \lambda_i / (\lambda_i^2 + \mu)$ ,  $\mu > 0$ )
  - more robust w.r.t. optics errors and mitigation of BPM noise/errors
    - allowed re-using same ORM for injection, ramp and 10+ squeeze steps



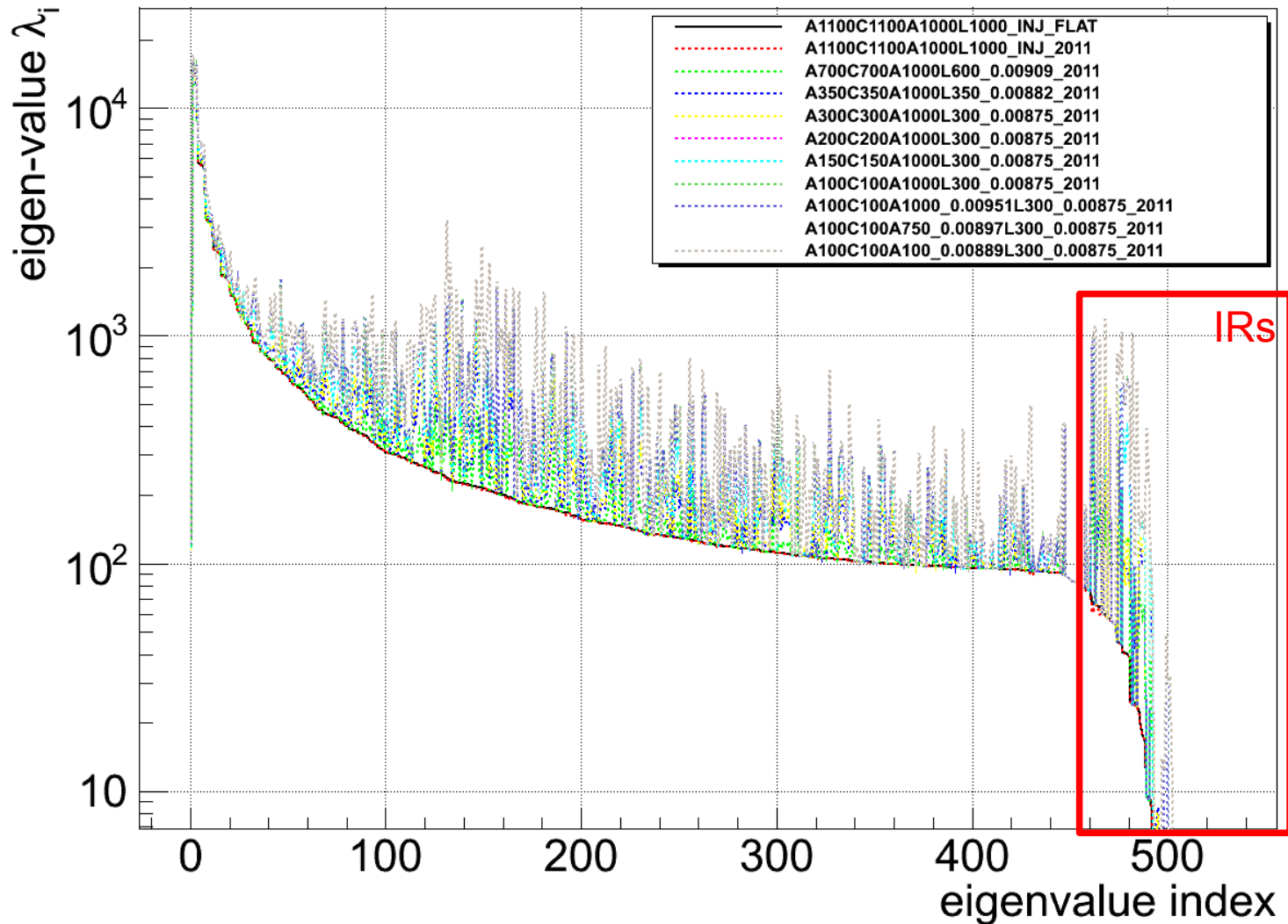


- Optics imperfections may deteriorate the convergence speed but do not affect absolute convergence (response functions are 'monotonic'):
  - for introduction: [http://en.wikipedia.org/wiki/Gradient\\_descent](http://en.wikipedia.org/wiki/Gradient_descent)
- Example: 2-dim orbit error surface projection

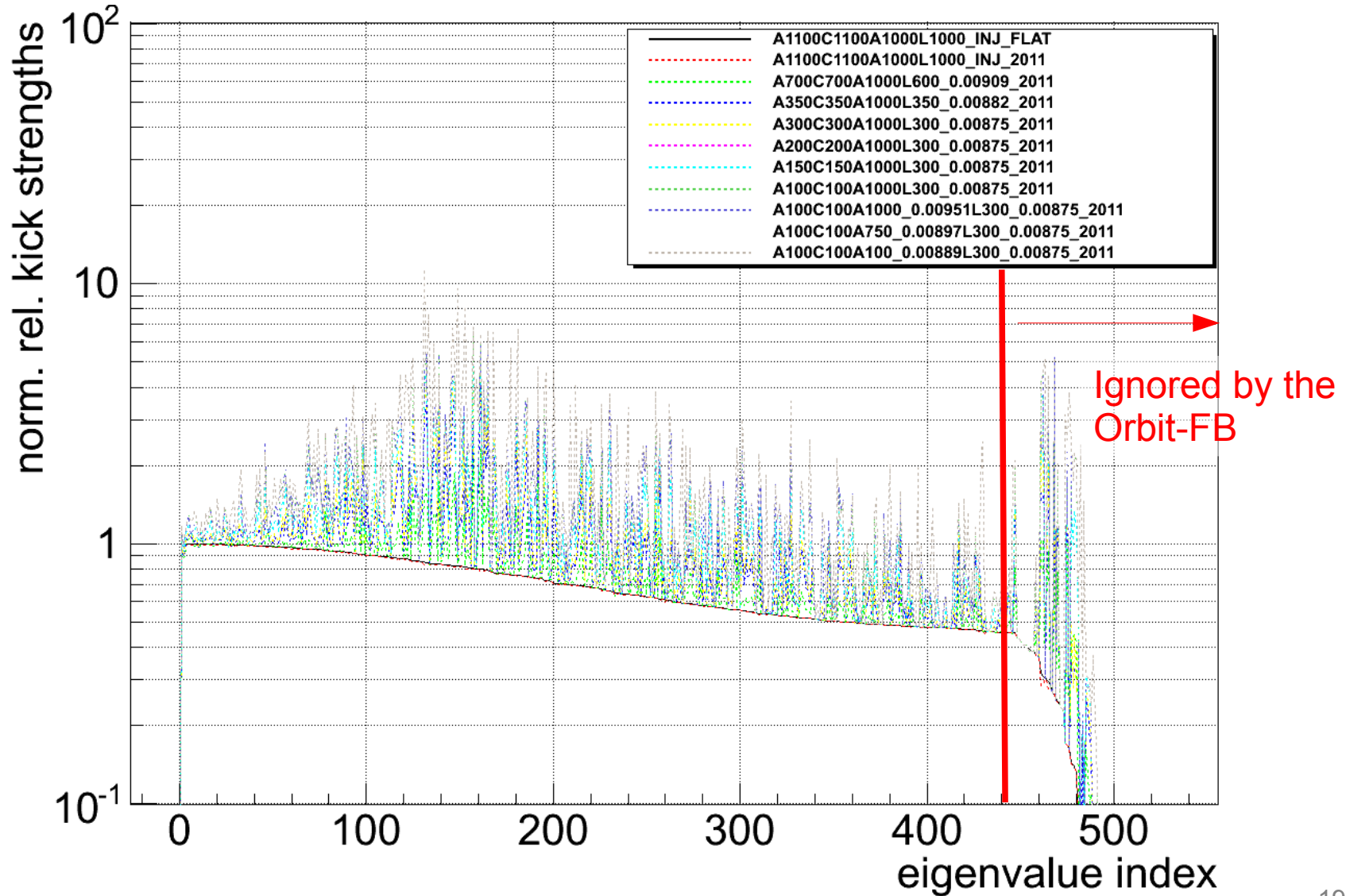


- LHC feedbacks are fairly insensitive to optics (= beta-beat) errors
  - However, pickup and corrector magnet polarities are crucial

