Experimental Physicist Approach to: Improving Frequency Resolution of FFT based Data

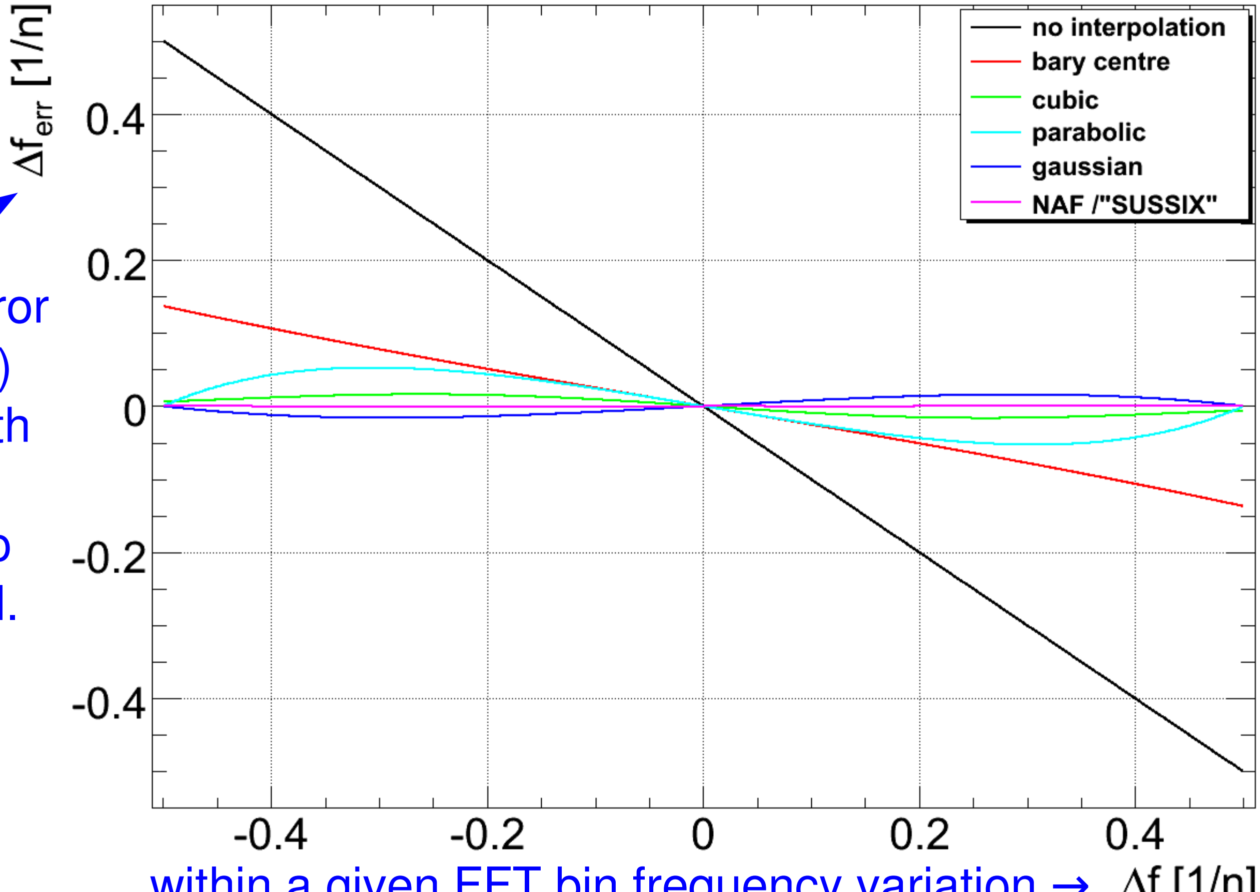
- Problem:
  - What is an appropriate (the best) method to increase the tune frequency resolution using FFT based Interpolation algorithms?
  - constrain: using "realistic" beam oscillation data?
- Some candidates:
  - No interpolation: $$q \approx \frac{k}{N}$$

    k: index of highest bin
    N: total number of turns
    M_k: magnitude of bin k
  - Barycentre (n=1) & cubic (n=3) fit: $$q \approx \frac{M_{k-1}^n (k-1) + M_k^n (k) + M_{k+1}^n (k+1)}{N\left(M_{k-1}^n + M_k^n + M_{k+1}^n\right)}$$
  - Parabolic fit: $$q \approx \frac{k}{N} + 0.5 \cdot \frac{M_{k+1} - M_{k-1}}{2\mathrm{M}_k - M_{k-1} - M_{k+1}}$$
  - Gaussian fit: $$q \approx \frac{k}{N} + 0.5 \cdot \frac{\log\left(M_{k+1} / M_{k-1}\right)}{\log\left(M_k^2 / \left(M_{k-1} M_{k+1}\right)\right)}$$
  - NAFF/"SUSSIX": $$q \approx \frac{k}{N} \pm \frac{1}{\pi} \cdot atan\left(\frac{|M_{k\pm1}| \sin\left(\frac{\pi}{N}\right)}{|M_k| + |M_{k\pm1}| \cos\left(\frac{\pi}{N}\right)}\right)$$
- Test case: controlled oscillation at a given frequency which is varied within one bin, normalised to sampling frequency, no integer part (obvious?)

- 1024 turns: perfect sinusoidal oscillation & within one bin varying frequency
  - introducing some

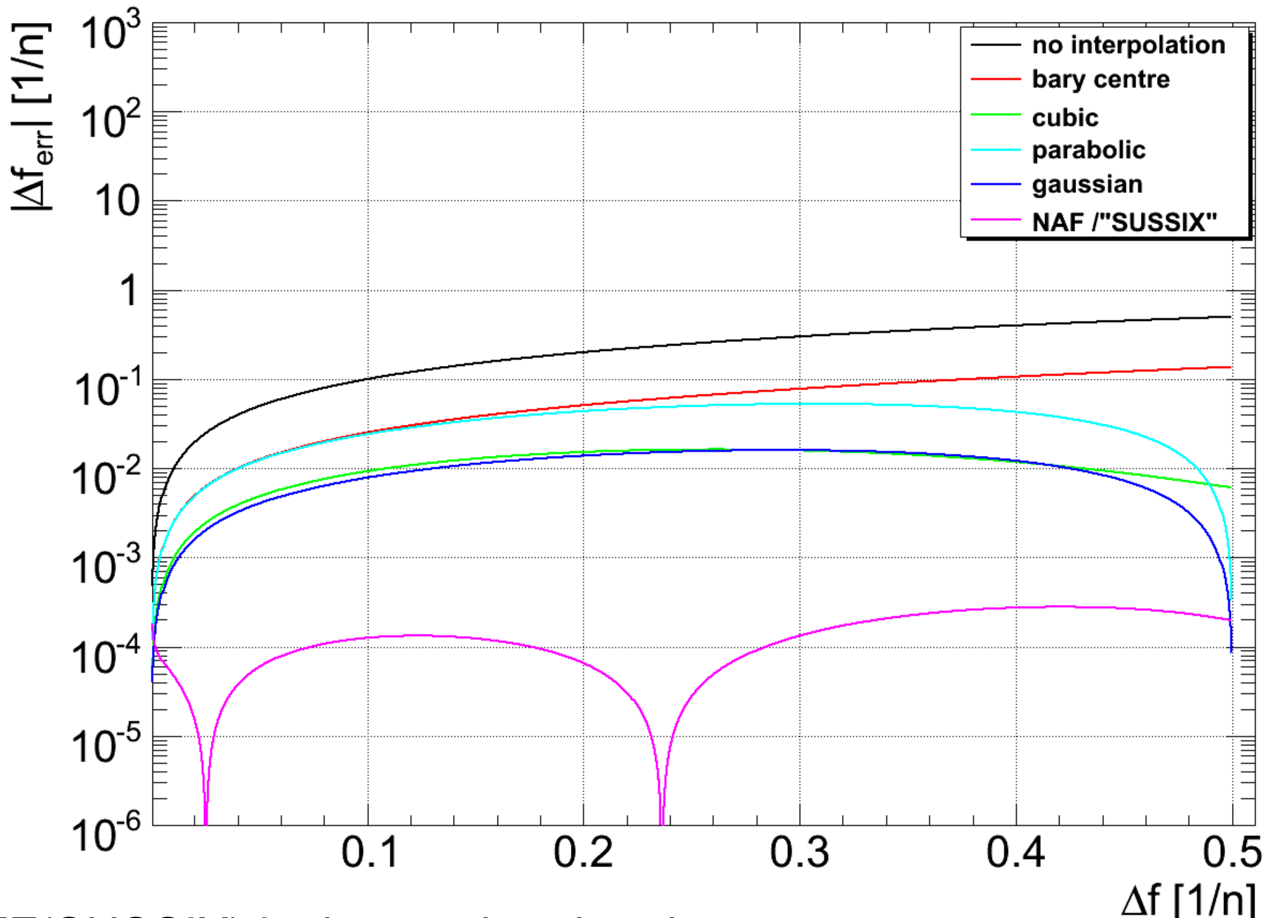
frequency error (sim. vs. det.) w.r.t. bin width

closer to zero = better resol.



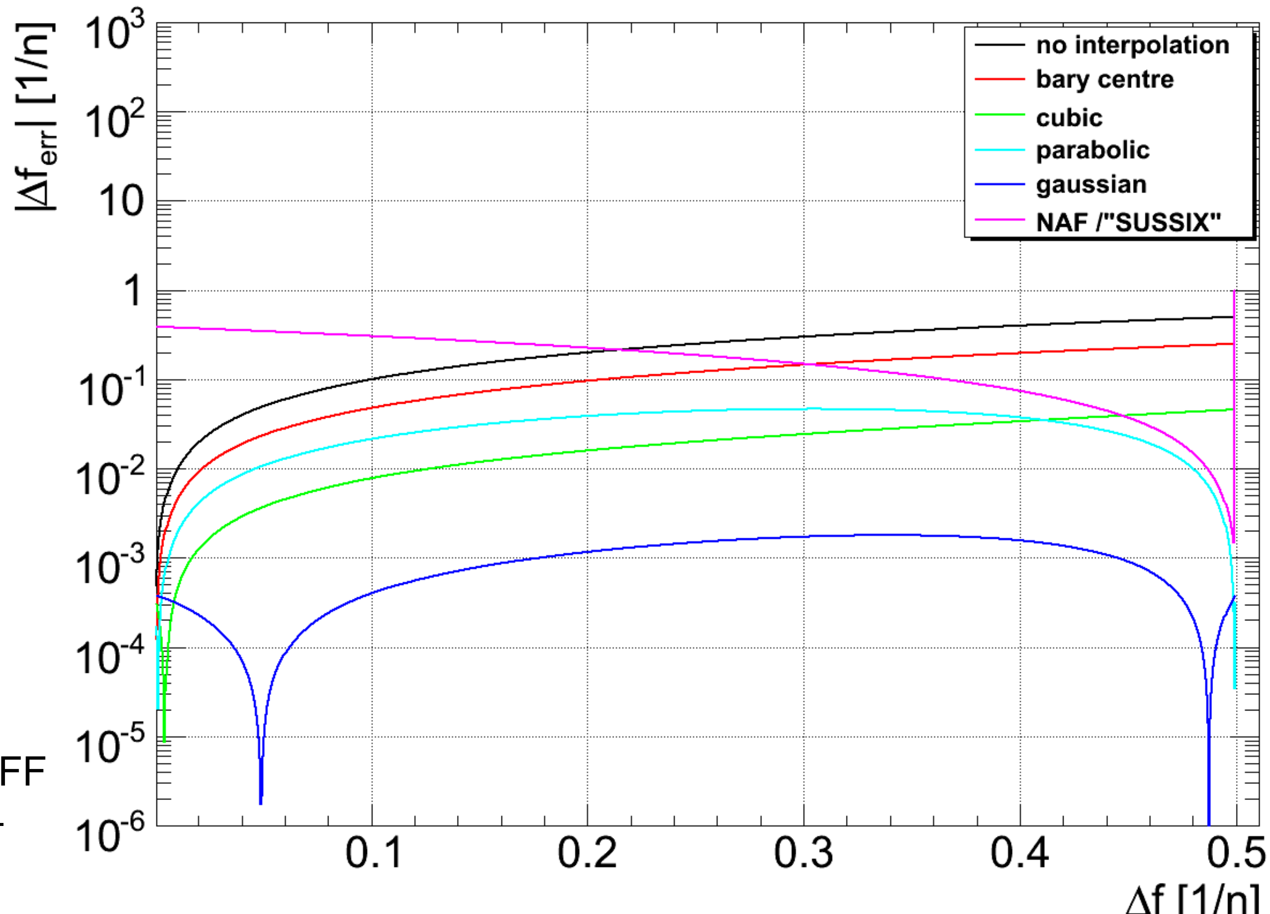
within a given FFT bin frequency variation →

- same plot as before but: absolute error, logarithmic scale and considering frequency only within half a bin width (symmetry!)



  – NAFF(SUSSIX) looks good ... done!
    - .... wait a moment.... what about damping and kick offset?

- Perfect sinusoidal with exponential damping (200 turns) and kick starting after 100 turns (amplitude := 1), no additional noise

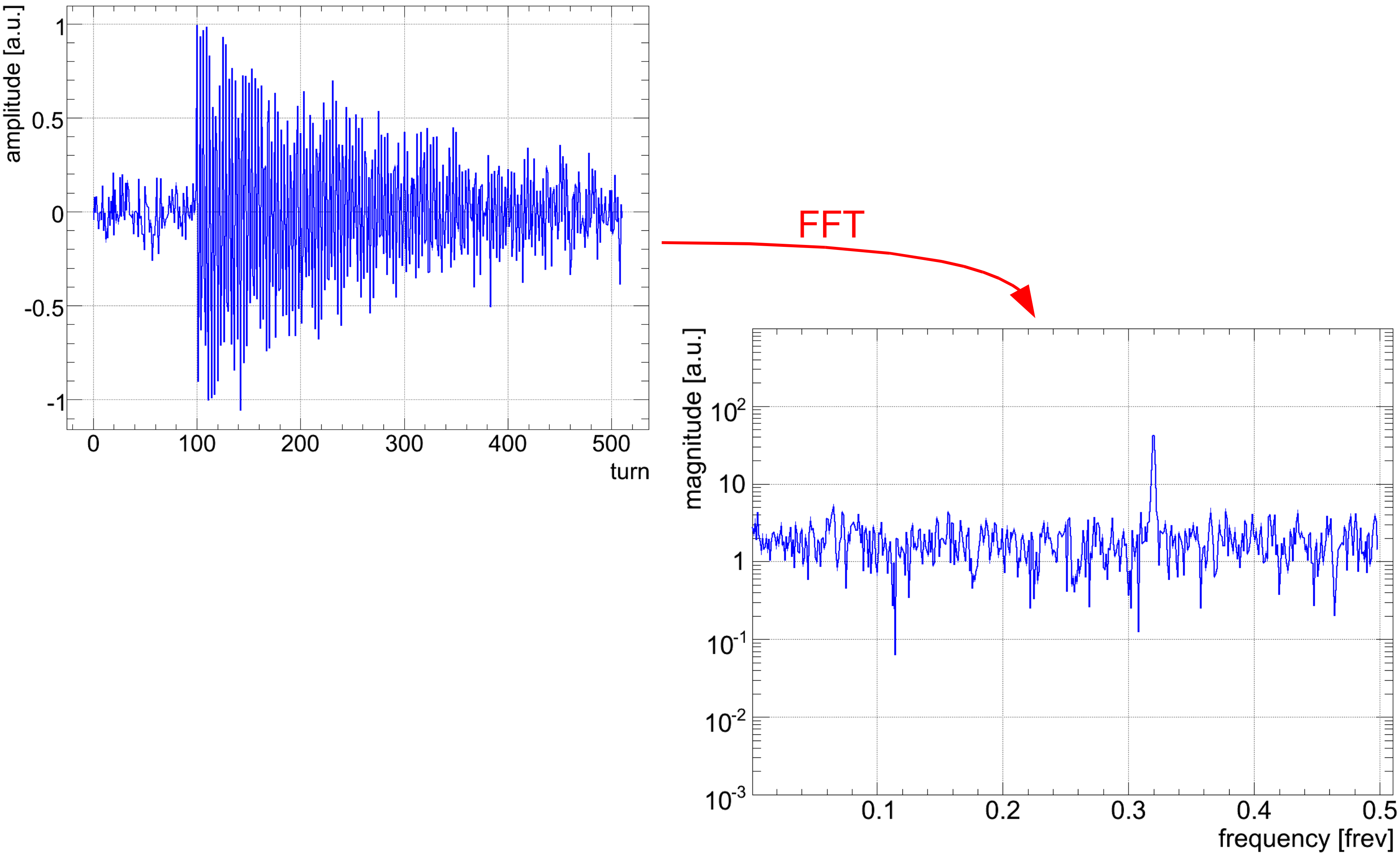


N.B. cause: NAFF assumes quasi-periodic signals

  – Still perfect signal! Normal beam oscillation data is affected by noise
    • common: choose kick ~ 10 times higher than measurement noise

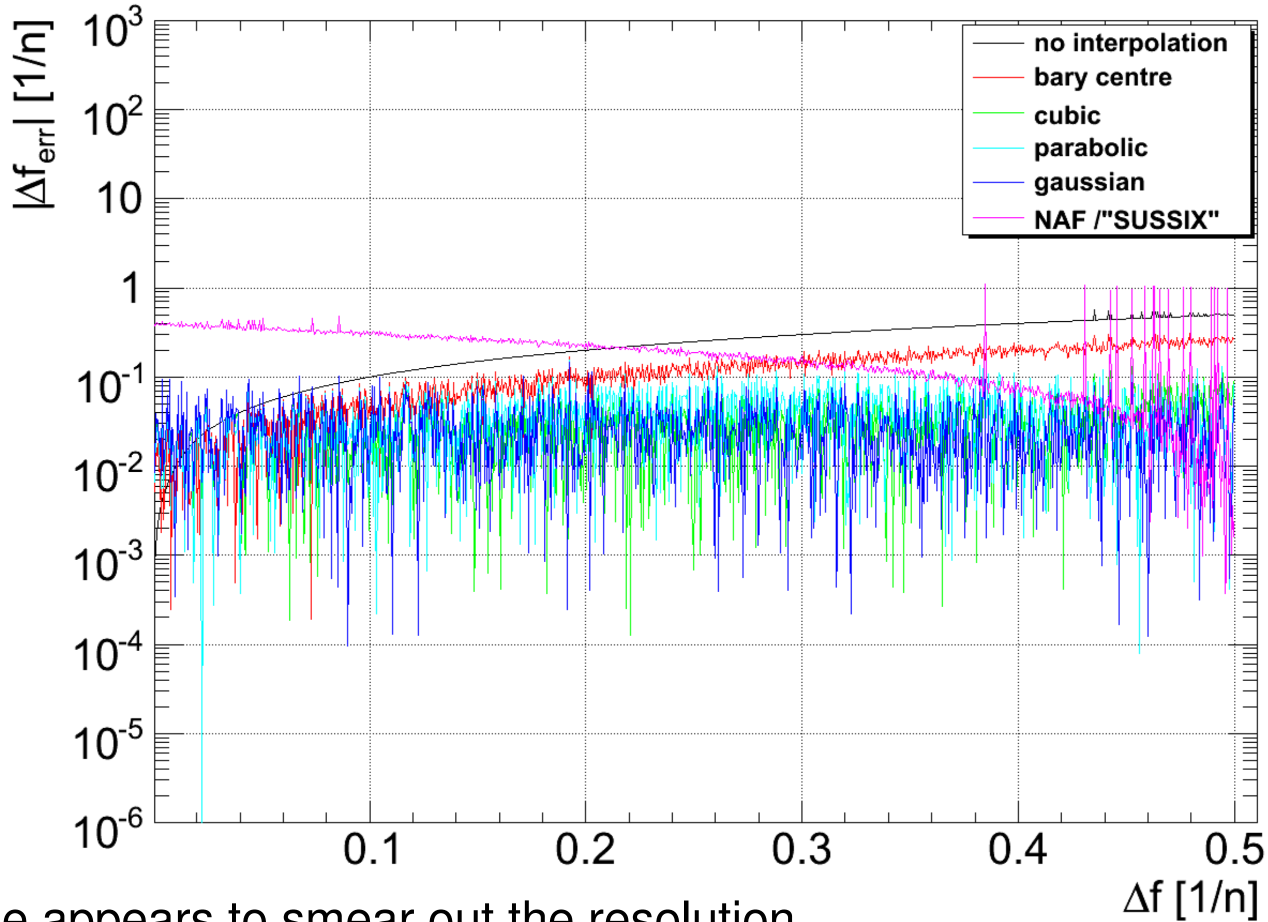
Improving Frequency Resolution of FFT based Data Time Domain Signals

- .... how an kick-induced beam oscillation usually looks like (no sync. beating)
  - still some kind of optimistic ... beam data is often worse.



Improving Frequency Resolution of FFT based Data
Perfect Sinusoidal + Damping + Kick Offset + Noise

- same as before + 0.1 r.m.s. noise vs. kick amplitude of '1'



  - noise appears to smear out the resolution
  - Cubic, barycentre, parabolic and Gaussian interpolation seem to yield similar performance... your conclusion???