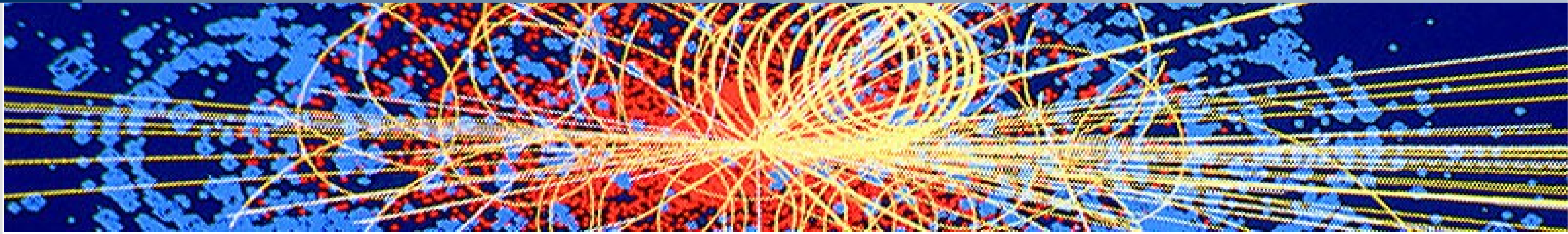
Disclaimer: this is not a lecture but aim to revise some of the key-aspects:

- Luminosity, energy and their limits
  - Concepts
  - Examples
- Beam Cooling
  - Concepts
  - Examples
- Q&A and advanced lumi topics
  - Crossing angle & 'Hour Glass' effect
  - Magic of integrated luminosity

Accelerator Collider Optimisation Considerations
Where every high-energy-physics particle quests starts:



Event Rate → the frequency a given particle is created per second
Physics detectors

cross-section → probability that a given particle is created
Mother Nature defines that for us and typically depends on energy

$$\dot{N}_{event} = L \cdot \sigma_{physics}$$

Luminosity → the frequency of how often the particles are brought in to collisions
Accelerator design and operation

- Push maximum peak luminosity
  - essentially: increase number of particles inside the machine and squeeze them to a confined space to increase the probability of a collision
- Push achievable energy E:
  - Minimise synchrotron radiation losses: e^+e^- → hadrons collider ($p^+p^\pm$, ...)
  - Choice: linear vs. circular
    - Optimise RF cavities + normal conducting magnets (CLIC,ILC)
    - Standard RF cavities + superconducting magnets (Tev,RHIC,LHC)

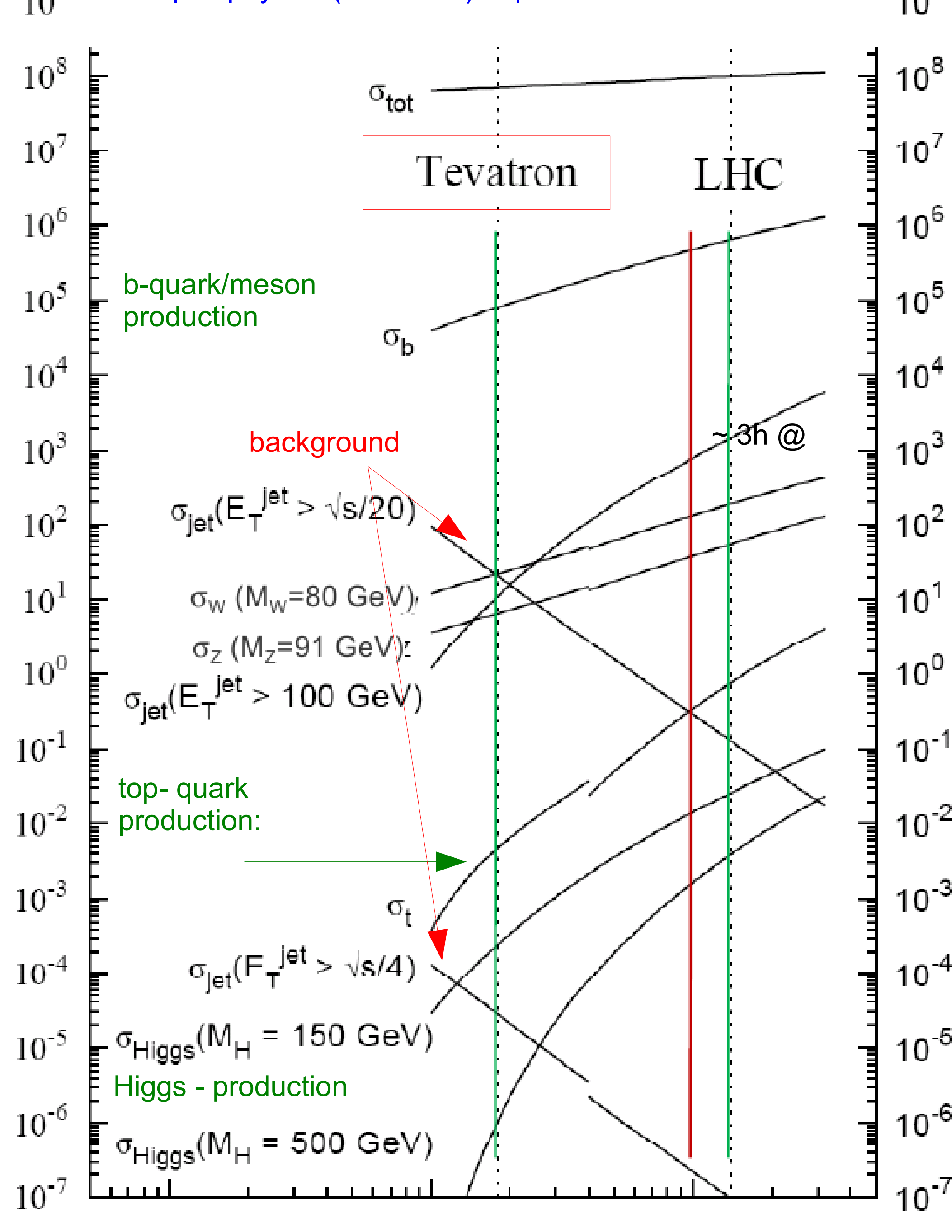
Why maximum Multi-TeV Collision Energies?