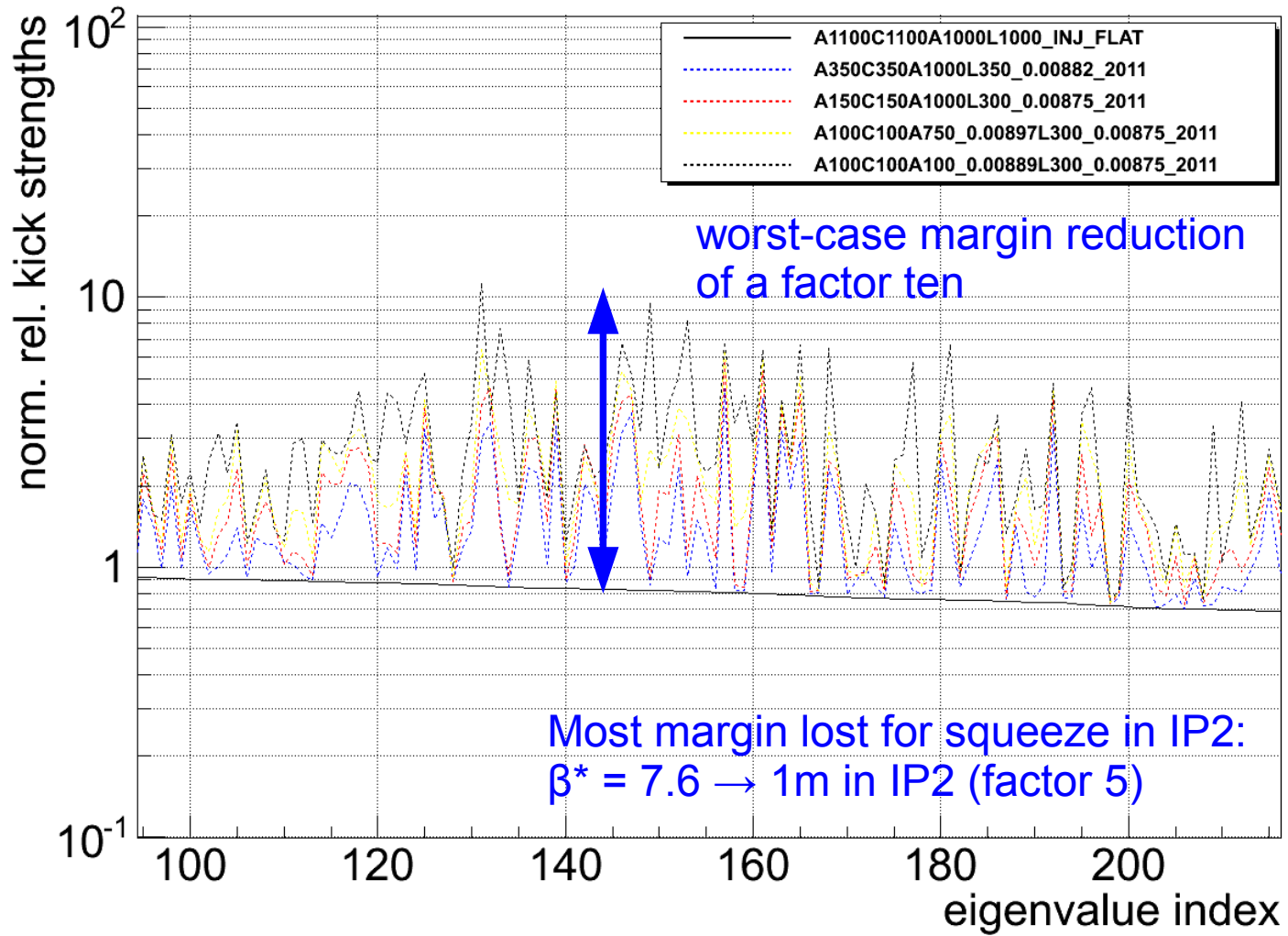
- Eigenvalue spectrum w.r.t. the same injection base 'V':
  - Errors are creeping in, particular for patterns around IRs



- Bandwidth modifier w.r.t. eigenvalue index

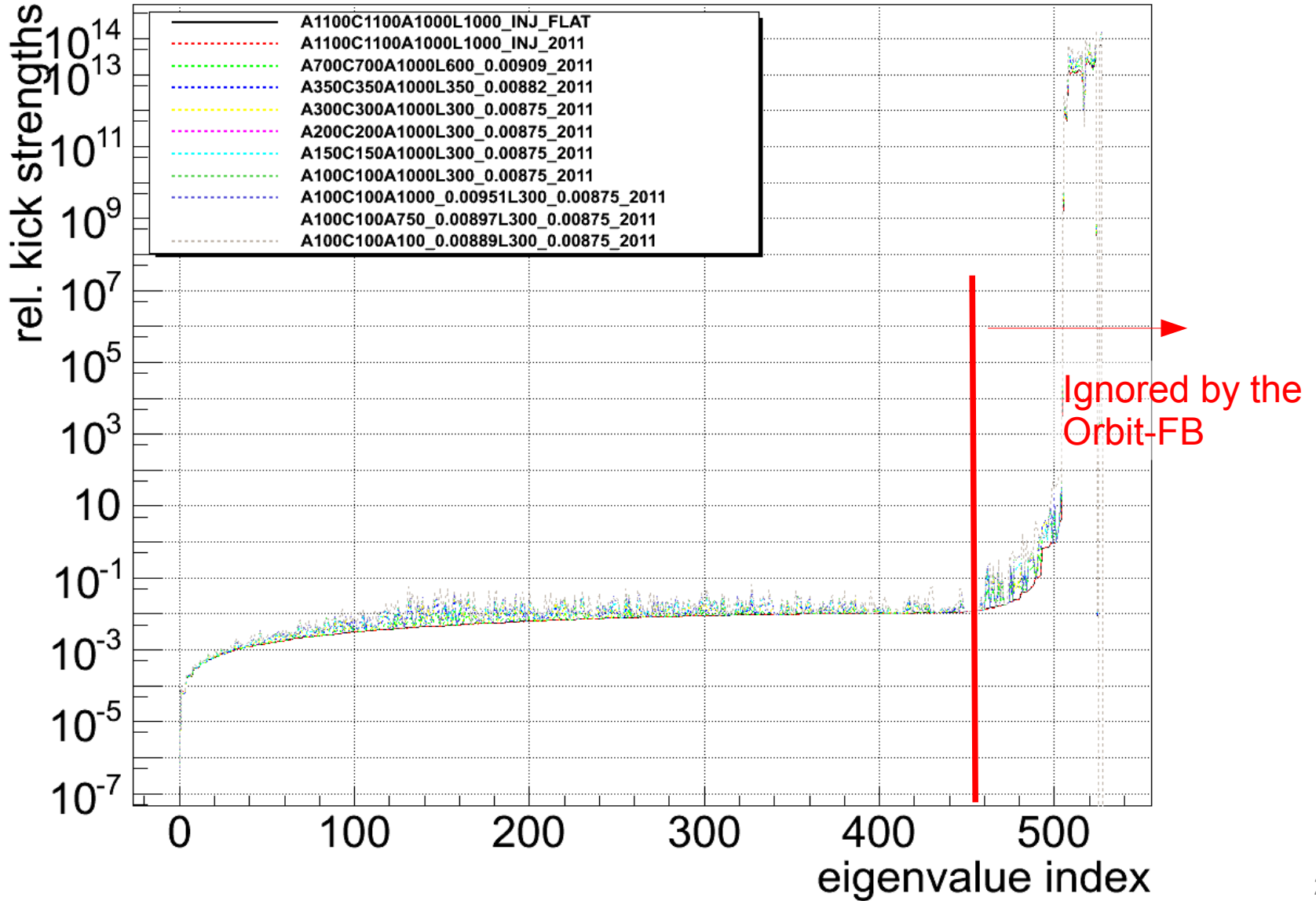


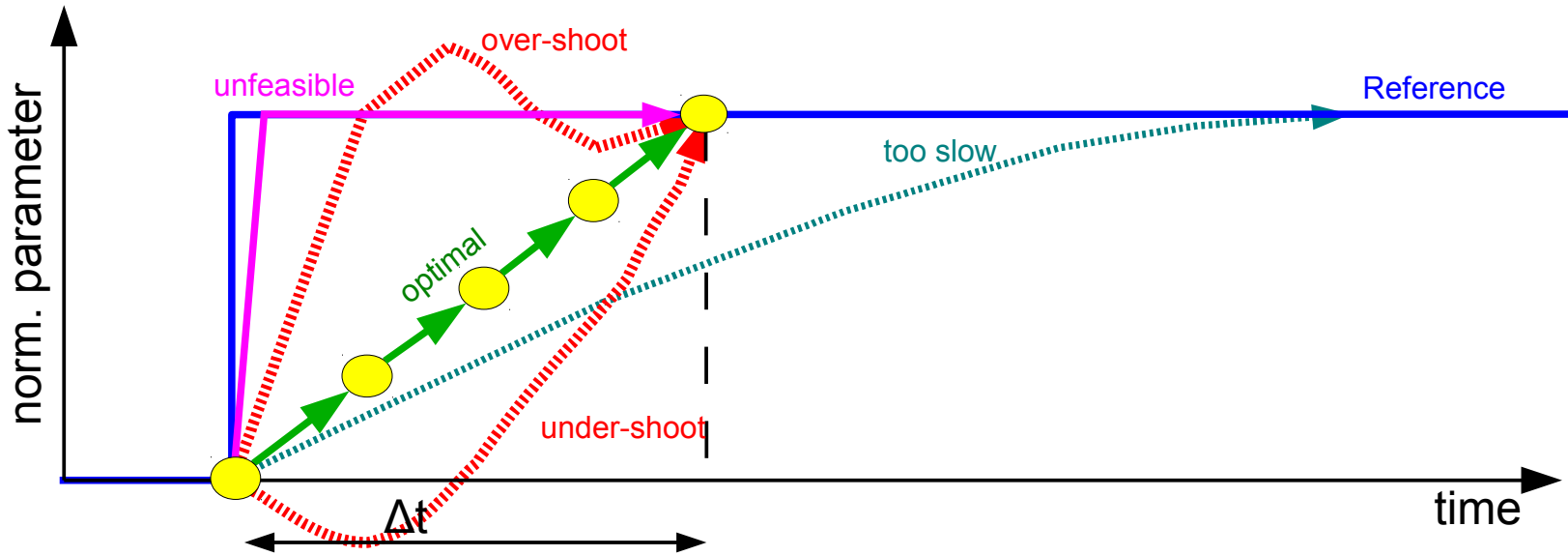
- Bandwidth modifier w.r.t. eigenvalue index – ZOOM



- However, Orbit-FB does not act on a sample-by-sample basis  
 → Closed-Loop Bandwidth « Sampling Frequency!!

- Kick strength profile:





- Optimal control [or design] ...

*“... deals with the problem of finding a control law for a given system such that a **given optimality criterion is achieved**. A control problem includes a cost functional that is a function of state and control variables.”*

- Common criteria: **closed loop stability**, minimum bandwidth, minimisation of action integral, power dissipation, ...

- classic closed loop:  $T_0(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} \longrightarrow$  “this tells me???”

- Using Youla's method: “design closed loop in a open loop style”:
- Youla showed<sup>1</sup> that all stable closed loop controllers  $D(s)$  can be written as:

$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)} \quad (1)$$

- Example: first order system

$$G(s) = \frac{K_0}{\tau s + 1} \quad \text{with } \tau \text{ being the circuit time constant} \quad (2)$$

- Using for example the following ansatz:

$$Q(s) = F_Q(s) G^i(s) = \frac{1}{\alpha s + 1} \cdot \frac{\tau s + 1}{K_0} \quad (3)$$

– Response/optimality can be directly deduced by construction of  $F_Q(s)$

–  $G^i(s)$ , pseudo-inverse of the nominal plant  $G(s)$

$$\rightarrow T_0(s) = \frac{1}{\alpha s + 1}$$

- (1)+(2)+(3) yields the following PI controller:

$$D(s) = K_p + K_i \frac{1}{s} \quad \text{with} \quad K_p = K_0 \frac{\tau}{\alpha} \quad \wedge \quad K_i = K_0 \frac{1}{\alpha}$$

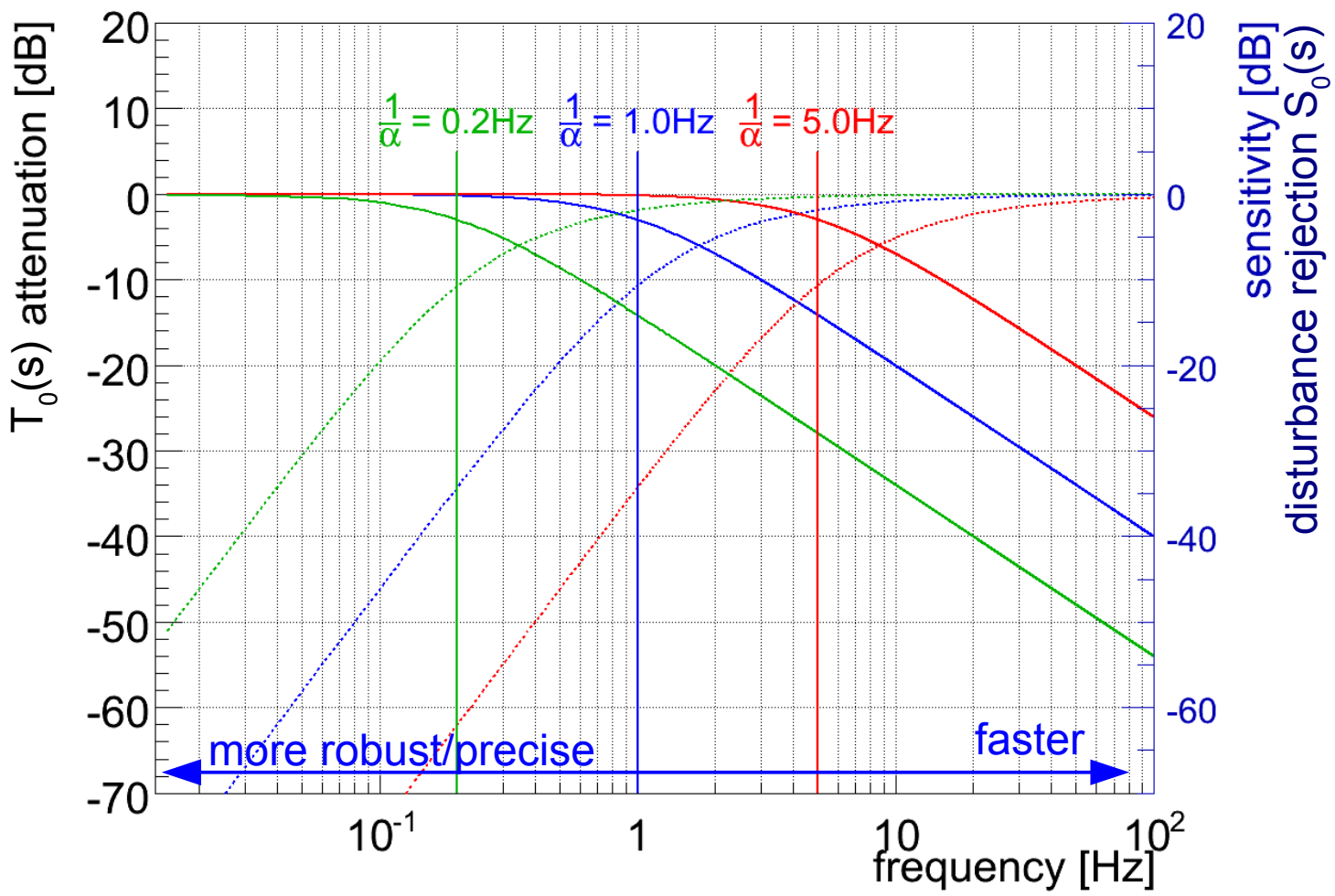
<sup>1</sup>D. C. Youla et al., “Modern Wiener-Hopf Design of Optimal Controllers”, IEEE Trans. on Automatic Control, 1976, vol. 21-1, pp. 3-13 & 319-338

# Time-Domain: Optimal Controller Design

## Example: PLL Closed Loop Controller - Bandwidth

- $\alpha > \tau \dots \infty$  facilitates the trade-off between speed and robustness
  - operator has to deal with one parameter  $\rightarrow$  enables simple adaptive gain-scheduling based on the operational scenario!

$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)}$$

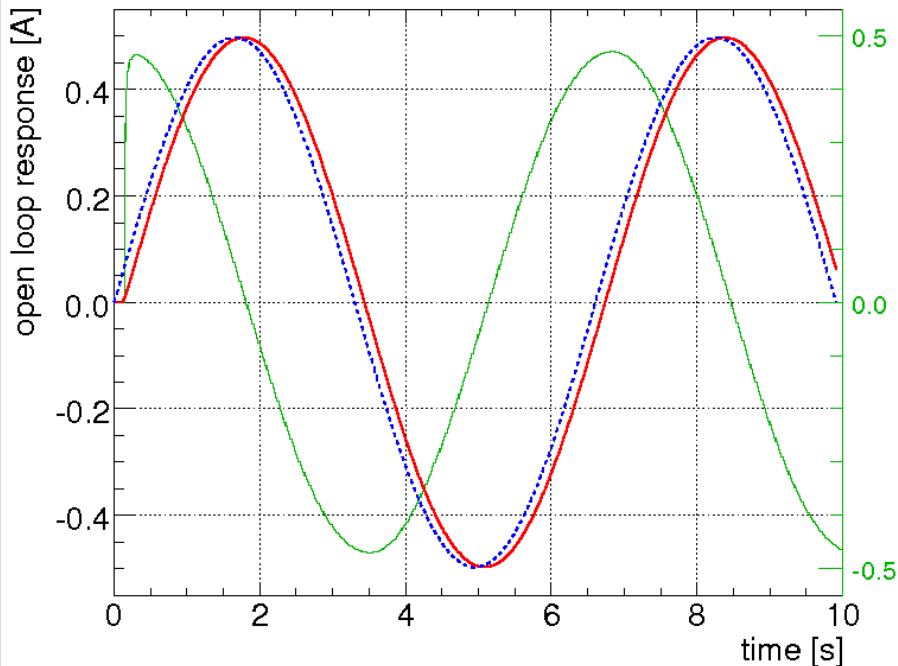




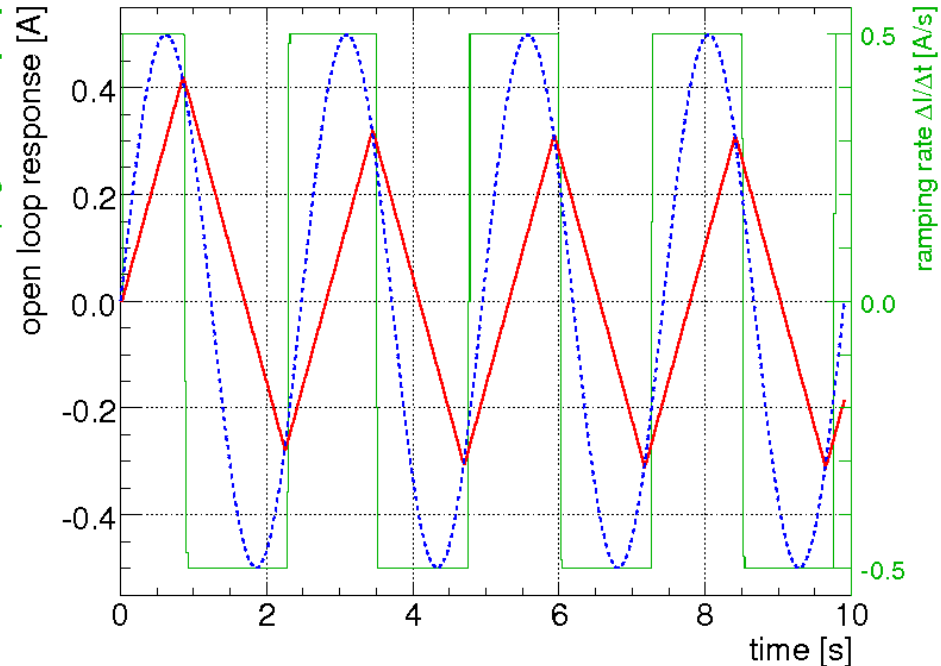
Two common non-linear effects in accelerators:

- Delays: computation, data transmission, dead-time, etc.
- Rate-Limiter: limited slew rate of corrector circuits (due to voltage limitations)
  - e.g. LHC:  $\pm 60\text{A}$  converter:  $|\Delta I/\Delta t|_{\max} < 0.5 \text{ A/s}$

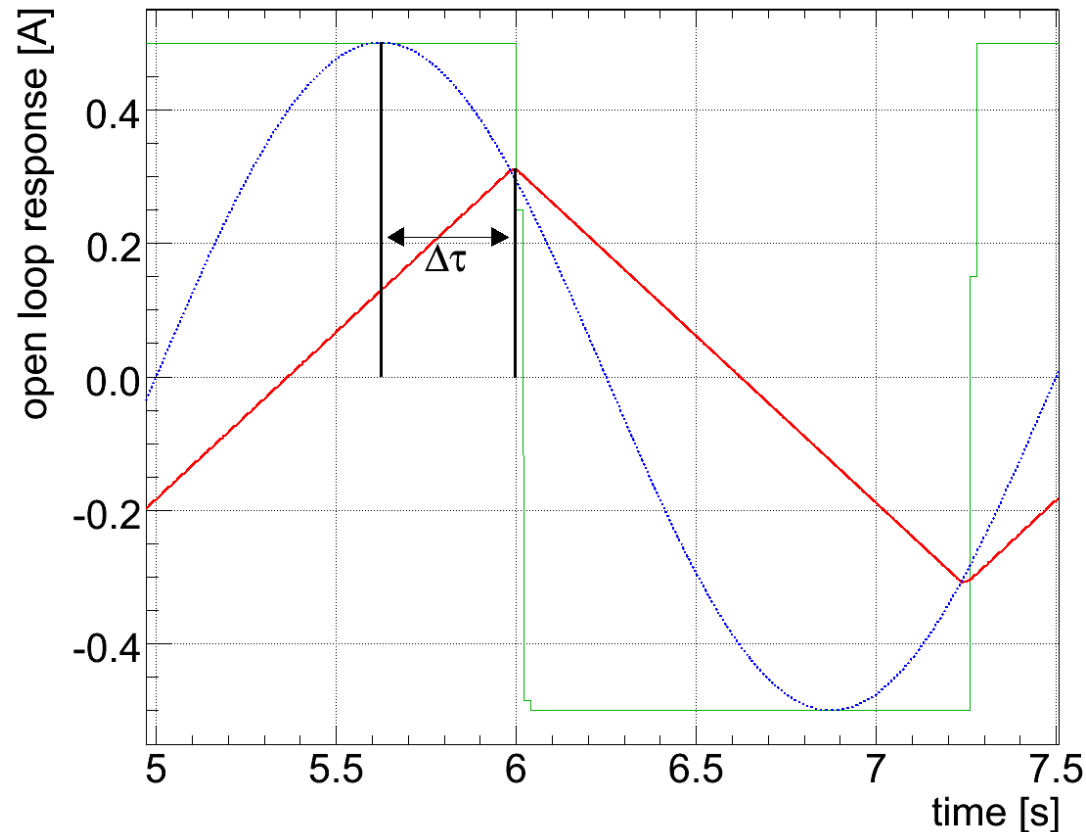
slow perturbation: perfect tracking



fast perturbation: saw-tooth

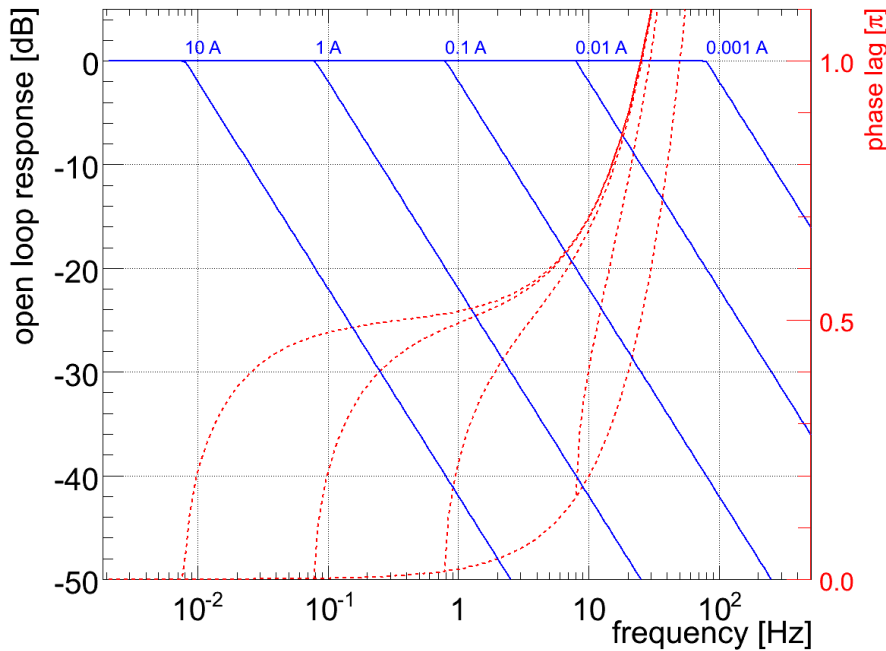


- Rate-limiter in a nut-shell:
  - additional time-delay  $\Delta\tau$  that depends on the signal/noise amplitude
  - (secondary: introduces harmonic distortions)



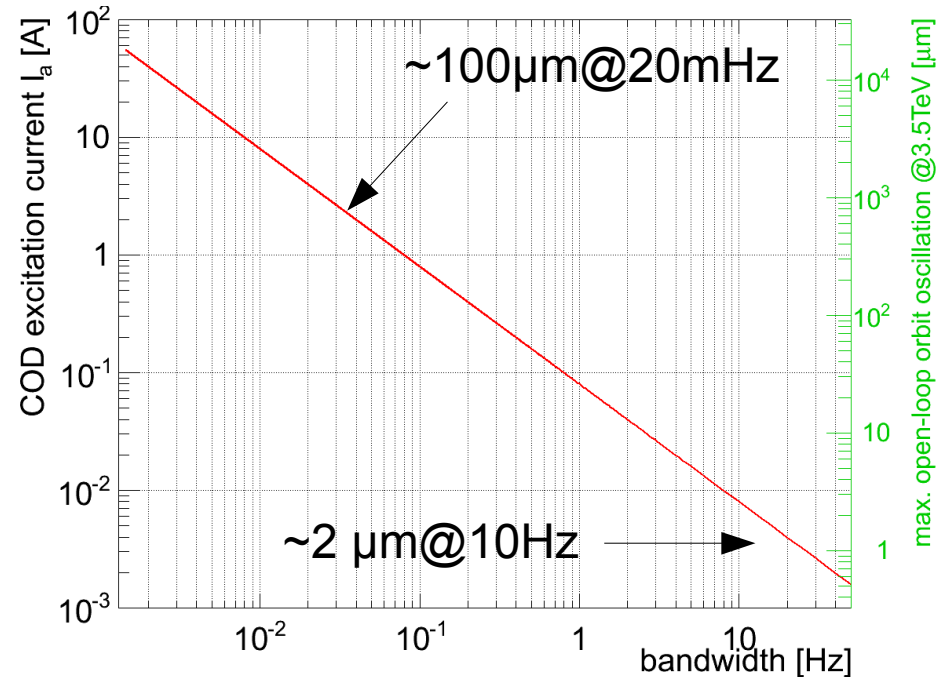
- N.B. Orbit-FB only knows about strength/rate change of RT trims
  - no info on rate-limits underlying LSA feed-forward functions

- Open-loop circuit bandwidth depends on the excitation amplitude:
  - + non-linear phase once rate-limiter is in action...



$$\Delta I = 0.1 \text{ A} \leftrightarrow \Delta x \approx 32 \text{ } \mu\text{m} @ \beta = 180 \text{ m}$$

- Consider  $\sim 32 \mu\text{m} @ 1 \text{ Hz}$  as effective bandwidth @ 3.5 TeV



$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)}$$

- ... cannot a priori be compensated.
  - however, their deteriorating effect on the loop response can be mitigated by taking them into account during the controller design.
- Example: process can be split into **stable** and **instable 'zeros'/components**

$$G(s) = \frac{A_0(s)A_u(s)}{B(s)} = G_0(s) \cdot G_{NL}(s) \quad \text{e.g.} \quad G(s) = G_0(s) \cdot \underbrace{e^{-\lambda s}}_{\lambda: \text{delay}}$$

- Using the modified ansatz ( $F_Q(s)$ : desired closed-loop transfer function):

$$Q(s) = F_Q(s) \cdot G^i(s) = F_Q(s) \cdot G_0^{-1}(s)$$

- yields the following closed loop transfer function

$$\rightarrow T(s) = Q(s)G(s) = F_Q(s) \cdot \underbrace{G_{NL}(s)}_{\text{here:}} = F_Q(s) \cdot e^{-\lambda s}$$

- Controller design  $F_Q(s)$  carried out as for the linear plant
- Yields known classic predictor schemes:
  - **delay** → **Smith Predictor**
  - **rate-limit** → **Anti-Windup Predictor**

$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)}$$

- If  $G(s)$  contains e.g. delay  $\lambda$  & non-linearities  $G_{NL}(s)$

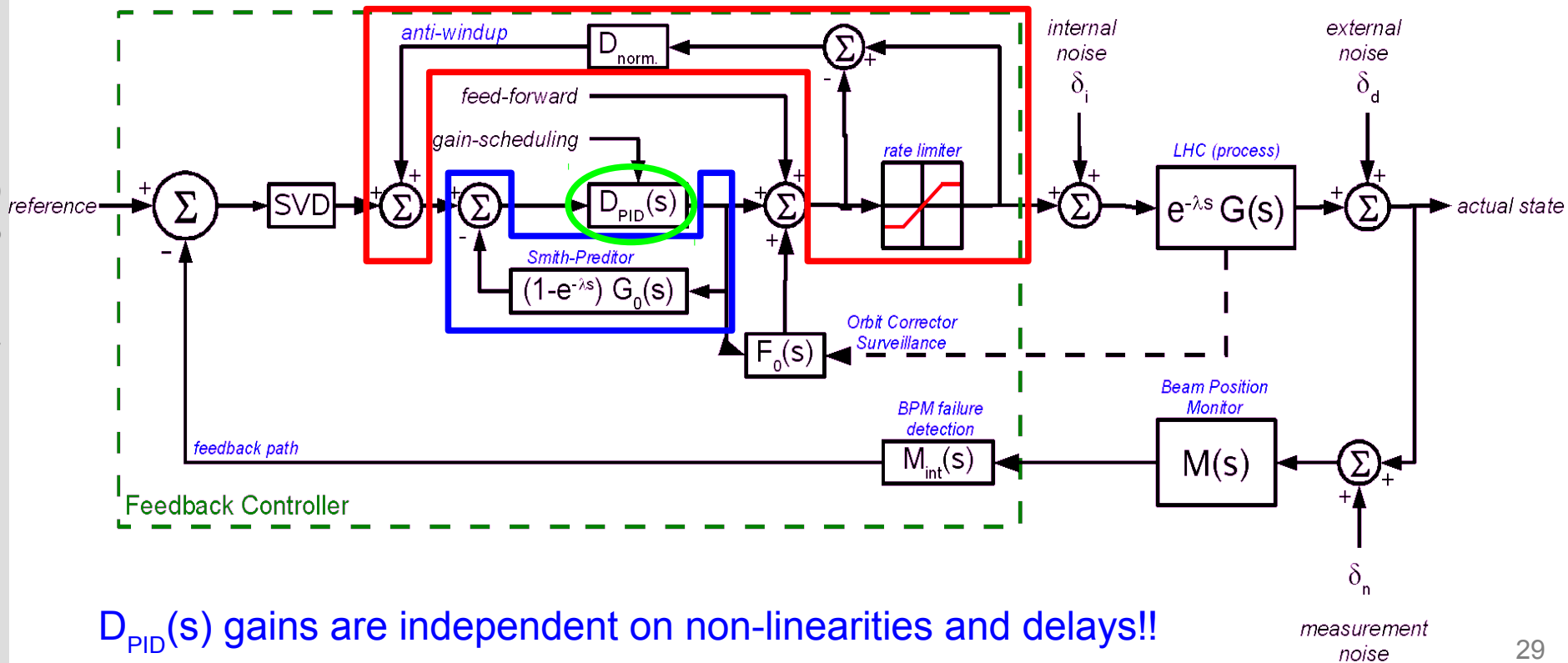
$$G(s) = \frac{e^{-\lambda s}}{\tau s + 1} G_{NL}(s)$$

- with  $\tau$  the power converter time constant and

$$G^i(s) = \frac{\tau s + 1}{1}$$

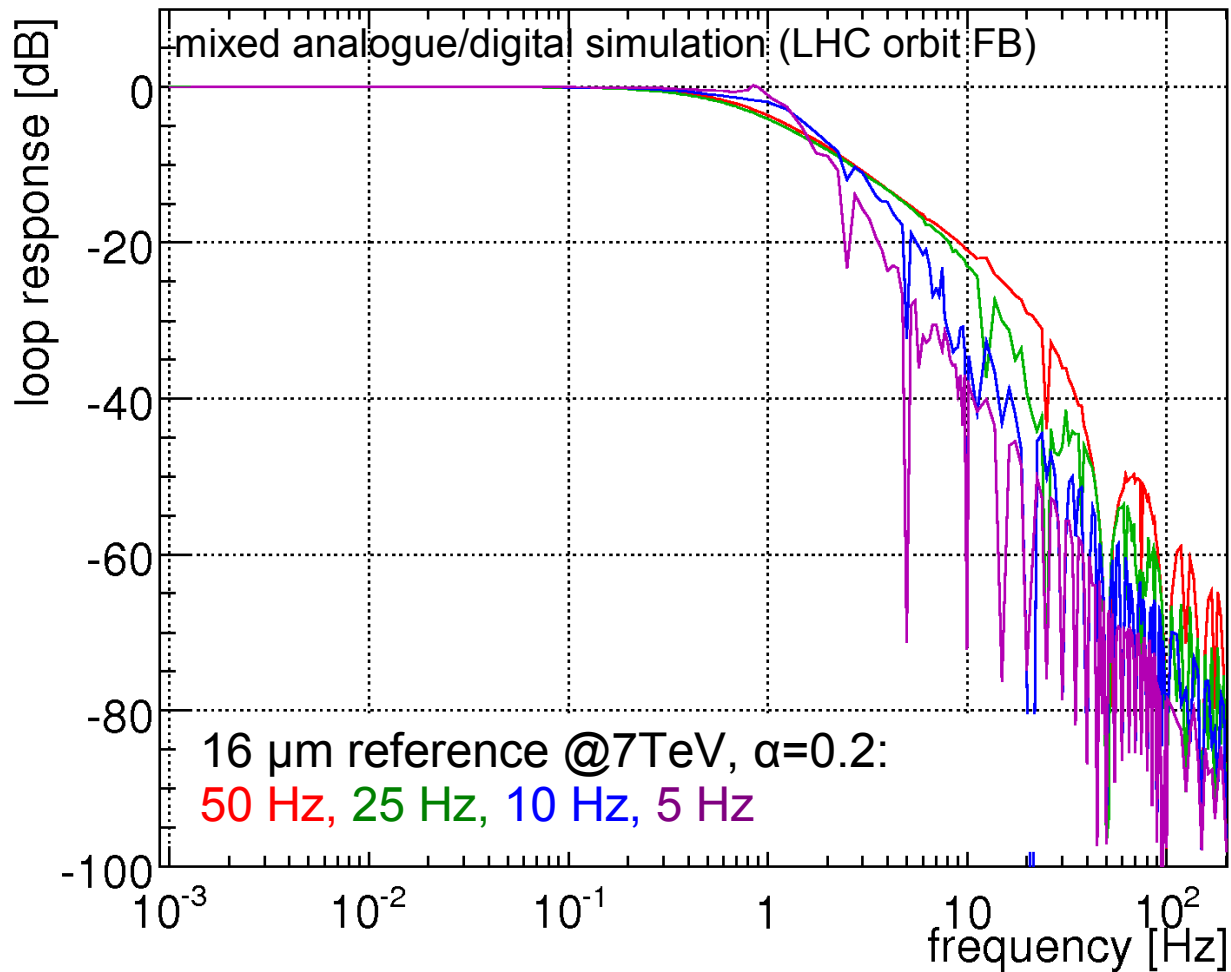
- yields **Smith-Predictor** and **Anti-Windup** paths:

$$T(s) = F_Q(s) \cdot e^{-\lambda s} G_{NL}(s)$$



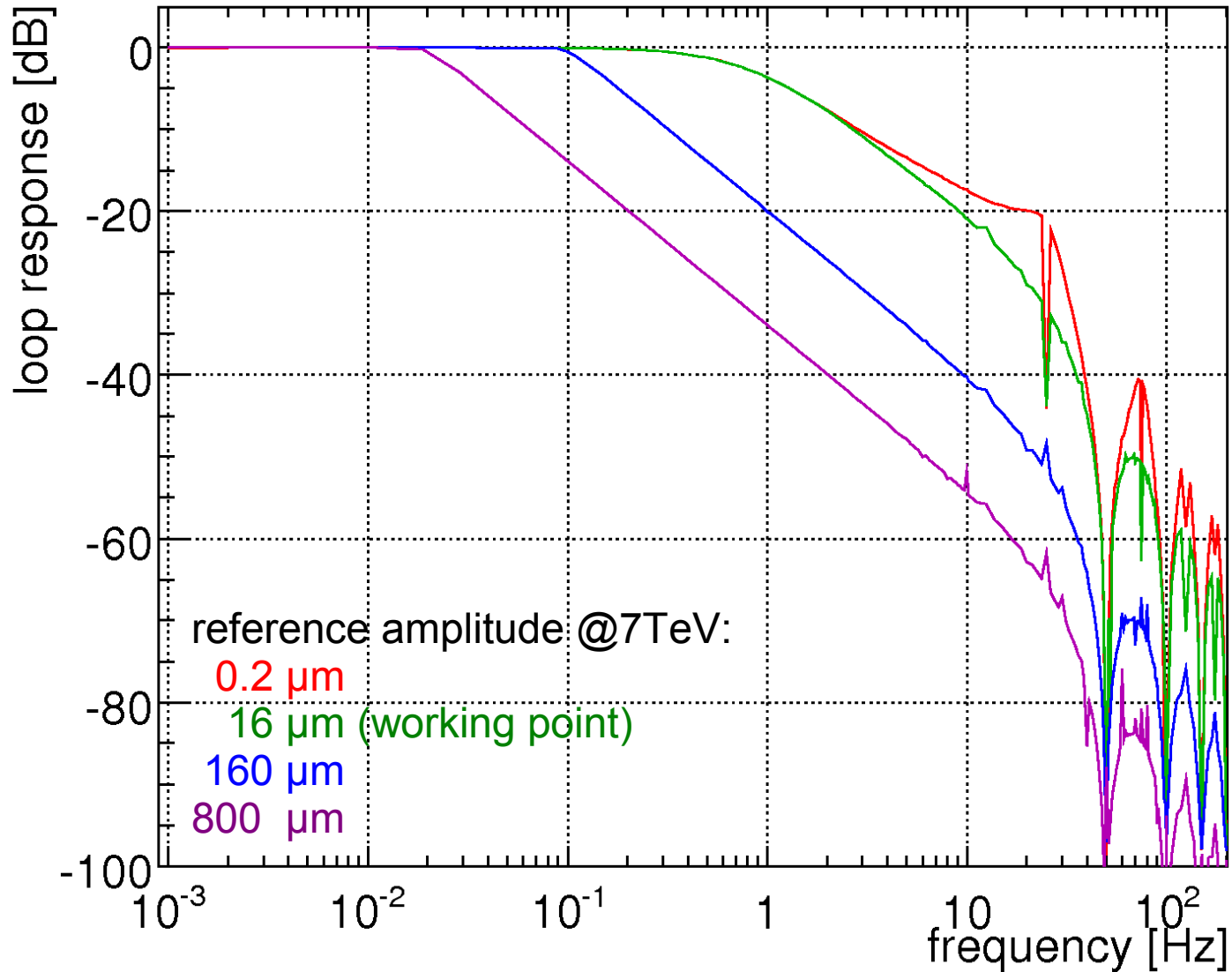
$D_{PID}(s)$  gains are independent on non-linearities and delays!!

# Loop Bandwidth $\neq$ Sampling Frequency

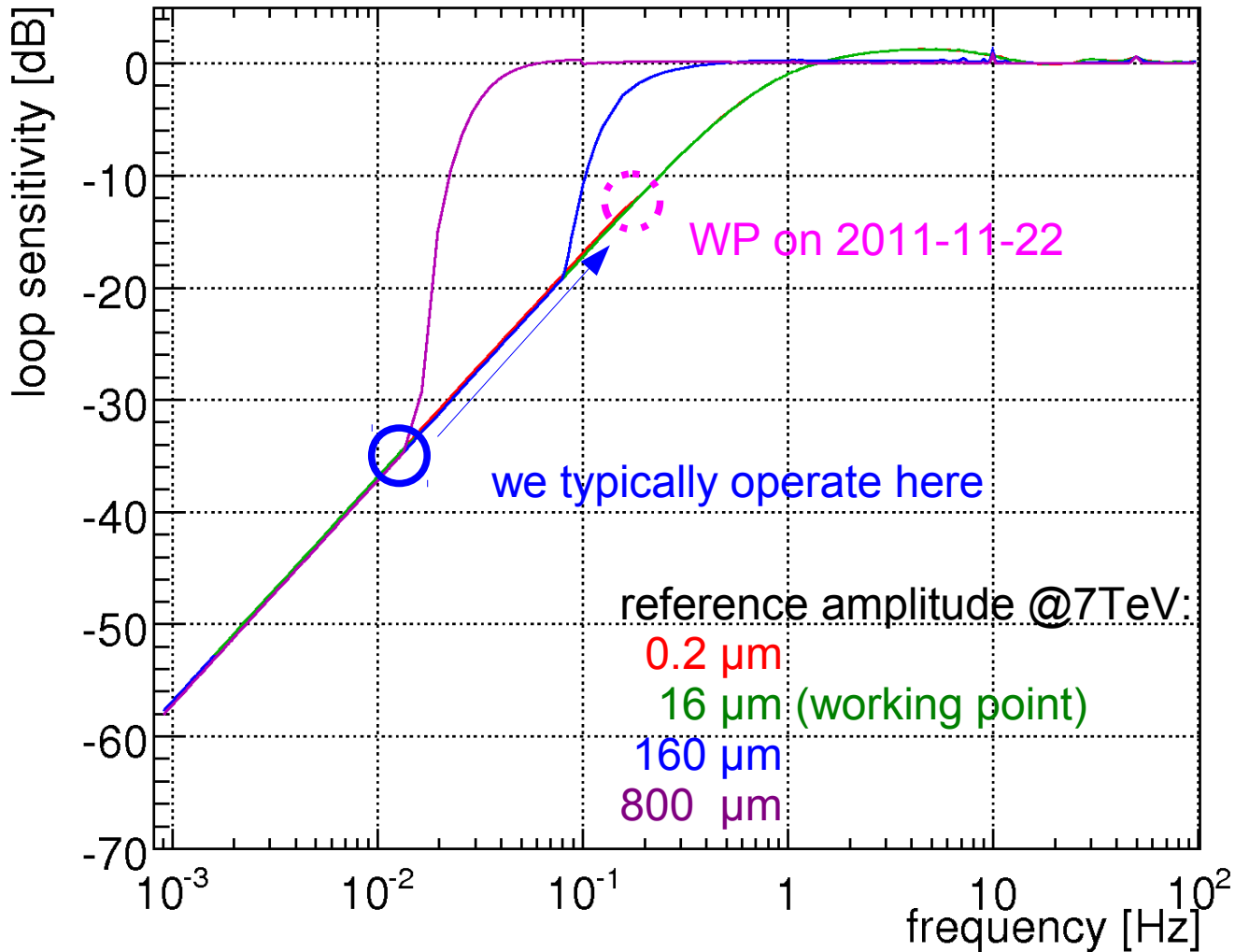


- ... a theoretic limit assuming a perfect system (no noise, model errors)!
- common sense/advise:  $f_s > 25 \dots 40 \times$  desired closed-loop bandwidth  $f_{\text{BW}}$

- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)



- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)

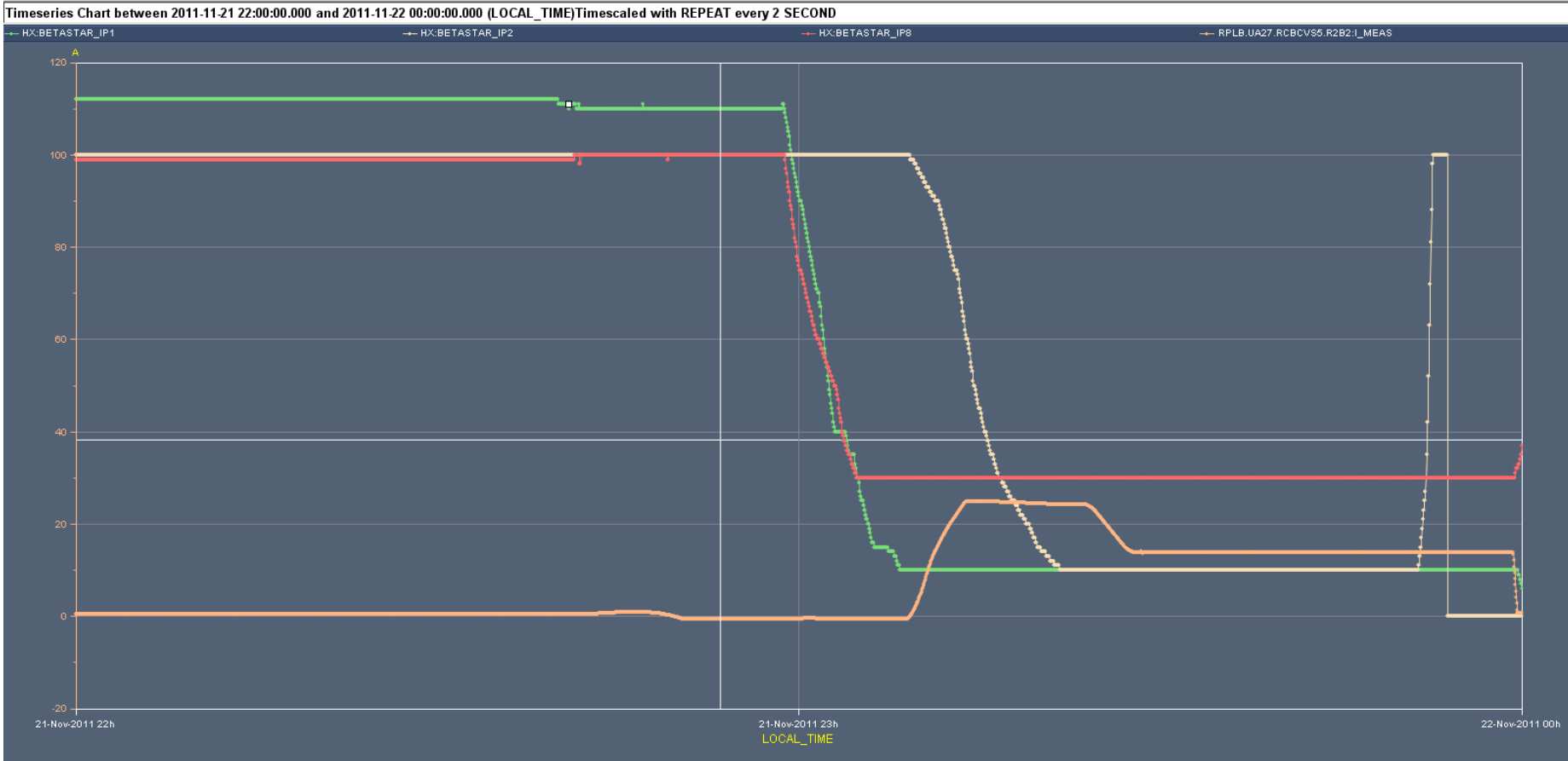


- Factor 10 margin but radically lost if encountering rate-limit or delays  
→ should be validated early-on in 2012!



- Feedback bandwidth can certainly be increased during dedicated special periods such as MDs, setup of new ramps, squeezes, etc.
- However, need to respect intrinsic trade-offs of increasing bandwidth
  - more BPM noise propagated onto the beam
  - limited by CODs and delays → non-linearities become important
- Using the right optics during the squeeze is not mandatory but certainly improves the bandwidth and stability margin
  - should investigate and test the impact of using half- or fully squozen optic
- Specific Orbit-FB wish list for re-commissioning next year:
  - we should aim at verifying the actual orbit feedback stability margin during every/most new squeeze steps → need proper allocated time
  - Perform feed-forward more often/sooner (start of run?)

■ e.g. RPLB.UA27.RCBCVS5.R2B2 at 0.1 A/s



# Last squeeze:

