Tune and Chromaticity Diagnostics

Part II

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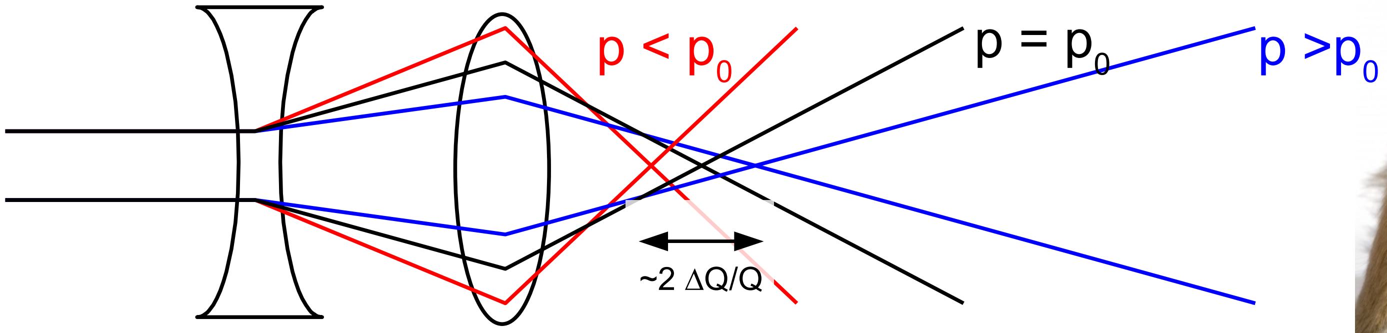
R. Jones, H. Schmickler, C.Y. Tan (FNAL)

Beam Chromaticity - Primer

- Light optics analog: chromatic error

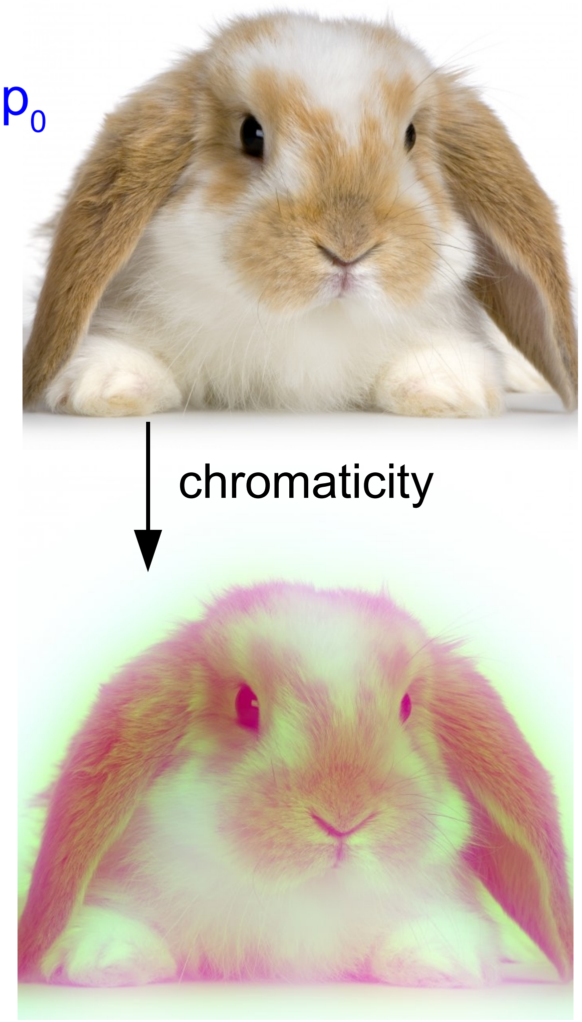


- Tune spread ΔQ/Q dependence on momentum spread Δp/p:

$$\boxed{\Delta Q := Q' \cdot \frac{\Delta p}{p} \quad \text{or:} \quad \frac{\Delta Q}{Q} := \xi \cdot \frac{\Delta p}{p}}$$

  – defines: (normalised) 'chromaticity' Q' (ξ)

→ also 1st order measurement principle



Outline

Part I:

- Recap: What the .... is 'Q', Oscillations Dampening
  - Perturbation Sources, Requirements
- Tune Diagnostics
  - Classic Fourier-Transform Based
    - Detectors: BPMs, Diode-Peak-Detection, (Schottky → F. Casper)
  - Phase-Locked-Loop (PLL) Systems
- Advanced Topic

Part II: → now

- Recap: Definitions, Requirements & Constraints
- Classic Chromaticity Diagnostics
  - Momentum shift Δp/p based Q' tracking methods → LHC examples
- Collective Effects
  - Head-tail phase shift
  - De-coherence based methods: PLL Side-Exciter

Non-Linear Chromaticity

- Tune-shifts may depends not only linearly but also quadratically on Δp/p

  → Second order Chromaticity Q''

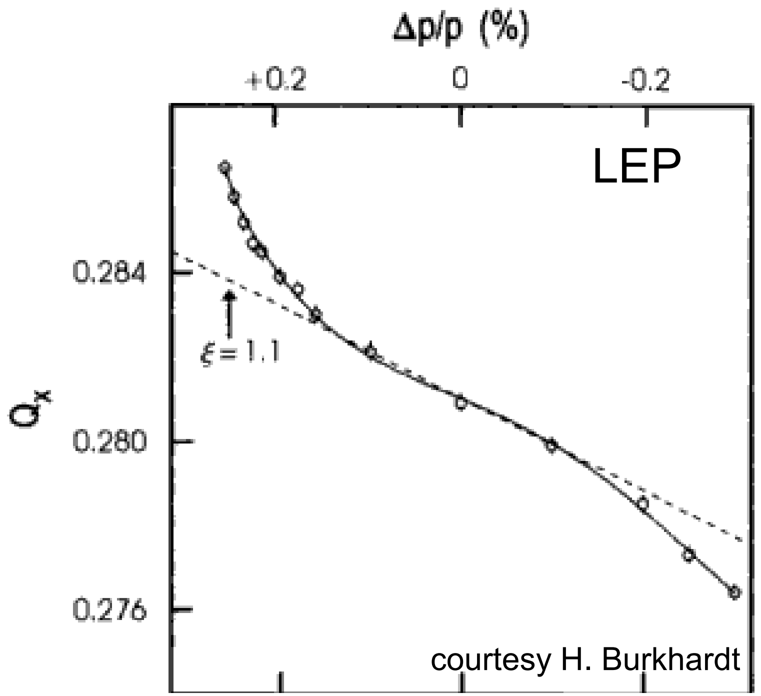
$$\Delta Q = Q'' \cdot \left(\frac{\Delta p}{p}\right)^2$$

- Can be generalised to higher orders Q'''... $Q^{(n)}$:

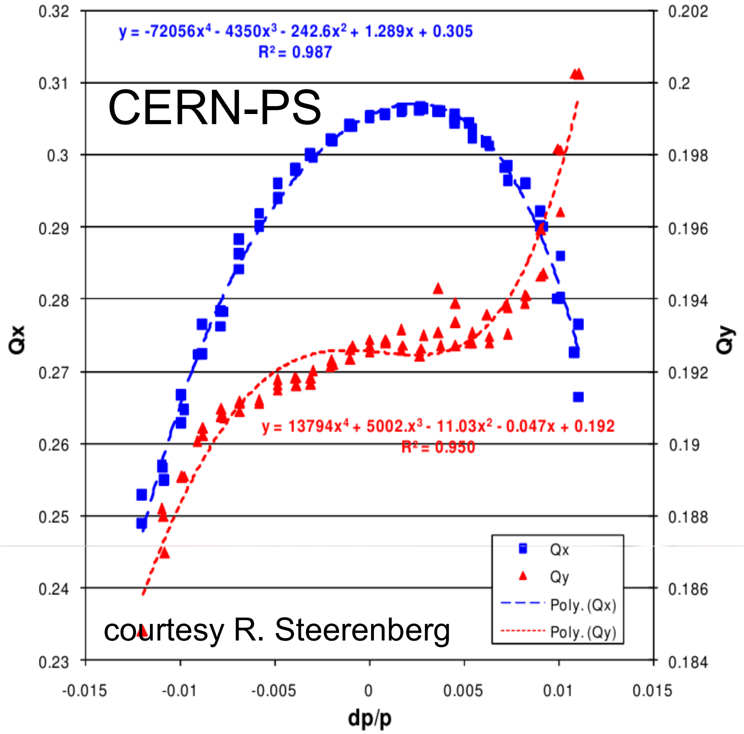
$$\boxed{Q^{(n)} = \frac{\partial^{(n)} Q}{\partial \delta^{(n)}} \quad with \;\; \delta := \frac{\Delta p}{p}}$$

  – Principle stays the same:
    - Measure Q as a function of Δp/p
    - Fit n-th order polynomial to the tune shift
      – returns: Q, Q', Q'', Q''', ...

- However: correction is highly non-trivial!!