- Physicists want as high as possible
  - useful (competitive) HEP

Need to find a balance between:

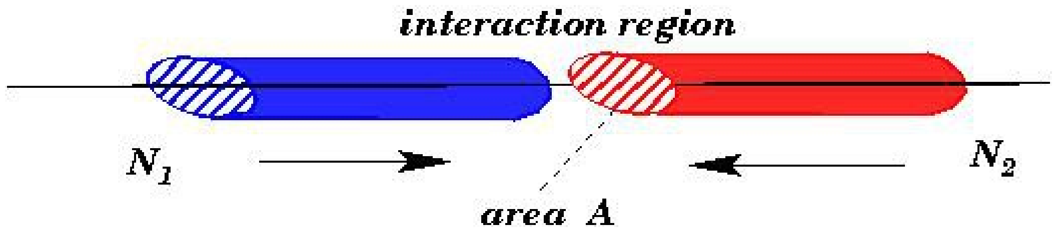
- Dipole field which can be reached
- Risks associated with operating at that field
  - splices stability, thermal runaway, splice detection
- Operational efficiency of other systems
  - e.g. cryo recovery time:
    5TeV vs. >6h @ 7TeV)

Example: physics (cross-sec) dependence on E



Luminosity: $dN_{event}/dt = L\ s_{process}$

- Collider design:



$$L = \frac{N^2 k\, f_{rev}}{A} \cdot F_{corr.} = \frac{N^2 k\, f_{rev}}{4\pi\,\sigma_x \sigma_y} \cdot F_{corr.} = \frac{N^2 k\, f_{rev}}{4\pi\,\beta^* \epsilon} \cdot F_{corr.}$$

  - *N*: number of particles per bunch,
  - *k*: total number of bunches,
  - σ_x, σ_y: hor./vert. r.m.s. beam size in IR,
  - f_{rev}: revolution or repetition frequency,
  - $F_{corr.}$: numerical correction factors (hour-glass, crossing angle, ...),

$$\sigma_{x,y} = \sqrt{\epsilon\beta(s) + D(s)\cdot\frac{\Delta p}{p}}$$

$$I_{stored} = Nk$$

- Warning: hadron accelerators tend to express emittance as normalised emittances!!
  - Facilitates monitoring of blow-up through the injector chain
  - N.B. e-machines: emittance given by energy, damping, lattice rather than injectors

$$\epsilon^* = \frac{\epsilon}{\gamma}$$

Exercise I
– The present, from a circular Accelerator far far away...

1. Calculate the luminosity for a beam of 10^{11} protons per bunch circulating at 7 TeV in a ring of C=26.658 km circumference, assuming $\varepsilon(\beta\gamma)$ = 3.1 μm rad and β^*= 0.55 m. Emittance is defined for 1σ and there are 2808 bunches per beam. (assume $A=4\pi\sigma^2$)
2. Calculate the beam–beam tune shift per crossing for the above.
3. Assuming that ΔQ_{bb} should be < $8.3\cdot10^{-4}$, adjust the number of bunches and intensity to maximize the luminosity within this limit. Assume $N\,f_b$ remains constant.

Exercise I – Solution
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- What is needed:
  - Proton mass at rest: $m_p \approx 2000\ m_e \approx 0.938$ GeV/c^2
  - At 7 TeV protons are ultra-relativistic → β ≈ 1 → f = c/C (c ≈ $3\cdot10^5$ km/s)
  - Beam-beam tune shift (N.B. $r_0 \approx 0.842$ fm):

$$\Delta Q_{bb} \approx \frac{r_0 \cdot \beta^* \cdot N}{4\pi\gamma\sigma^2} = \frac{r_0}{4\pi\gamma}\cdot\frac{N}{\epsilon} = \boxed{\frac{r_0}{4\pi}\cdot\frac{N}{\epsilon^*}}$$

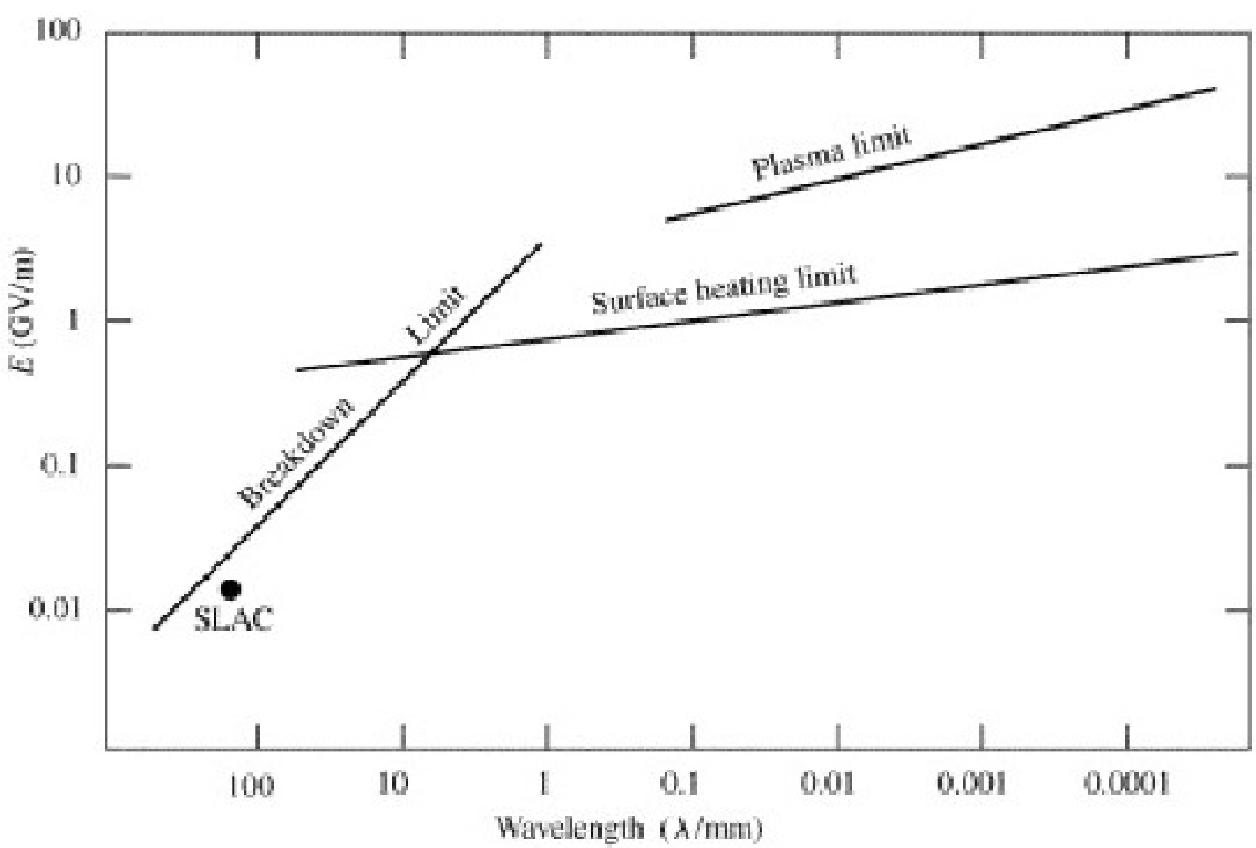
Independent of β* and energy!!