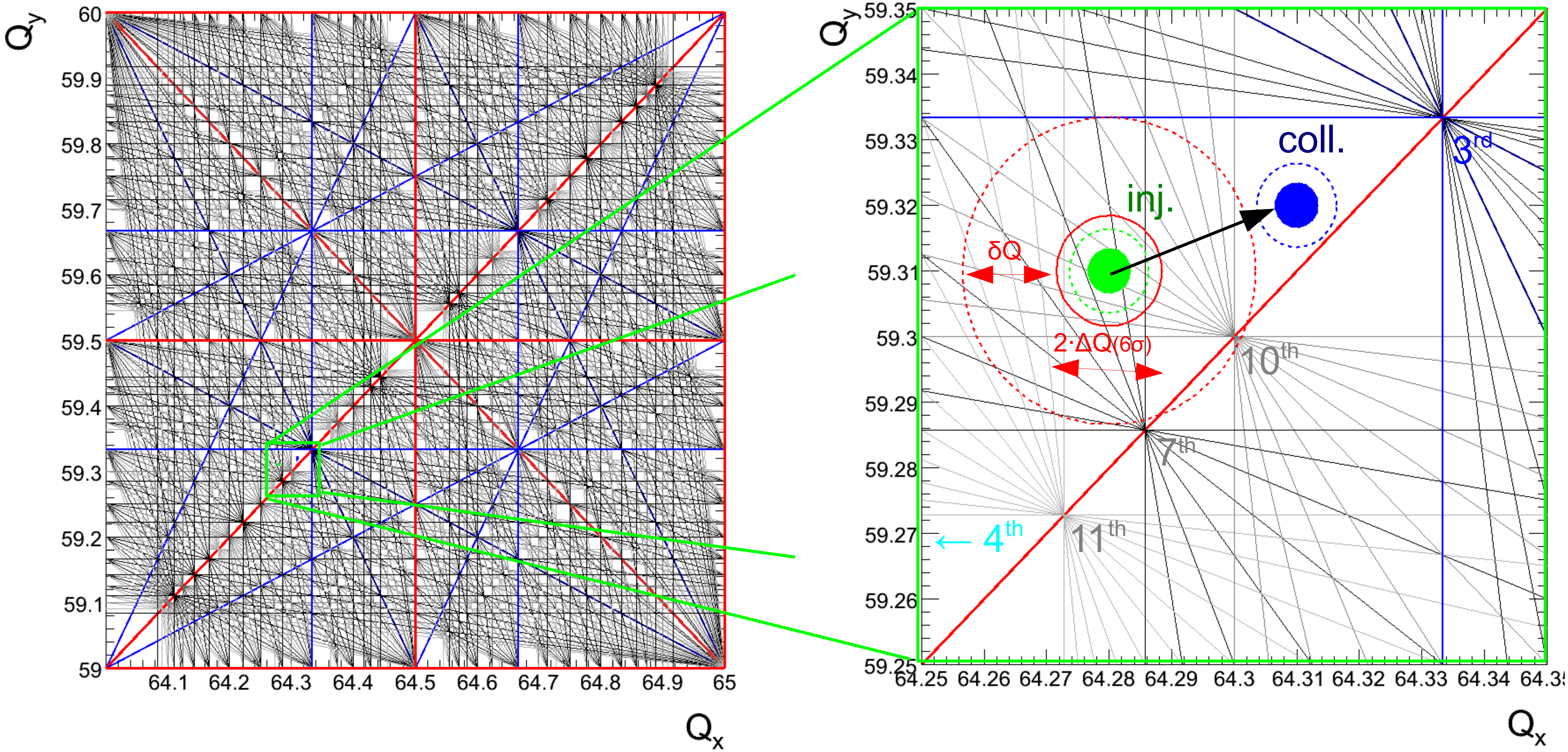
courtesy H. Burkhardt



courtesy R. Steerenberg

Chromaticity Stability Requirements I/III
Why bother about measurement, stability & control of Q', Q'', ...?

- Increases footprint in Q diagram and causes resonances for off-momentum particles
  - Example LHC (RF cavities 'off'):



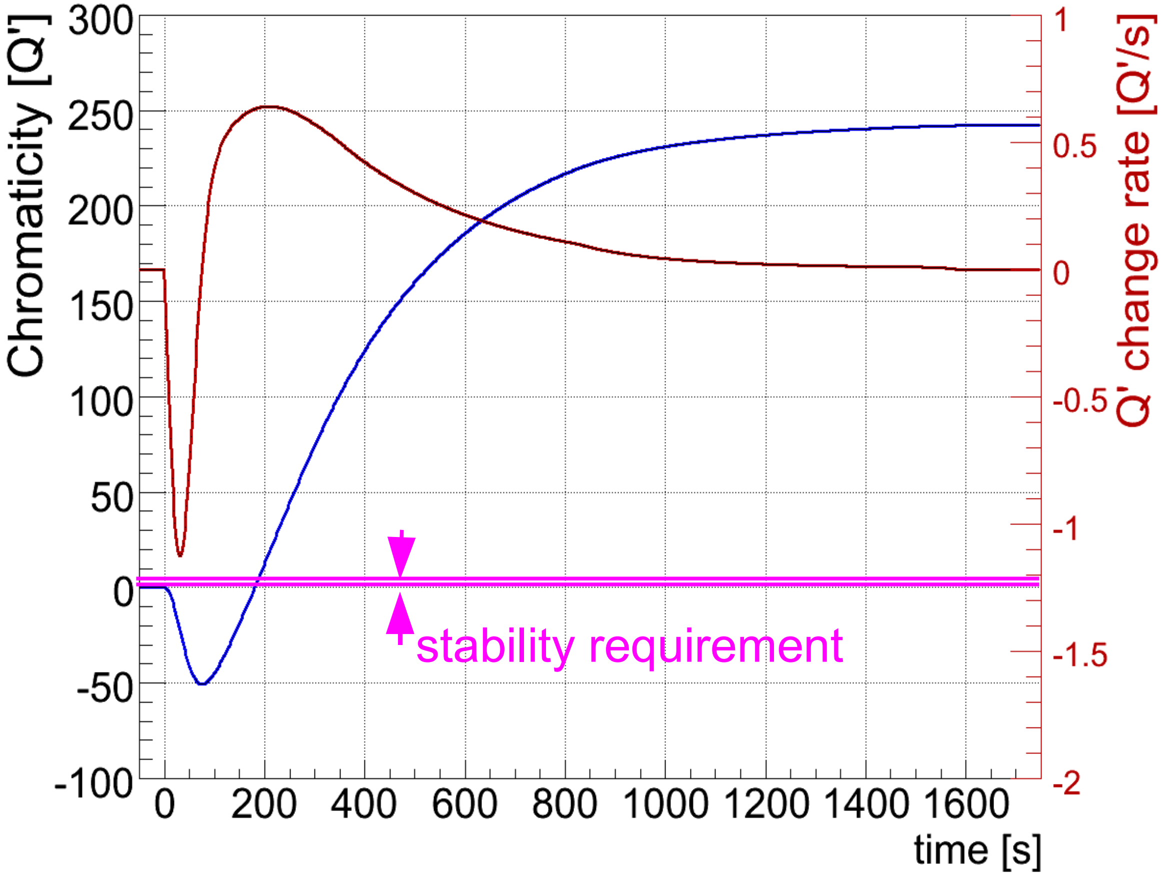
  - need to obey this if we want to have more than one particle in the machine.

- Head-Tail instability → requires positive chromaticity for machines above transition
  - practically all lepton accelerators (e^+e^- collider, light sources, ...)
  - high-energy proton accelerator (Tevatron, RHIC, SPS, LHC, ...)

→ more on this later

What drives Chromaticity Changes?

- Main Q' error sources: quadrupoles' natural chromaticity, field errors on higher order magnets (sextupoles, octupoles, ...) and CO. feed-down of higher order magnets
  - LHC: main dipoles' sextupole error (decay and snapback, persistent currents)



  - Exp. perturbations are about 200 times than required stability!
    - Chromaticity: $\Delta Q'/\Delta t|_{max}$ < 2 s^{-1} ← the critical/difficult LHC beam parameter
    - first machine that requires an active beam-based control of Q'

LHC Base-Line Q/Q' Diagnostics Overview – Q'

- RF momentum modulation

$$Q' = \frac{\Delta Q}{\Delta p / p}$$

← *measured tune change*

← *RF induced momentum change (known)*

  - Measurement procedure (manual – human driven):
    1. Step: measure tune Q_1
    2. Step: change Δp/p (RF cavities), measure tune Q_2 → $\Delta Q = Q_2 - Q_1$
    3. Step: enter ΔQ & Δp/p into above definition → Q'

- Kicked Head-Tail Phase-Shift
  - Q' driven phase shift of bunch head- versus tail-oscillation

- Tune-width and de-coherence based methods
  - PLL Side-exciter & higher order fits

} collective effects
- handle with care

Q' trough RF momentum modulation based method

$$Q' = \frac{\Delta Q}{\Delta p / p}$$

the measured tune change

the RF induced momentum change (known)

- Q' tracking perfomance is essentially given by:
  - Ability to track tune both accurately ΔQ_{res} & fast Δt_{res}:
    - Fourier based (fast) or Phase-Locked-Loop based (trade-off: fast ↔ precise)
  - Limits on allowed momentum modulation:
    - RF momentum acceptance (typically ~ 1 %) & RF power consumption
    - Example LHC: collimation/protection limit excursions < 20 μm → $\Delta p/p \leq 10^{-5}$
      - new regime for Q' tracking loops
- Also intrinsic to this problem: $\boxed{\Delta Q_{res}^{(,)} \cdot \Delta t_{res} = const.}$
  - Example LHC:
    - Tune: $\Delta Q/\Delta t|_{max} < 10^{-3}\ s^{-1}$
    - Chromaticity: $\Delta Q'/\Delta t|_{max} < 2\ s^{-1}$

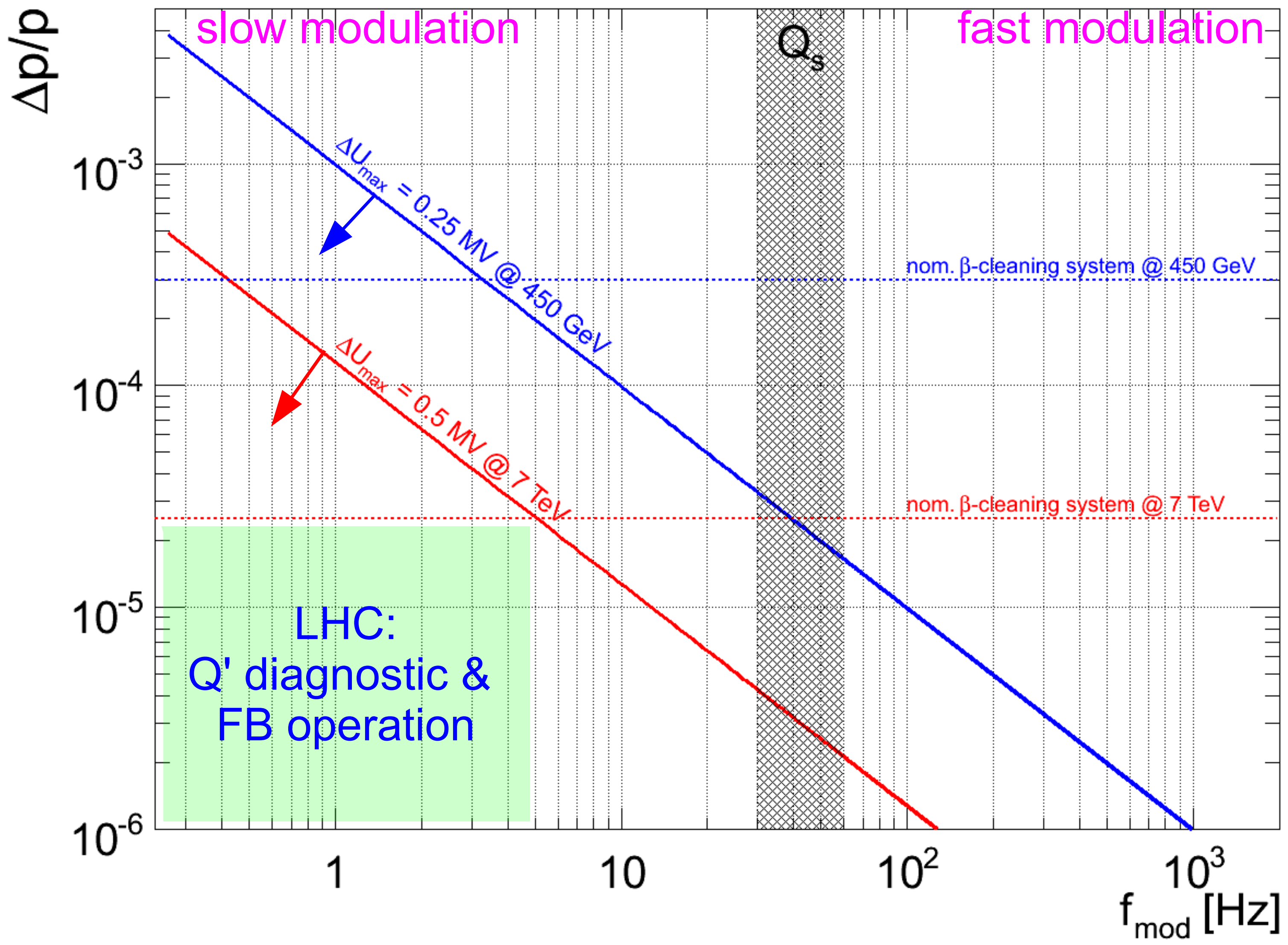
“slow” compared to Q/Q' drifts e.g. in the SPS/CPS/PSB

→ Choose to tackle the Q/Q' measurement in the high accuracy limit.
→ very small but slow (few Hz) Δp/p modulation while tracking Q with a PLL

Q' through RF momentum modulation based method
Example: LHC

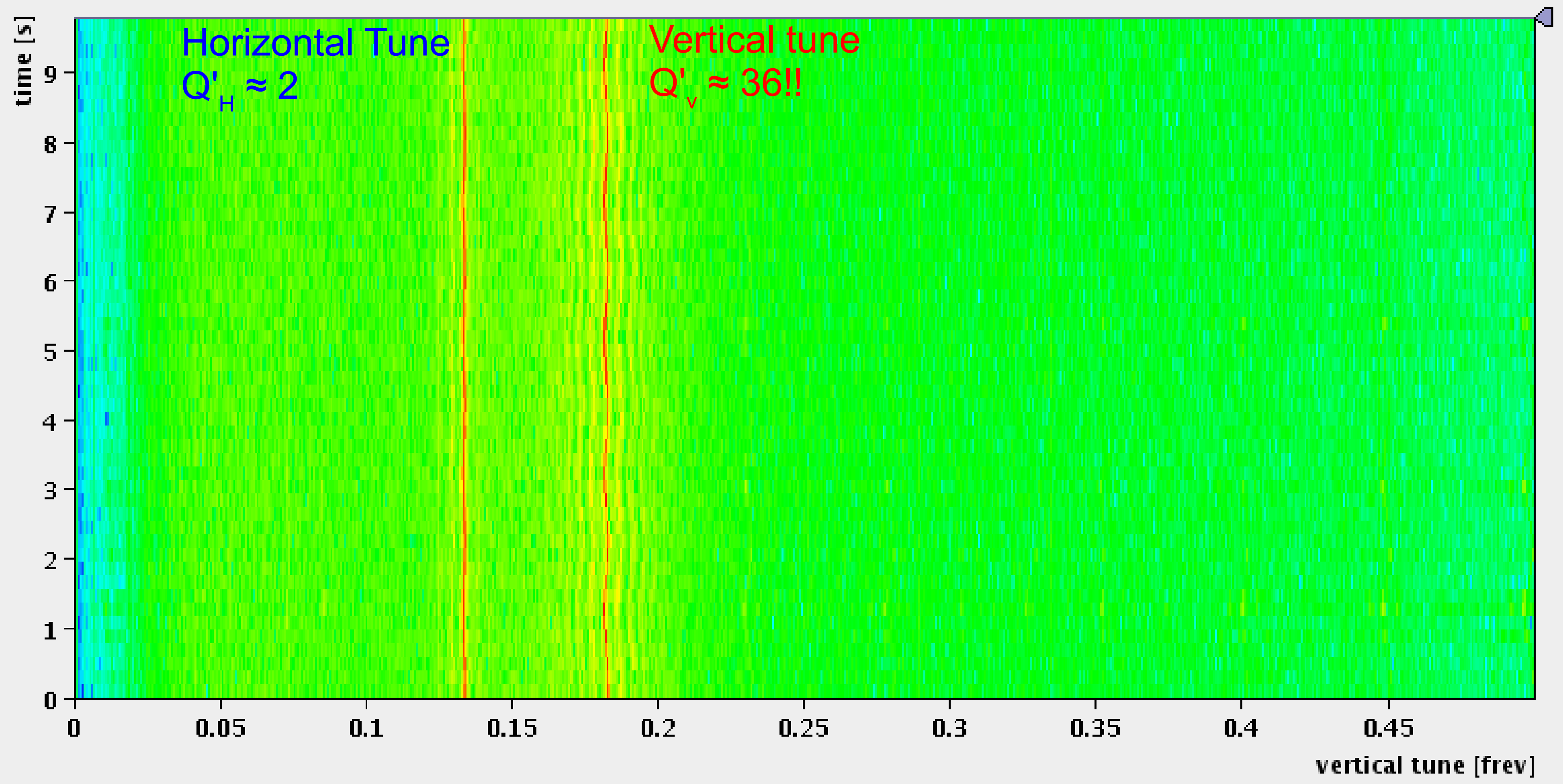
- There are multiple but similar detection techniques:
  - modulation below Qs → classic schoolbook example
  - modulation above Qs → Brüning's and/or McGinnis' method

$$Q' = \frac{\Delta Q}{\Delta p / p}$$



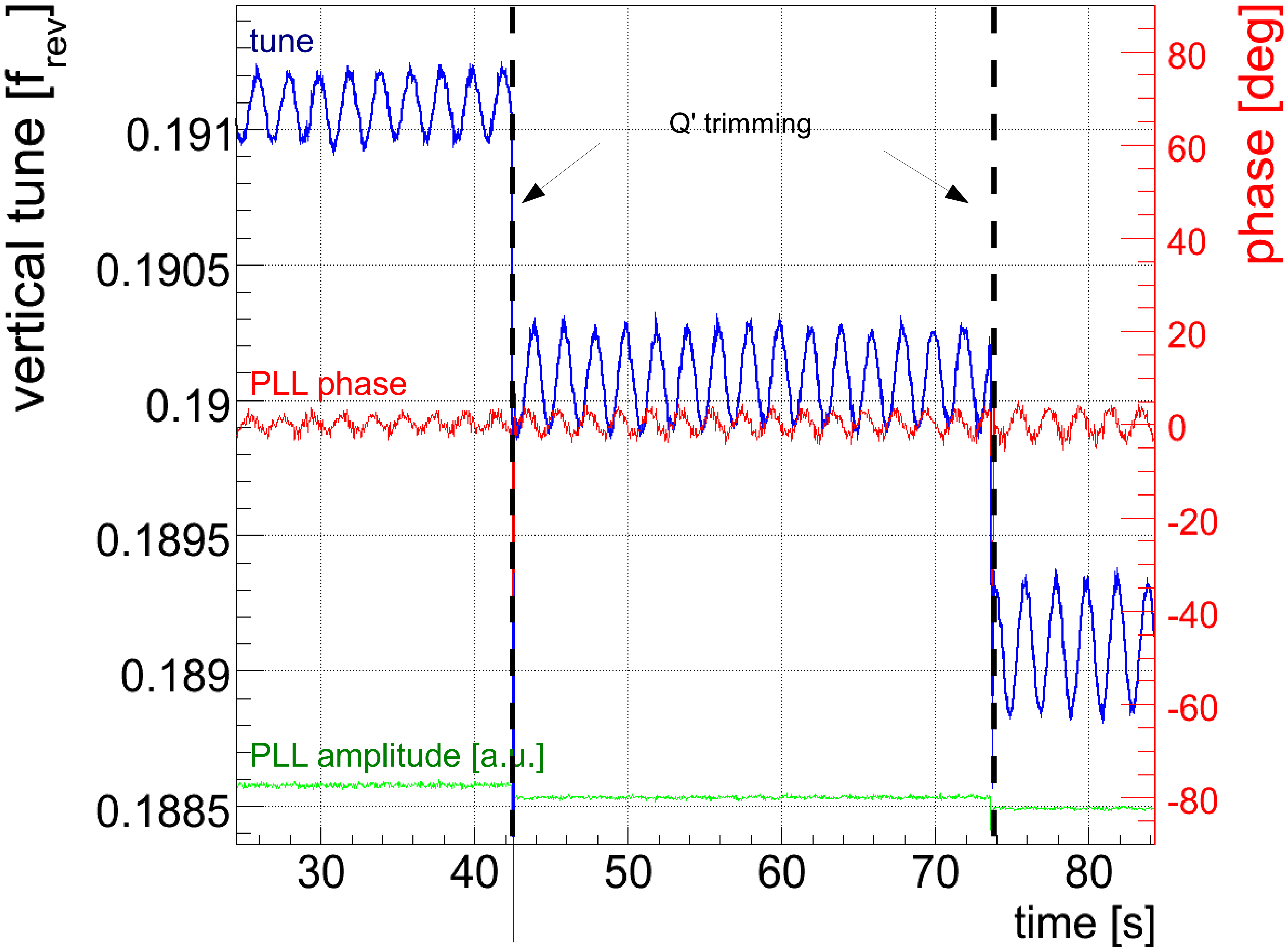
- LHC: RF power permits only slow modulation

Example: FFT based Q/Q' tracking study at the CERN-SPS Modulation Amplitude: $\Delta p/p \sim 2\cdot10^{-5}$ sinusoidal Modulation

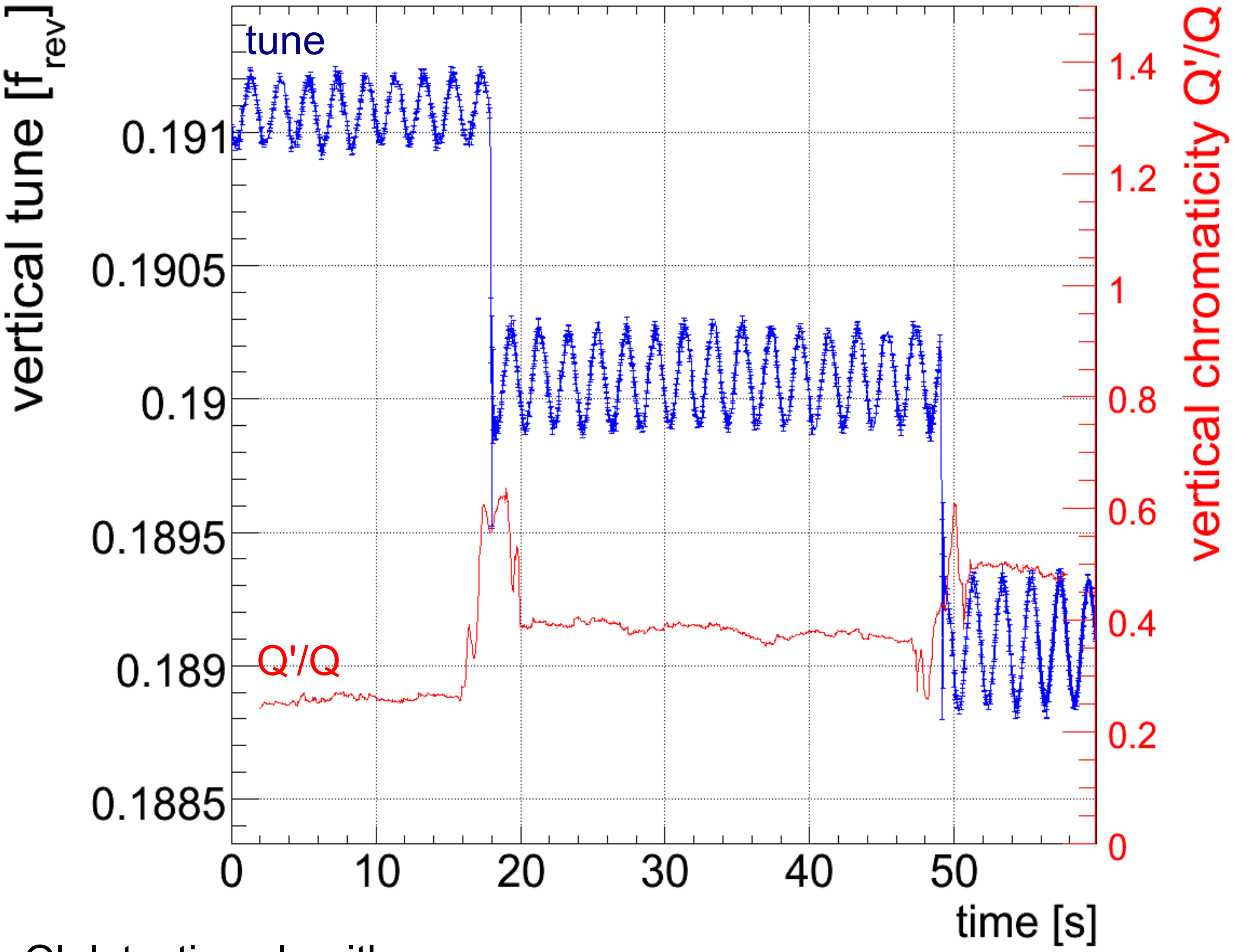


- Q' resolution is limited by tune resolution
  - large Q' increases frequency spread → Landau damping
  - can be improved by e.g. using a PLL
    - achievable frequency resolution $\Delta Q_{res} \sim 10^{-5}...10^{-6}$

Example: LHC-PLL based Q/Q' tracking study at the CERN-SPS
Modulation Amplitude: Δp/p ≈ $1.85 \cdot 10^{-5}$



Example LHC-PLL based Q/Q' tracking study at the CERN-SPS Modulation Amplitude: $\Delta p/p \approx 1.85 \cdot 10^{-5}$



- real-time Q' detection algorithm:
  - Q' resolution ≈ 1 unit
- N.B. tracking transients: ΔQ' feed-down on ΔQ (non-centred orbit)
  - $\Delta Q/\Delta t$ >> $\Delta Q'/\Delta t$ → SPS specific, LHC: $\Delta Q/\Delta t|_{max} < 10^{-4}$/s

Intermezzo
Collective Effects Head-Tail and De-coherence based Q' Diagnostics

Recap: Transverse Beam Dynamics
Chromaticity Perturbation Sources – Quadrupole Driven

- Recap: tune shift due to quadrupole error: $\boxed{\Delta Q=\frac{1}{4\pi}\oint\beta(s)\cdot\Delta k(s)\,ds}$
  - quadrupolar force:

$$f_x(s)=k(s)\cdot x \quad with \quad k(s)=\frac{q}{\boxed{p}}\frac{\partial B}{\partial x}$$

    - focusing: deflection depends on transverse position in the magnet
- Off-momentum particle p → p + Δp: $\left(\quad\frac{1}{1+x}=1-x+x^2+h.o.\quad\right)$

$$f_x(s)\ =\ k(s)\cdot x\approx k_0(s)\cdot x-\underset{\to\Delta k(s)}{\boxed{k_0(s)\cdot\frac{\Delta p}{p}}}\cdot x+\underbrace{k_0(s)\left(\frac{\Delta p}{p}\right)^2}_{\sim Q''}\cdot x$$

  - Inserting 'Δk' into tune shift formula yields 'natural chromaticity' Q'_{nat} definition:

$$\boxed{\Delta Q=-\frac{1}{4\pi}\left[\oint\beta(s)\cdot k(s)\,ds\right]\cdot\frac{\Delta p}{p_0}:=Q'_{nat.}\cdot\frac{\Delta p}{p_0}}$$

    - ~ number of quadrupoles (~ accelerator circumference)
    - always negative (since β(s) > 0)
      - drives head-tail instability (all lepton and hadron colliders above transition)

→ needs to be compensated for nearly all (big/high intensity) machines

Transverse Beam Dynamics
Chromaticity Perturbation Sources – Sextupole Driven

Hill's equation

$$\boxed{z'' + k(s)\cdot z = f(z)}$$

- Sextupolar field:

$$f_{x/y}(s) = \begin{Bmatrix} +\frac{1}{2}m(s)\cdot\left(x^2 - y^2\right) \\ -m(s)\cdot x\cdot y \end{Bmatrix} \quad with \quad m(s) = \frac{q}{p}\frac{\partial^2 B}{\partial x^2}$$

  - Off-Momentum particle passage through sextupole (assume y=0): $x \rightarrow D\cdot\frac{\Delta p}{p} + x_\beta$
    - keep only relevant order (estimate: D ~ m, Δp/p ~10^{-4} & x_β ~ 10^{-4} m)

$$\begin{aligned} f_x(s) &= +\frac{1}{2}m(s)\cdot\left[\left(D\cdot\frac{\Delta p}{p} + x_{beta}\right)^2\right] \\ &= +\frac{1}{2}m(s)\cdot\left[\left(D\cdot\frac{\Delta p}{p}\right)^2 + 2\left(D\cdot\frac{\Delta p}{p}\right)\cdot x_{beta} + x_{beta}^2\right] \\ &= \underbrace{+m(s)\left(D\cdot\frac{\Delta p}{p}\right)}_{\sim Q'}\cdot x_{beta} + \underbrace{\frac{1}{2}m(s)\cdot\left(D\cdot\frac{\Delta p}{p}\right)^2}_{\sim Q''} + \underbrace{\frac{1}{2}m(s)\cdot x_{beta}^2}_{\rightarrow \text{Landau damping}} \end{aligned}$$

    - linear natural chromaticity compensated if $m(s)\cdot D(s) = k_0(s)$
  - General linear chromaticity compensation relation:

$$\boxed{Q' = \frac{1}{4\pi}\oint\left[D(s)m(s) - k(s)\right]\beta(s)\,ds}$$

Chromaticity Stability Requirements
Head-Tail Instability I/II

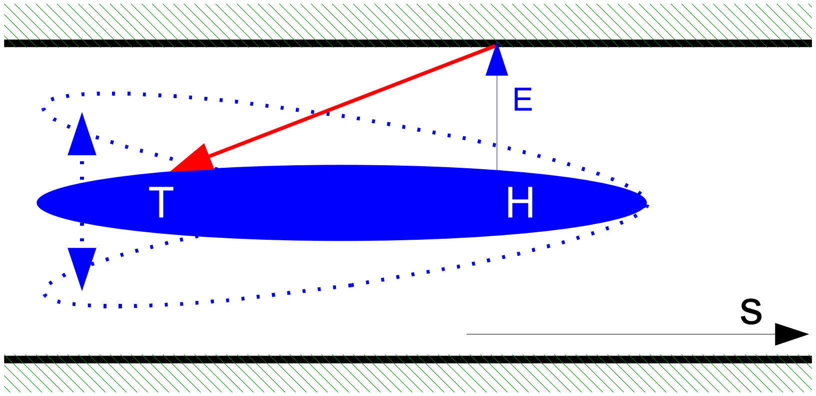
- First discovered at ACO and Adone (1969)[1]
  - mixing of longitudinal and transverse motion
    - Phase advance change:

$$\Delta Q := Q' \cdot \frac{\Delta p}{p}$$

    - Synchrotron oscillation:

$$\frac{\Delta p}{p} = \frac{\hat{\Delta p}}{p} \cdot \sin(\omega_s \cdot n + \phi_i)$$

→ Head and Tail swap position after half a sync. period!



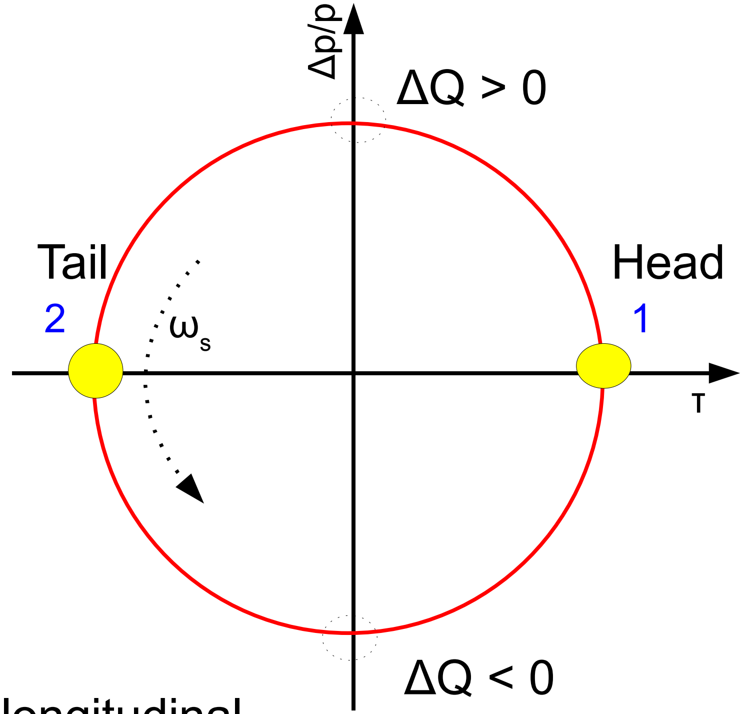
transverse

- Damping time constant (1st order):

$$\frac{1}{\tau_{HT}} \sim N_b \cdot \frac{\hat{\tau}\boxed{Q'}}{Q^2 \cdot \underbrace{(\alpha_c - 1/\gamma_{rel})}_{\eta}}$$

- Head-Tail motion becomes unstable if $\frac{1}{\tau} \overset{!}{<} 0$
  - above transition (η > 0) & Q' < 0

→ keep Q' slightly positive!



longitudinal

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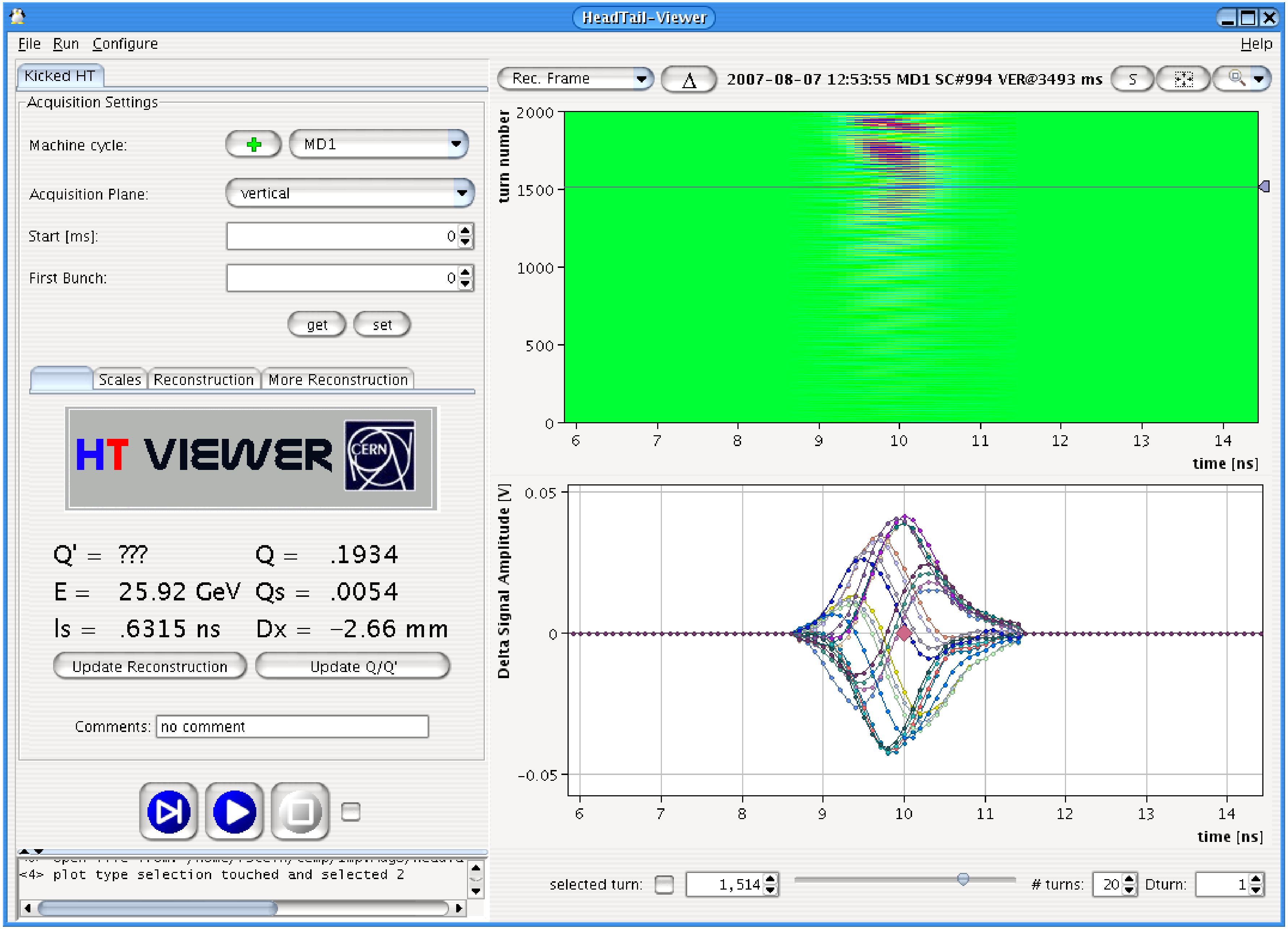
[1] M. Sands, *“The Head-Tail Effect: An Instability Mechanism in Storage Rings”*, SLAC-TN-69-008, 1969

Chromaticity Stability Requirements II/III
Head-Tail Instability II/II

- Measurement in the CERN-SPS (26 GeV, above transition)



2007-08-07 12:53:55 MD1 SC#994 VER@3493 ms

Machine cycle: MD1

Acquisition Plane: vertical

Start [ms]: 0

First Bunch: 0

Q' = ??? Q = .1934

E = 25.92 GeV Qs = .0054

ls = .6315 ns Dx = −2.66 mm

Comments: no comment

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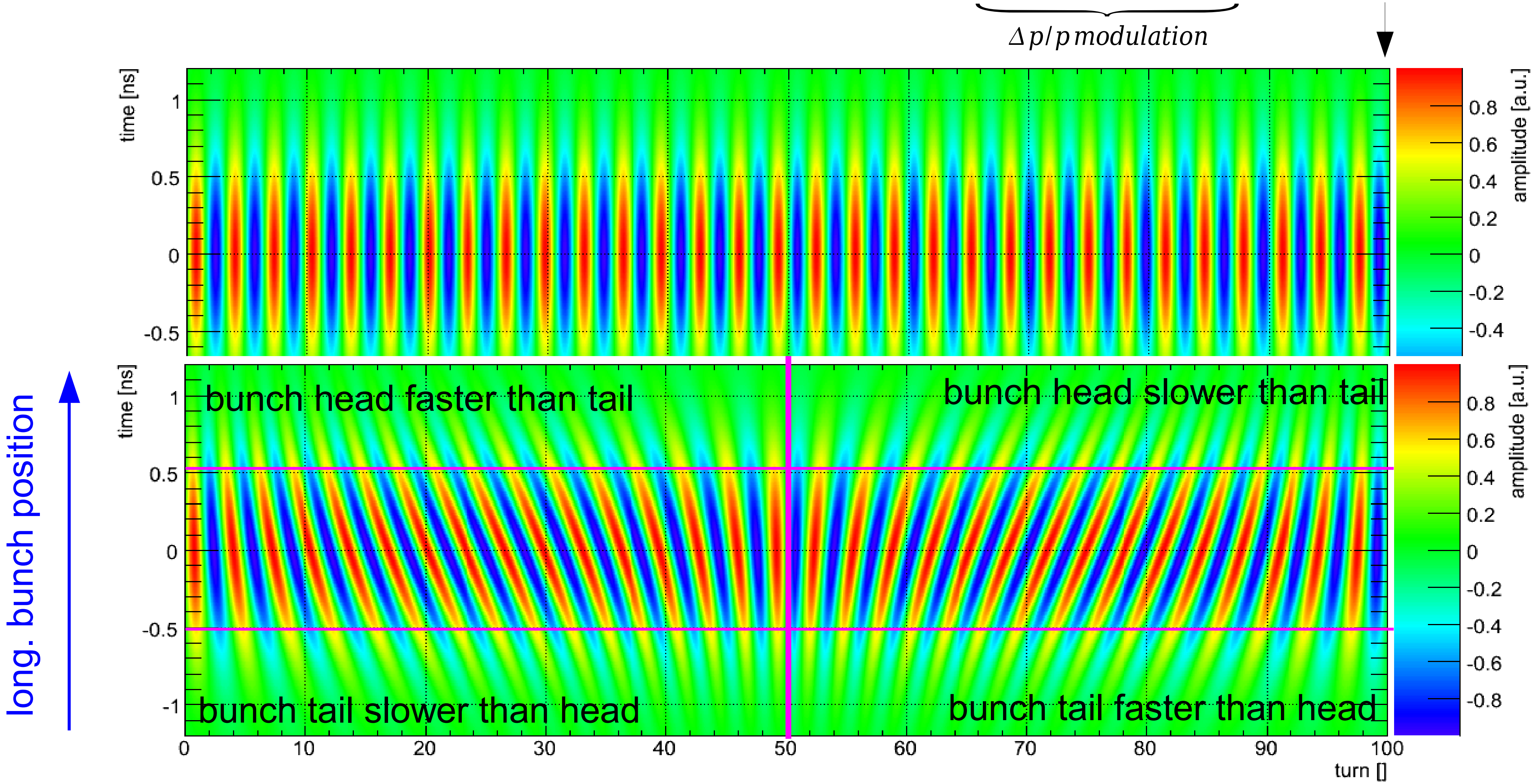
turns: 20 Dturn: 1

Head-Tail based Q' Diagnostic I/II

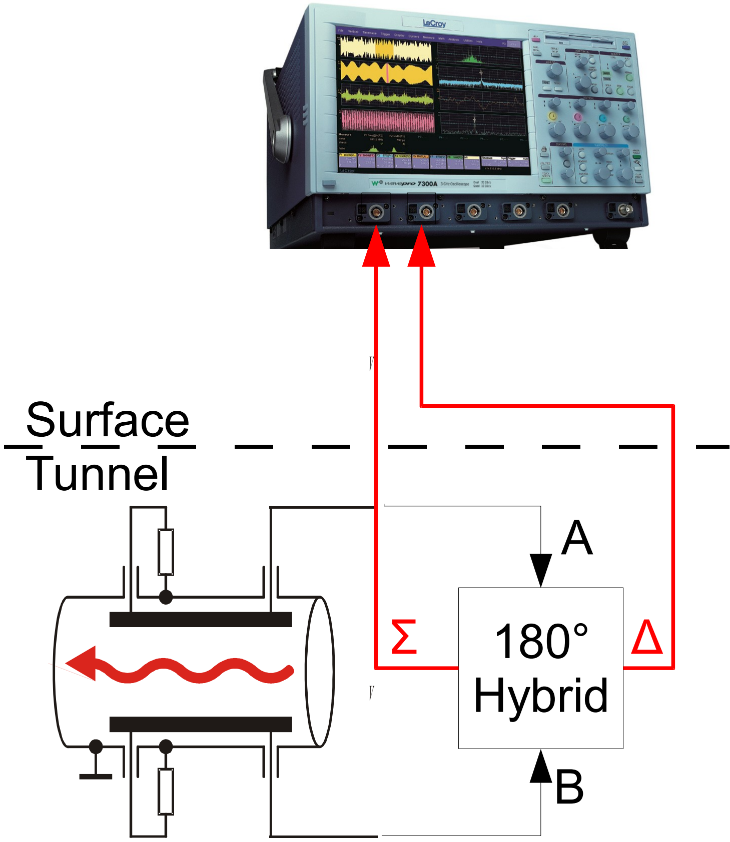
- Instability can be exploited to give in estimate on Q' in the first place:
  - track two slices, 'head' and 'tail' in the bunch distribution: $\Delta z_{HT}(n) \propto \sin(\psi_{HT}(n))$
    (tune: Q, long slice position: τ, synchrotron frequency: ω_s, turn: n)
    - Phase difference of betatron oscillations:

$$\psi_{HT}(n) = 2\pi Q\cdot n + \Delta\phi_\beta \quad with \quad \Delta\phi_\beta \approx Q'\cdot \underbrace{\frac{\omega_0\hat{\tau}}{\eta}\cdot\sin(\omega_s\cdot n)}_{\Delta p/p\, modulation}$$

one synchrotron period



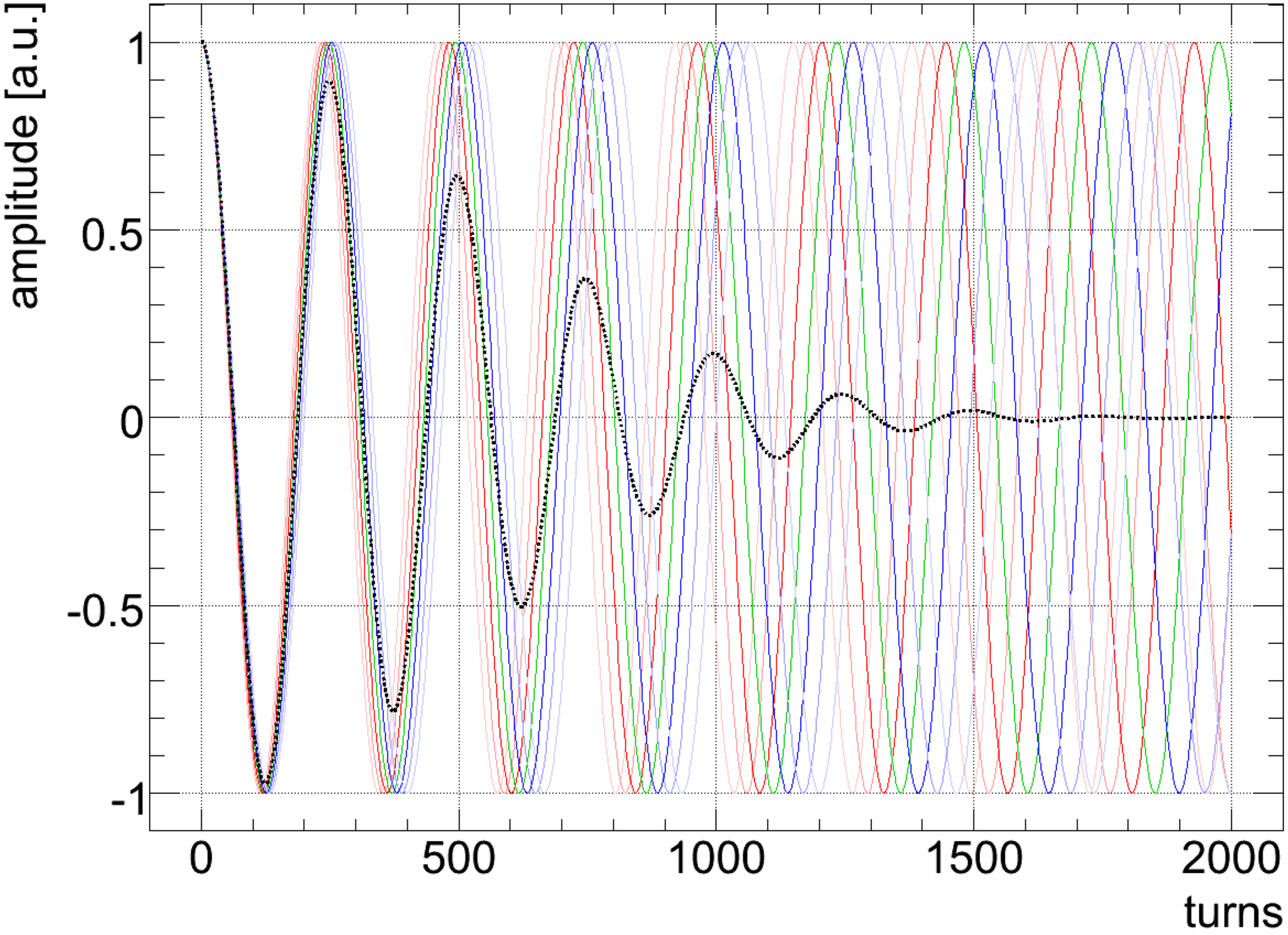
Head-Tail based Q' Diagnostic II/II



- Implemented/tested at CERN-SPS, Tevatron, LHC:
  - Long strip-line (60 cm, to avoid signal-reflection mixing)
  - Σ-Δ hybrid (removes common mode signal)
  - Fast-sampling to resolve bunch structure
    - ~ ns bunch length → GHz scope bandwidth
  - Need to compensates for non-beam effects:
    - pickup- & hybrid response, cable dispersion, ...
  - Phase detection: Hilbert transform (strips-off the sine)
- Limitations:
  - damping: synchrotron radiation, impedance, amplitude de-tuning and other high order effects driving HT instabilities
  - low synchrotron tune → de-coherence more dominant (damped signal)
  - RF bucket non linearities (dependence of synchrotron tune on amplitude)
  - Like BPM based Q systems: kick amplitudes (1..2 σ) → emittance blow-up
    → under investigation: Diode-detection based sampling (BBQ like),
    Challenge: fast GHz sampling with nm resolution

Tune-Width based Q' Diagnostic I/V
Recap: "Landau Damping"

- Individual bunch particles usually differ slightly w.r.t. their individual tune
  → Literature: "Landau Damping" (Historic misnomer: particle energy is preserved!)



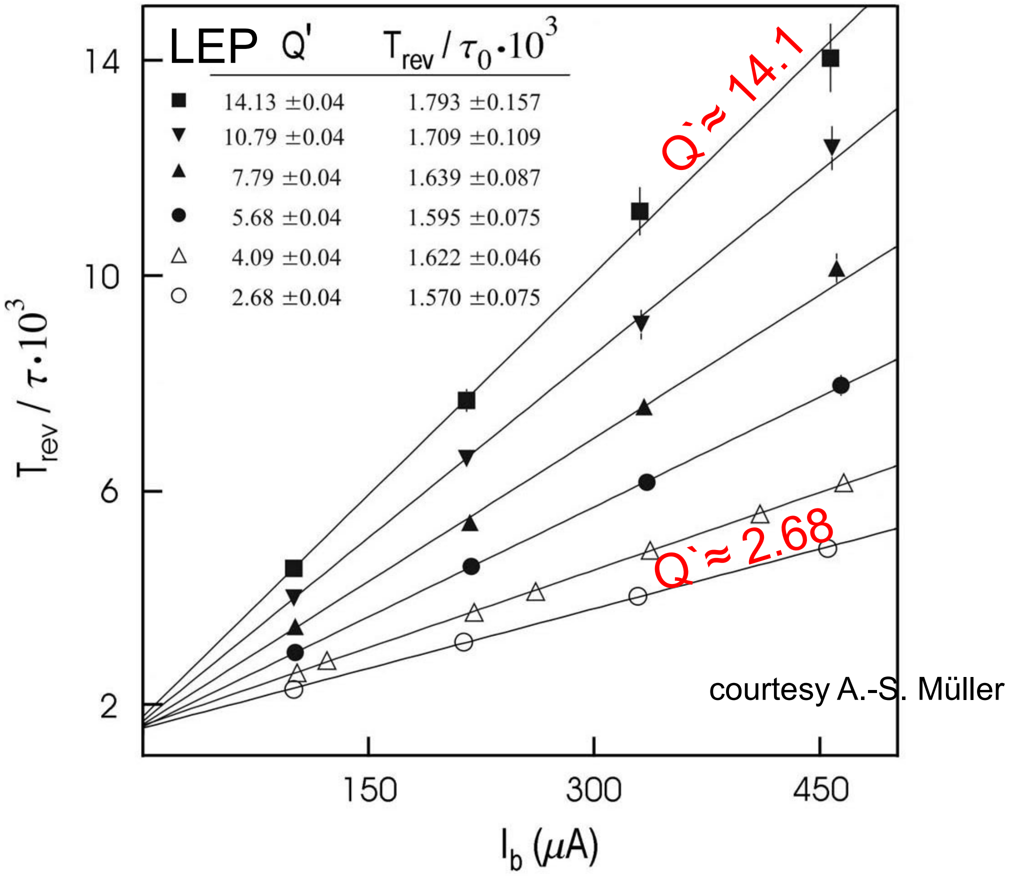
  - E.g. if f(ΔQ) is a narrow Gaussian distribution with with σ_Q << Q:

$$\overline{z}(t)=\overline{z}_0\cdot e^{-\frac{1}{2}\cdot\sigma_Q^2 n^2}\cdot\cos(2\pi Q\cdot n)$$

→ large tune spread ↔ fast damping of e.g. head-tail instabilities

Tune-Width based Q' Diagnostic II/V
Inverse Damping Time as a Function of Beam Current

- Two types of diagnostics:
  - – Kick excitation and measure of de-coherence of beam oscillations
    - limits: emittance blow-up (hadron), dependence on beam current
  - – Continuous width measurement using e.g. PLL (higher sensitivity, less emittance blow-up)

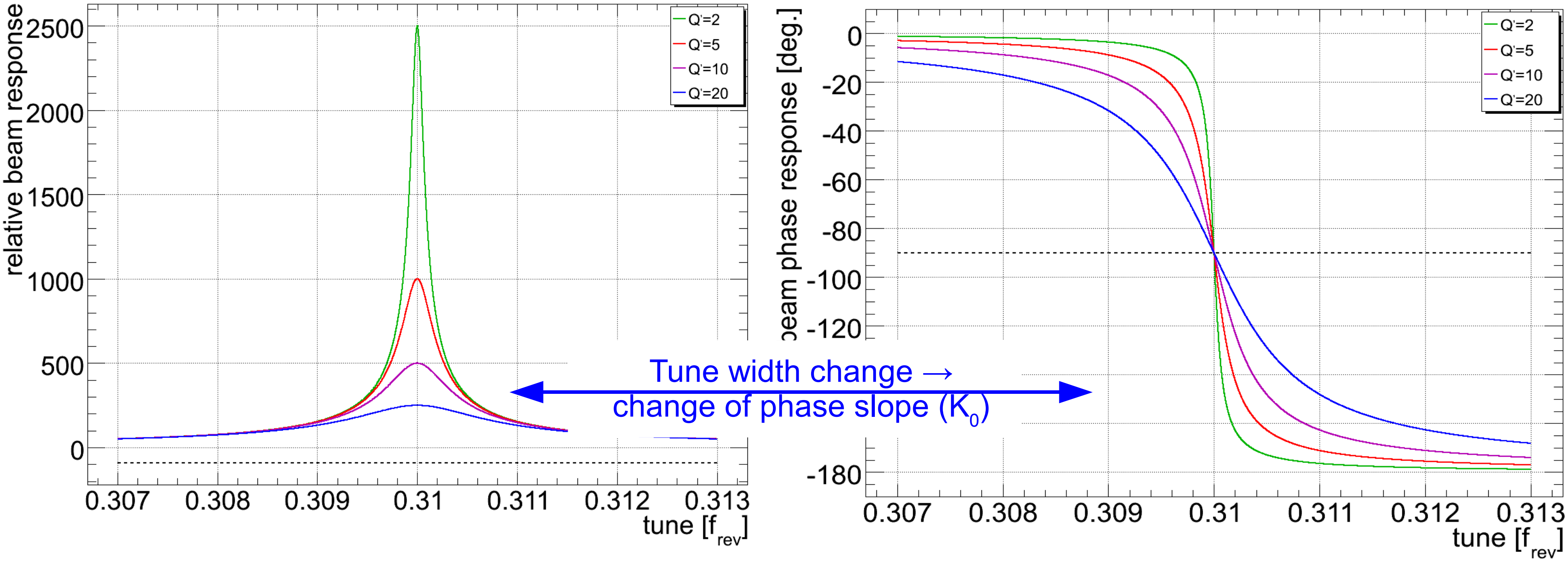


courtesy A.-S. Müller

~ inverse damping time

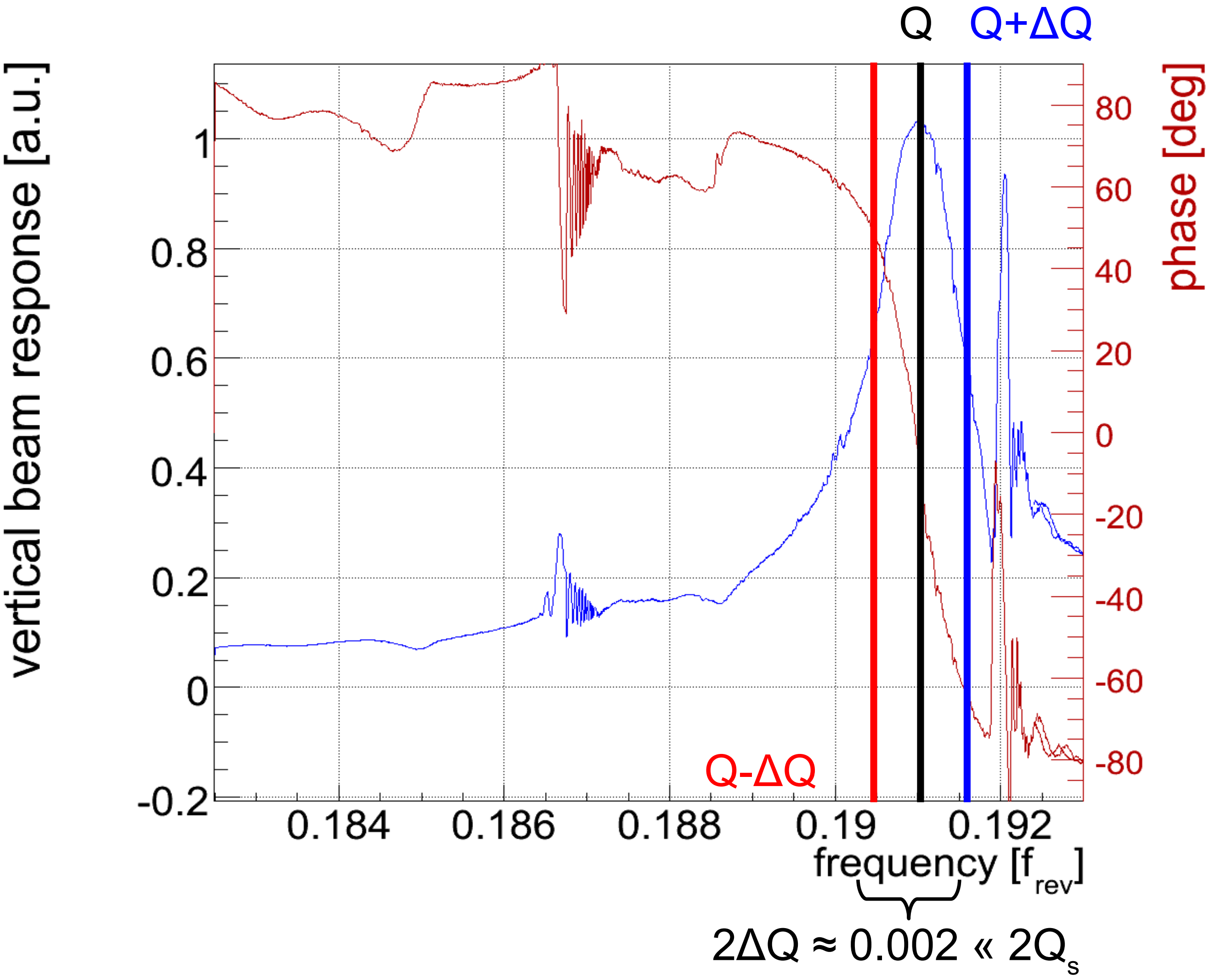
beam current ~ intensity

- Basic idea: larger damping ↔ larger tune width



Tune width change →
change of phase slope (K_0)

Tune-Width based Q' Diagnostic IV/V Measurement using PLL Side Exciter



$2\Delta Q \approx 0.002 \ll 2Q_s$

- Resonant phase change ↔ tune width change

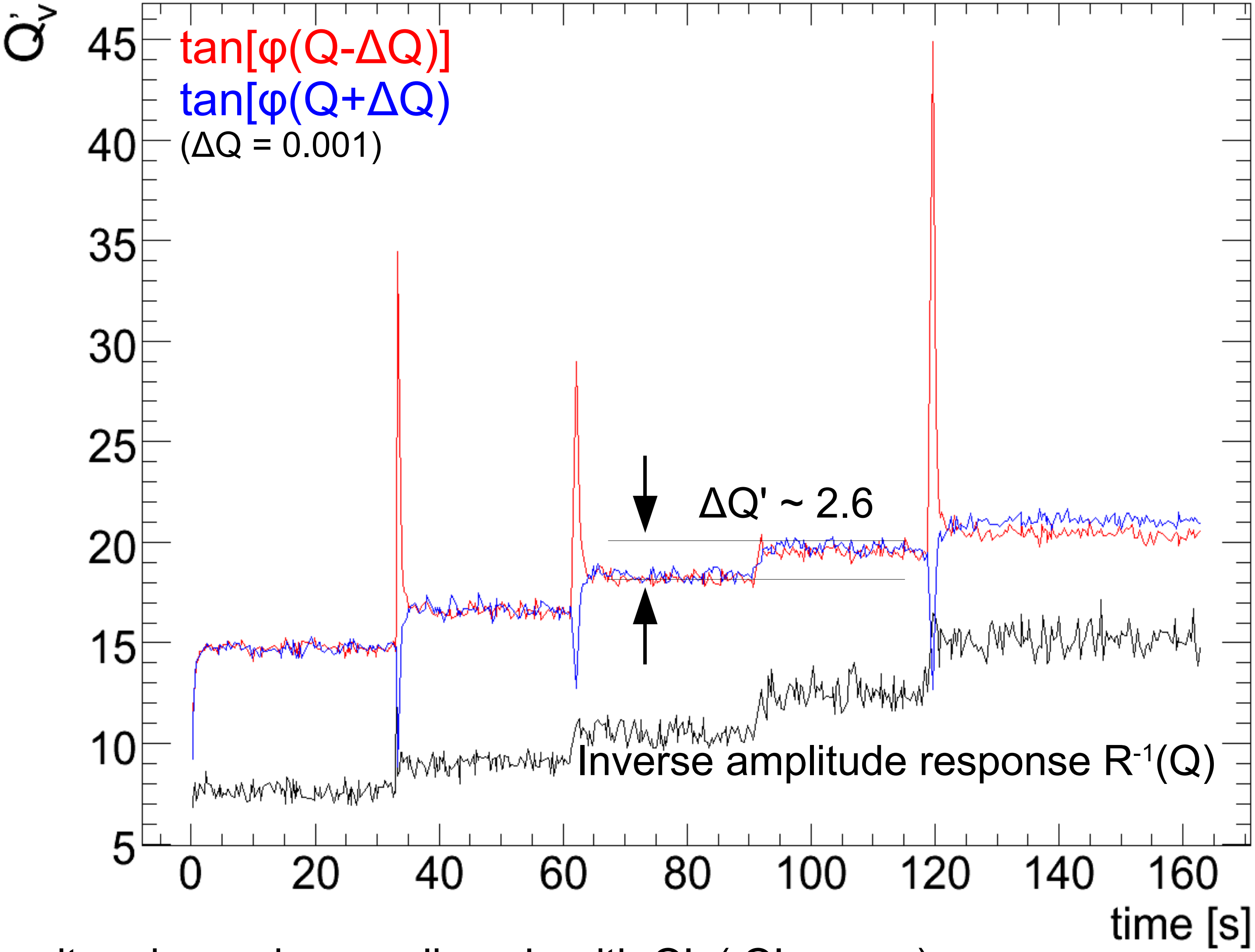
  → “free” real-time tune footprint measurement

  → measurable dependence of ΔQ ~ Q'

driven resonance:

$$\tan(\varphi) \approx \frac{\Delta Q \cdot \omega_Q \omega_D}{\omega_Q^2 - \omega_D^2}$$

Tune-Width based Q' Diagnostic V/V
CERN-SPS: 16th November 2006 ΔQ dependence on Q'



- Side-exciter phase changes linearly with Q' (Q' ~ m(s))
  - no additional momentum modulation
  - Absolute scale requires calibration w.r.t. to classic Q' measurement
  - can extended this to assess higher order non-linear effects (Q'', Q''', ...)

Conclusion

- Chromaticity Diagnostics depends on ability to track the tune within given limits:
  - precision, excitation amplitudes, speed
- Tune Diagnostics depends on ability to resolve small betatron-oscillations:
  - need to reject common mode due to: intensity, closed and dispersion orbit
    - could be be done with any turn-by-turn BPM (but with usally larger excitation)
    - favours passive (Schottky, BBQ) or very sensitive instrumentation (BBQ)
- For more details on Q, Q' diagnostics and instrumentation:

  Proceedings of CARE Workshop on 'Q, Q' and feedback Control', Chamonix, 07

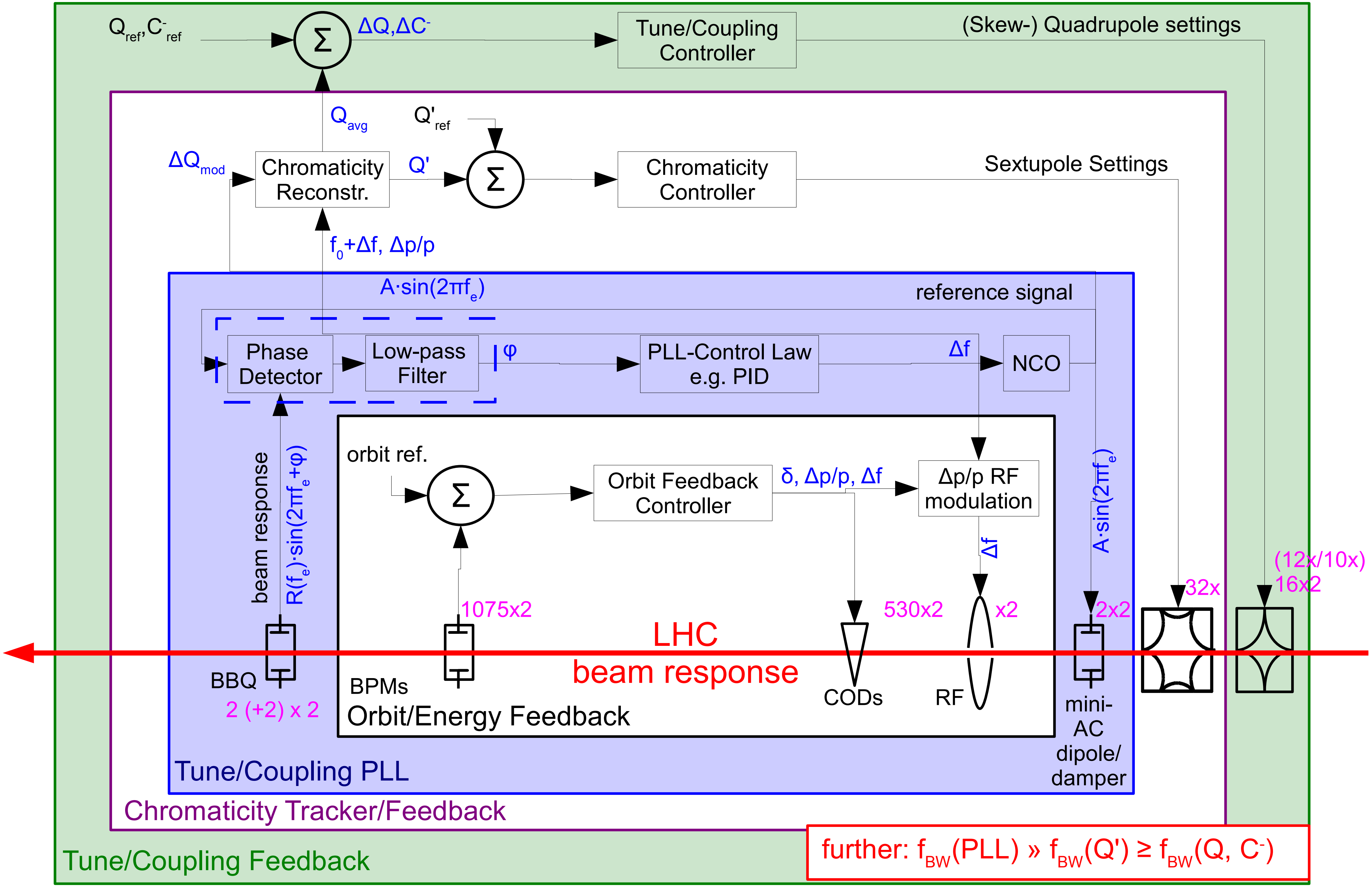
  and references therein.
- That's all – Questions?

Additional Slides

Additional Topic III:

Feed-Backs on Tune, Coupling and Chromaticity

Integration of Q/Q' Measurements for Q/Q' Control
Full LHC Beam-Based Feedback Control Scheme



LHC FBs: 2158 input devices, 1136 output devices → total: ~3300 devices!