    - Not fully "true" for LHC...
- Lumi – design (attention 'cm' vs. 'μm' definition):

$$L = \frac{N^2 k\, f_{rev}}{4\pi\sigma_x\sigma_y}\cdot F_{corr.}$$

Exercise II
– The future If you would design of the next Linear Collider...

- A linear collider seeks to achieve a centre-of-mass energy of 1000 GeV. Plot a curve of length versus field gradient and use the figure below to decide the frequency which fits a site of 25 km extent (assume a filling factor of 70%).
- Assuming a repetition frequency of 200 Hz and a mean beam radius ($\sigma_x = \sigma_y = 60$ nm), what beam intensity does the linear collider require to reach luminosity of 10^{34}cm^{-2}s^{-1}?
- What is the average beam power?



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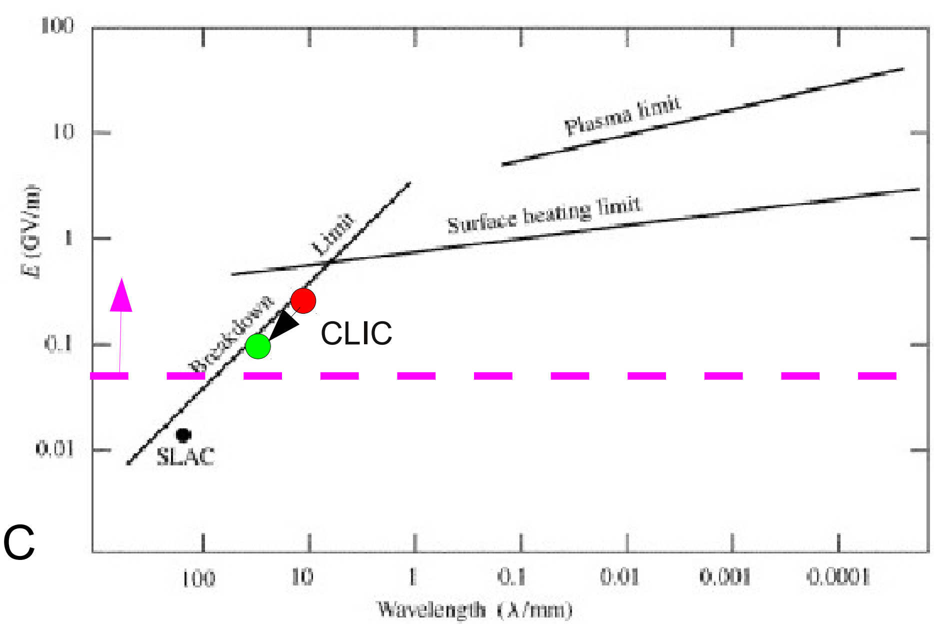
- Gradient = 2*500 GV/25 km > 0.04 GV/m
  - $f_{rf} = c/\lambda$, e.g. 30 cm → $f_{rf} \approx 1$ Ghz
- Lumi-design $$L = \frac{N^2 k\, f_{rep}}{4\pi\sigma_x\sigma_y}\cdot F_{corr.}$$
  - $k=1$ → $N \approx 1.5\cdot10^{11}$ charges = 24 nC
    - N.B. 1 C(oloumb) = 1As
    - Electron/Proton charge: $1.602\cdot10^{-19}$ C
  - Power $P = N\cdot E\cdot f_{rep}$,
    - here: $P = 1.5\cdot10^{11}\cdot1000\cdot10^{9}\cdot1.602\cdot10^{-19}$VAs·200Hz ≈ 5 MW (per bunch)



Exercise III
– The far future...

- Calculate the lifetime of a muon
  - at 50 GeV circulating in a storage ring and
  - at 4 TeV.
- Calculate the leading parameters (bending radius Q, number of periods, etc.) for a 6 T superconducting ring to store the muons.
- What would be a reasonable repetition (filling) rate for a muon collider at 4 TeV, assuming that a beam should be renewed when it has decayed by one exponential lifetime?

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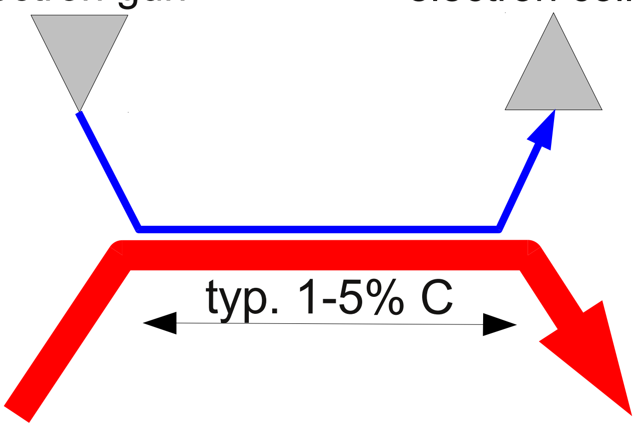
- What is needed:
  - muon mass and life-time at rest: $m_\mu \approx 200\ m_e \approx 105.7\ \mathrm{MeV}/c^2$, $\tau_\mu = 2.2\ \mu s$
  - A bit of special relativity $E = m\gamma c^2$ and time dilation $\Delta\tau' = \gamma\cdot\Delta\tau$
  - Bending radius of a dipole magnet: $p/e = (B\rho)$
  - From lattice design:
    - simple estimate $Q \approx \overline{R}/\overline{\beta}$ & $\beta_{min,max} = \frac{(1 \pm \sin(\mu/2))}{\sin\mu}\cdot L_{cell}$
      - e.g. (hadrons) $\Delta\mu = 90° \rightarrow \overline{\beta} \approx L_{cell}$

- Electrons → comes easily/naturally with synchrotron radiation $\sigma_{x,y}=\sqrt{\epsilon\beta(s)+D(s)\cdot\frac{\Delta p}{p}}$
- Protons → can be created in abundance → select those fitting to small ε
- Secondary beams or rare isotopes, difficult to produce large numbers, e.g.: ~10^6 protons on target to create one (!) anti-proton

- Electron Cooling:

electron gun electron collector

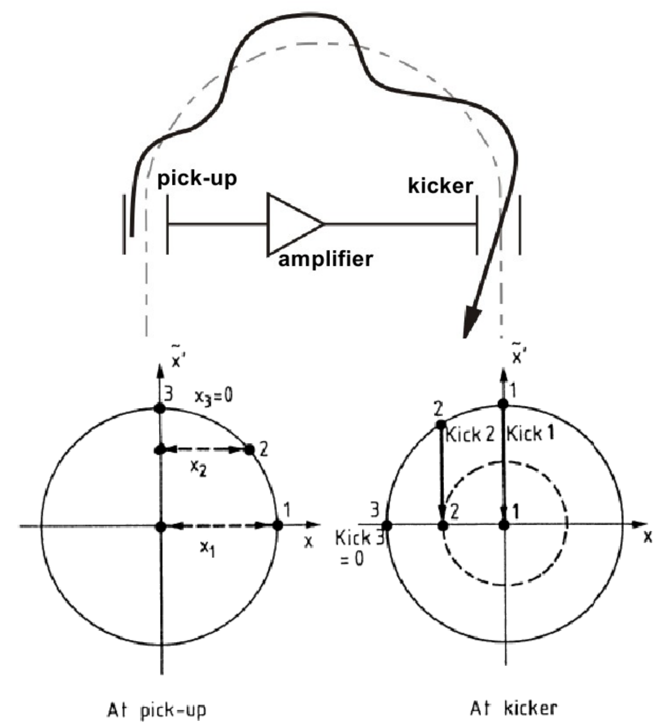


"hot" ion beam
  - watch out for: alignment, RF ripple, e-space-charge, recombination

$$\tau_z \propto \frac{A}{Q^2}\frac{1}{n_e\eta}\beta^3\gamma^5\theta_z^3 \begin{cases}\theta_\perp(s)=\sqrt{\frac{\epsilon}{\beta_\perp(s)}}\\ \theta_{||}=\beta\frac{\Delta p}{p}\end{cases}$$

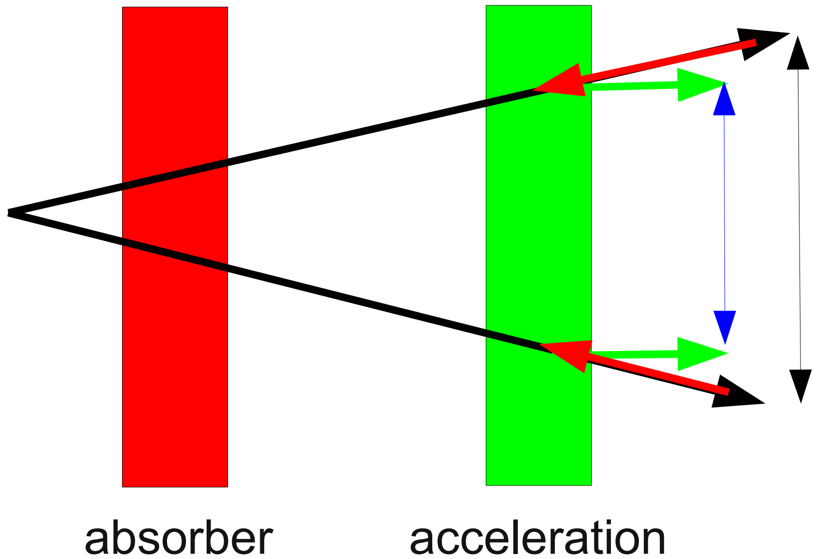
- Stochastic Cooling

Van der Meer & Rubbia → Nobel Prize in '84!



$$\frac{1}{\tau}=\frac{W}{N}\cdot\left(2g-g^2(1+\rho)\right)$$

- Muon cooling:

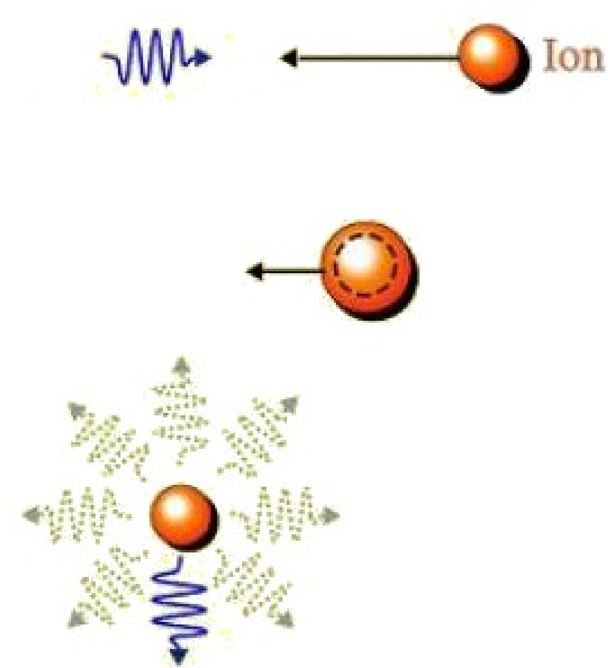


$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta\gamma\beta_\perp}{2}\frac{\langle\theta^2_{rms}\rangle}{ds}$$

$$= -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

- Laser cooling – uses 'Doppler' shift
  ...yet another Nobel Prize in 1997

$$\omega' = \omega\gamma\left(1-\beta\cos(\theta)\right)$$



  – Issue: doesn't work with too many ions...

Exercise IV
– Beam Cooling

- Use the following expression for the cooling rate, and obtain an optimum value for the gain g in the presence of a signal-to-noise ratio ρ. Write down the expression for the cooling rate for this optimum gain.

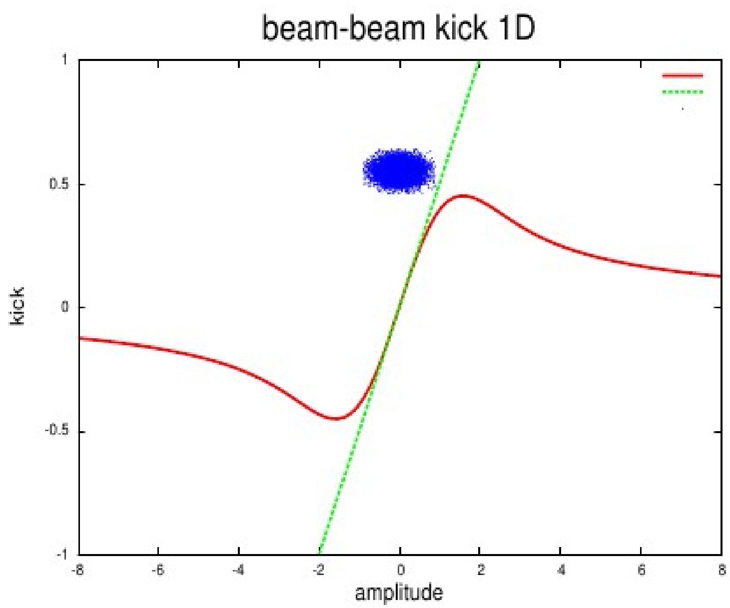
$$\frac{1}{\tau} = \frac{W}{N} \cdot \left(2g - g^2(1+\rho)\right)$$

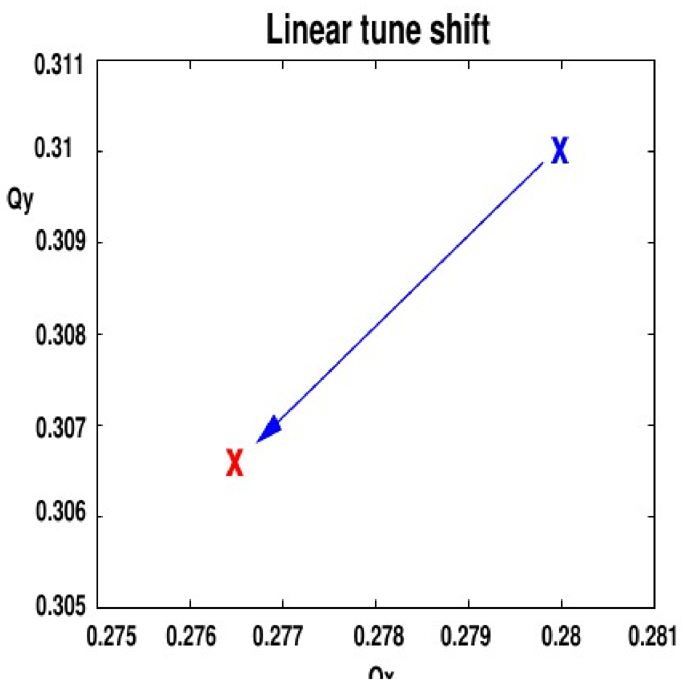
- A cooling system is designed with a central frequency of 300 MHz. What is the sample time and how many particles will be in any one sample of 10^6 a beam of particles circulating in a ring of 25 m radius? (Assume β = 0.96)
- An electron gun has a source potential of 60 kV. Calculate the momentum of protons with the same velocity
- We wish to cool a beam of protons 5 cm in diameter to an emittance of 40π mm mrad. What is the acceptable alignment tolerance on the electron beam?
- If the proton beam has a transverse emittance of 2π mm mrad. What would be its transverse velocity at a β of 10 m? What does this represent in terms of temperature?
- The transitional state of a 100 keV Li+ beam is excited by a laser frequency of 5485 Å. What is the energy level difference which is excited? (1Å = 0.1 nm, h = $4.136 \cdot 10^{-15}$ eVs)
- Write a short computer program to simulate stochastic cooling with and without mixing.

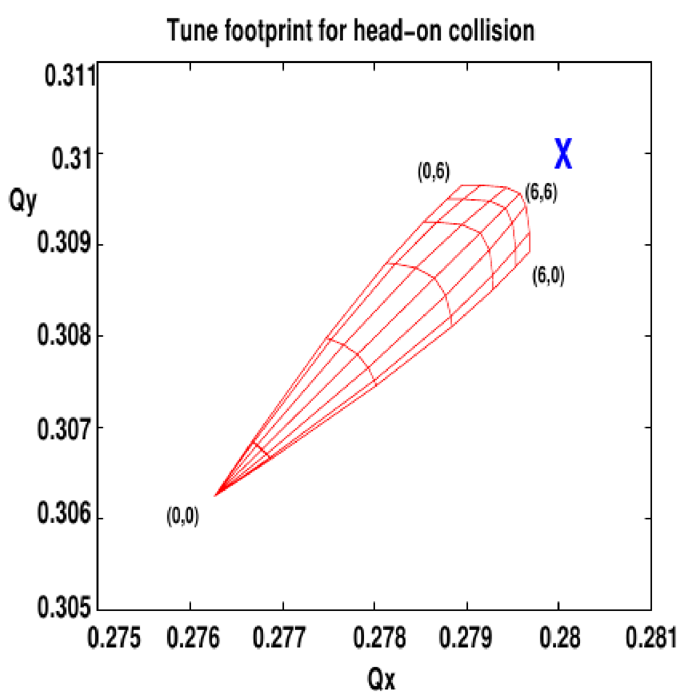
Questions & Answers

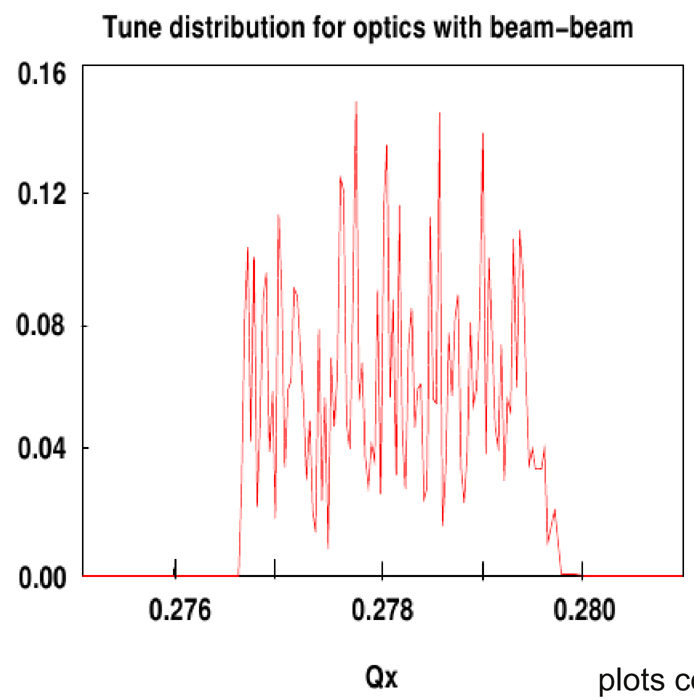
additional slides follow

Beam-Beam in a Nutshell – Head-On

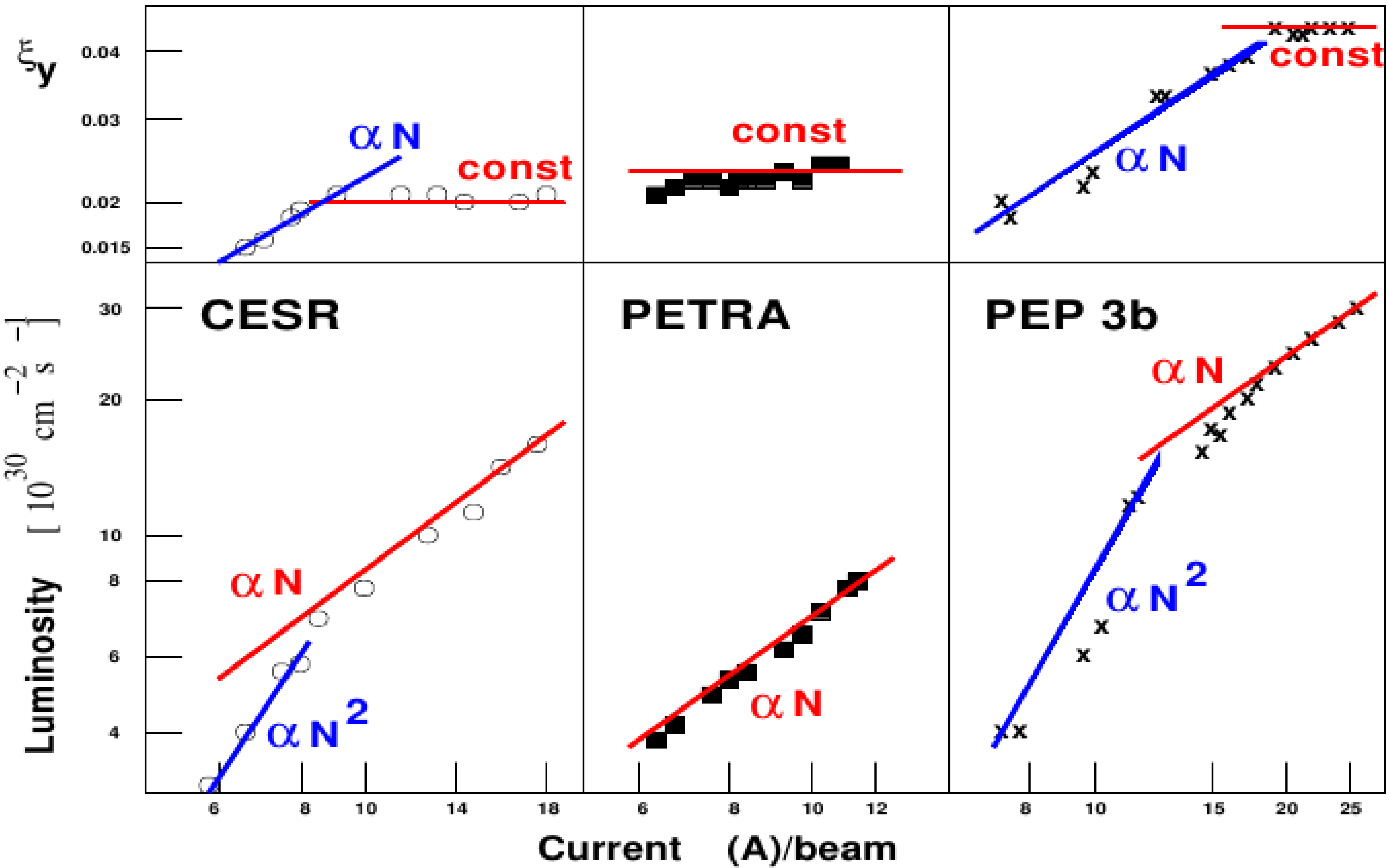








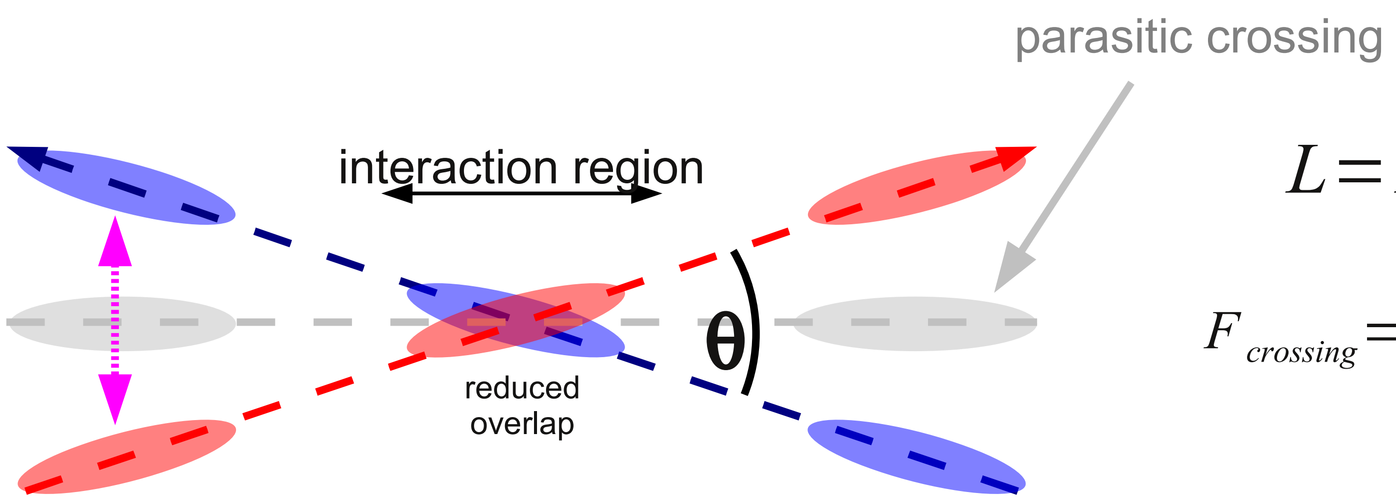
plots courtes W. Herr



ξy
0.04
0.03
0.02
0.015
α N
const
const
α N
const
Luminosity [10^30 cm^-2 s^-1]
30
20
10
8
6
4
CESR
PETRA
PEP 3b
α N
α N^2
α N
α N
α N^2
6
8
10
14
18
6
8
10
12
8
10
15
20
25
Current (A)/beam

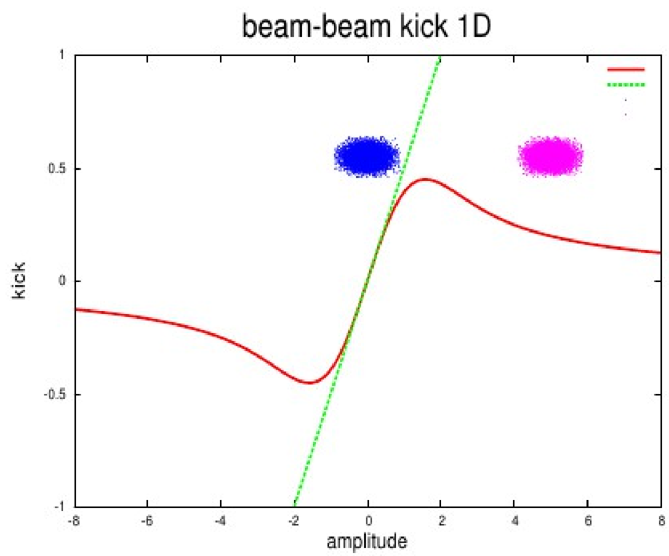
Beam-Beam in a Nutshell – Long Range

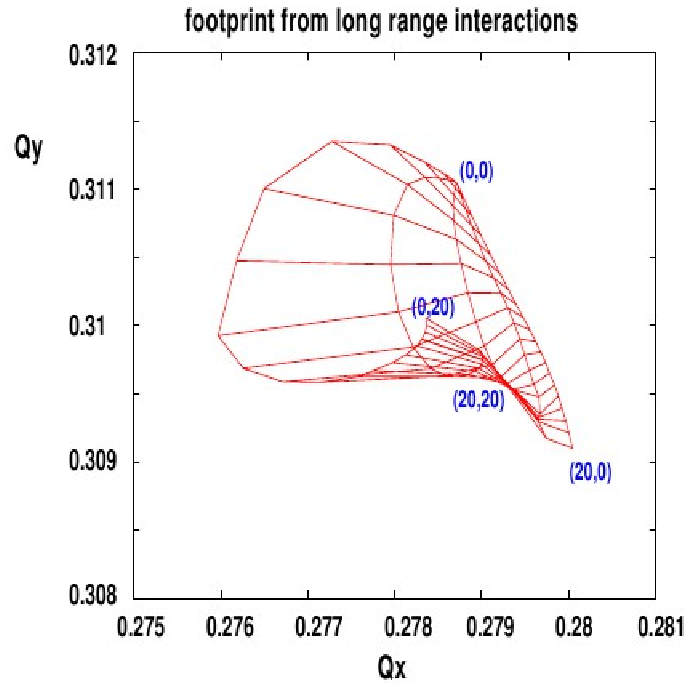
- Crossing angle θ to avoid additional parasitic crossings → reduced bunch overlap
- "crab cavities" to compensate this effect: rotate the bunches before and after the IR



$$L = L_0 \cdot F_{crossing} \cdot \ldots$$

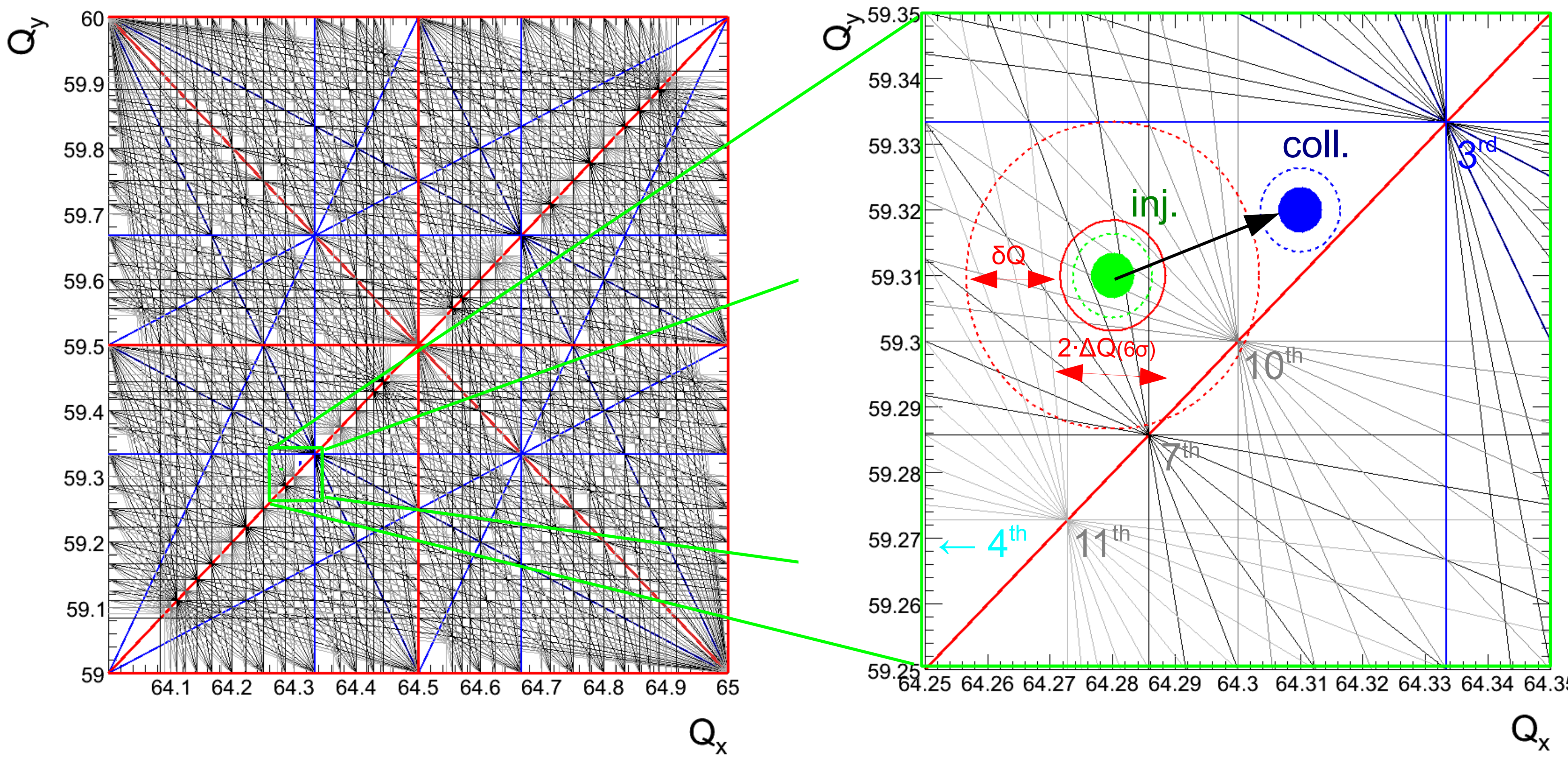
$$F_{crossing} = \frac{1}{\sqrt{1 + \frac{\sigma_s}{\sigma_{x,y}} \tan(\theta/2)}}$$





Beam-Beam Tune Footprint

- Example LHC:



- N.B. need to stay much further off these resonance lines due to
  - finite tune width: chromaticity, space charge, momentum spread, detuning with amplitude and resonance's stop band itself

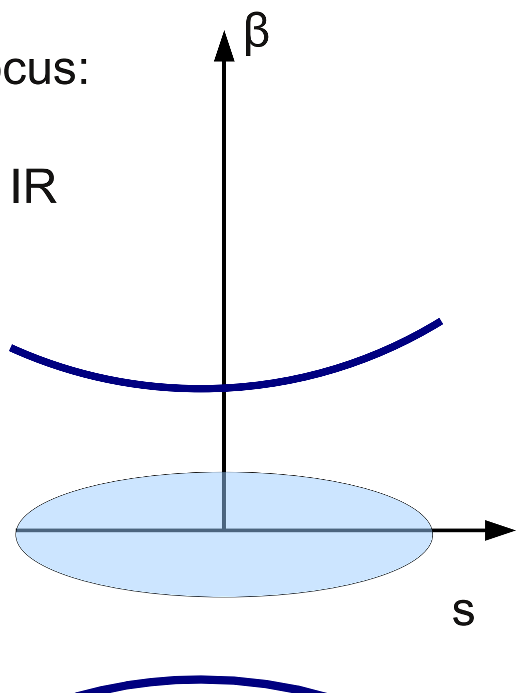
Luminosity: Hour Glass Effect

- Small beam sizes s(s) IR limited by final focus beta-function b_0. (LHC: b_0 = 0.55 m)
  - max possible beta function around the detector
    - large b_{max}: more sensitive to field errors and failures (many effects scale with β)
    - max available final focus quadrupole gradient
  - 'hour glass' effect if β^* similar to bunch length s_s:

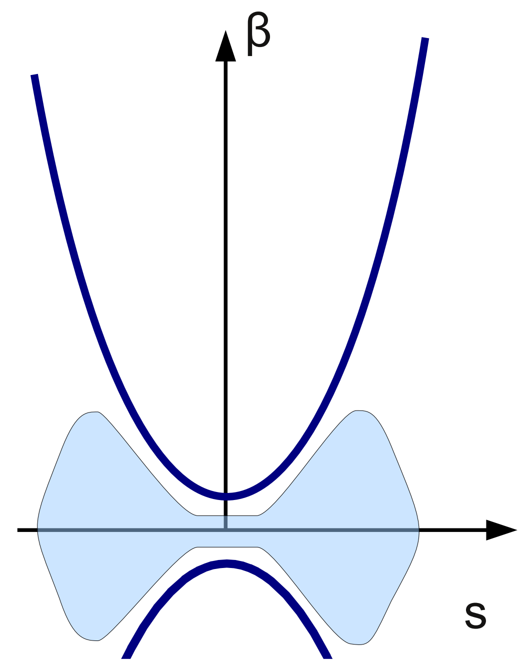
$$\sigma(s)=\sqrt{\frac{\epsilon\,\beta(s)}{\gamma_{rel}}}$$

$$\beta(s)=\beta_0\left(1+\left(\frac{s}{\beta_0}\right)^2\right)$$

Weak final focus:
$\beta_0 > s_s$
$s_s \sim$ const @ IR



Strong final focus:
$\beta_0 < s_s$
$s_s = s_s(s)$



counteract with shorter bunches:
protons: decrease e_s while keeping $e_{x/y}$ constant or decreasing ... (not trivial)

- Integrated Luminosity L_{int}

$$L_{int} = \iint_0^T L(s, \epsilon, \ldots, t)\, dt$$

1[st] order:

$$\langle L \rangle \approx L_0 \cdot \tau \cdot \frac{1 - e^{\frac{-t_r}{\tau}}}{t_r + t_p}$$

- run time $t_r \approx$ 10 hours ("free" parameter)
- preparation time t_p
  - magnet cycle
  - injectors,
  - detectors
  - ...
- beam lifetime τ
  - tune
  - tune spread
  - ...
  - (numerical aperture)
  - electron cloud

- Recipes to win the SUSY/Higgs Grandprix
  - optimise the machine (τ, t_p)

Picture of the Year: