Introduction to Feedback Controllers

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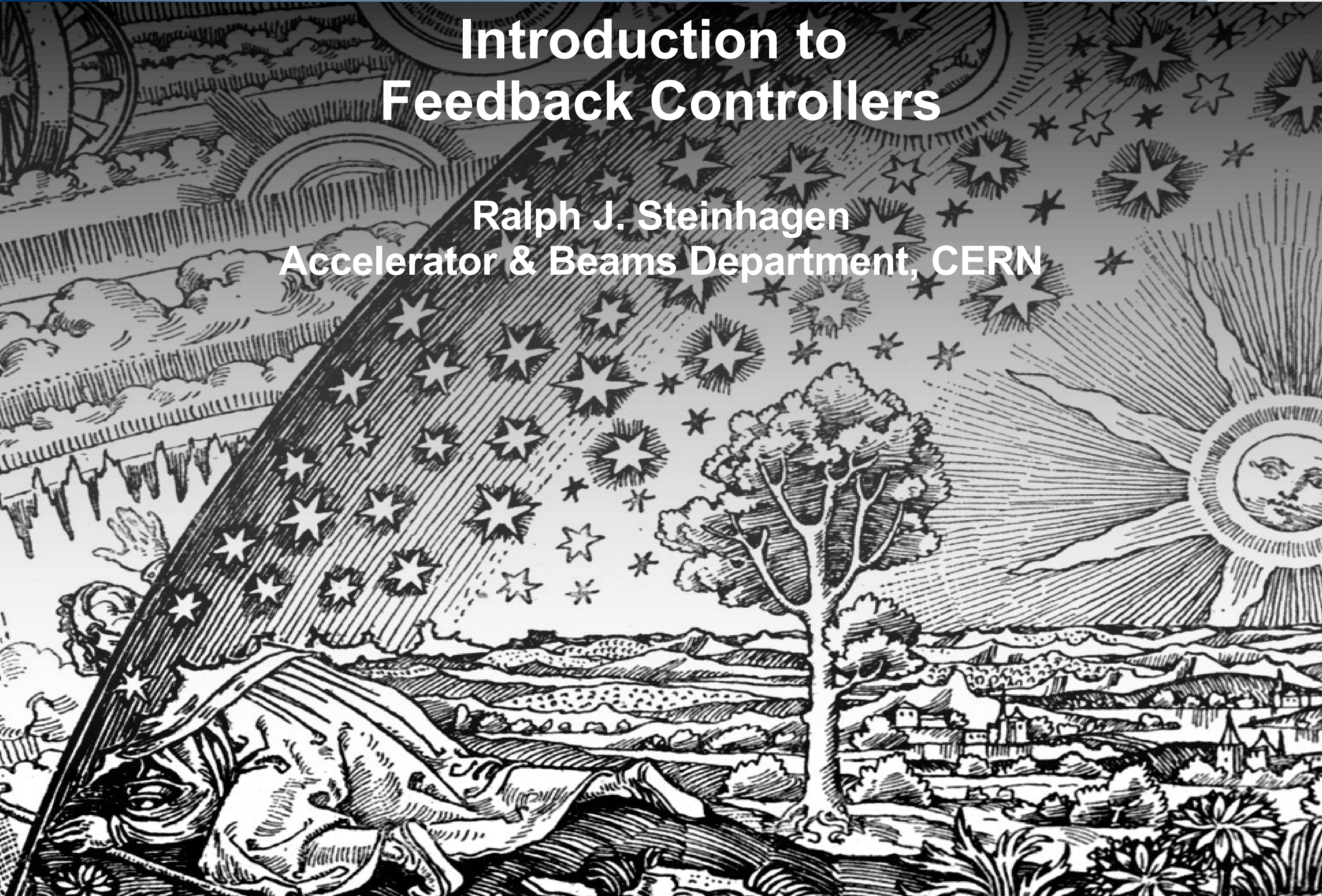


Overview

- Optimal Controller Design
  - Space Domain: MIMO inversion problem
    - orthogonality of parameters, blind parameters, singularities, ...
  - Time Domain: Youla (example), Kucera

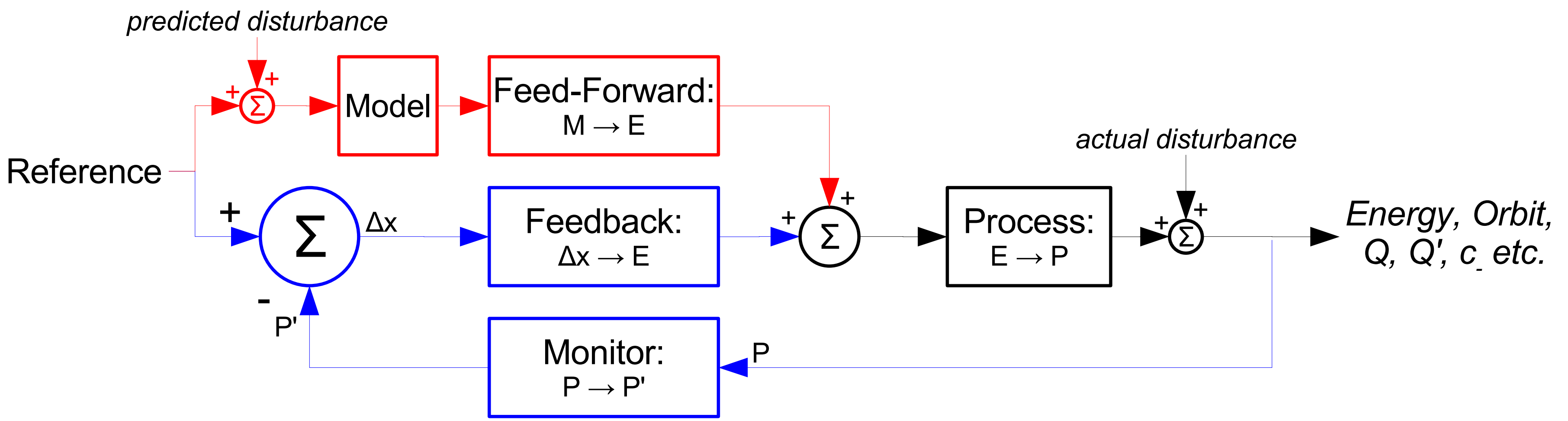
- Non-linear Systems
  - effect of delays, rate-limiter, sampling and their compensation

- Multiple FB Loops and Coupling Compensation

→ Presentation on LHC feedback architecture

Control Paradigms I/III

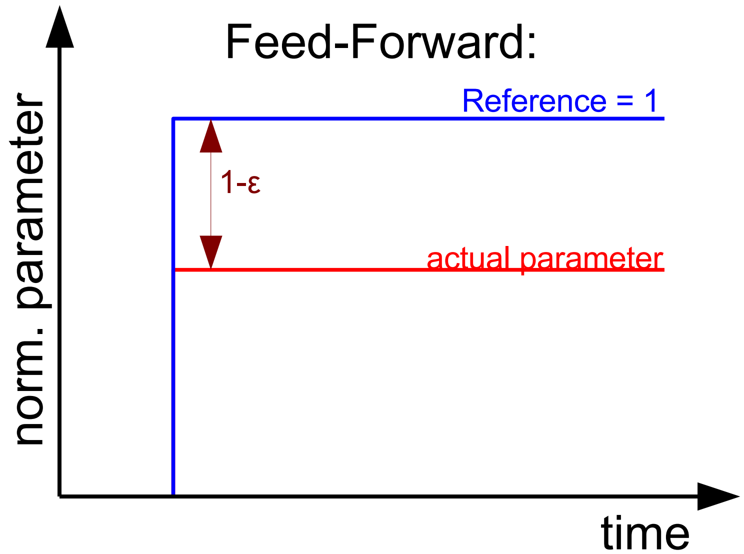
- Feed-Forward: (FF) - “set and forget”
  - Steer parameter using precise process model and disturbance prediction
- Feedback: (FB) - “set, verify, re-adjust, verify, ...”
  - Steering using rough process model and measurement of parameter
  - Two types: within-cycle or cycle-to-cycle

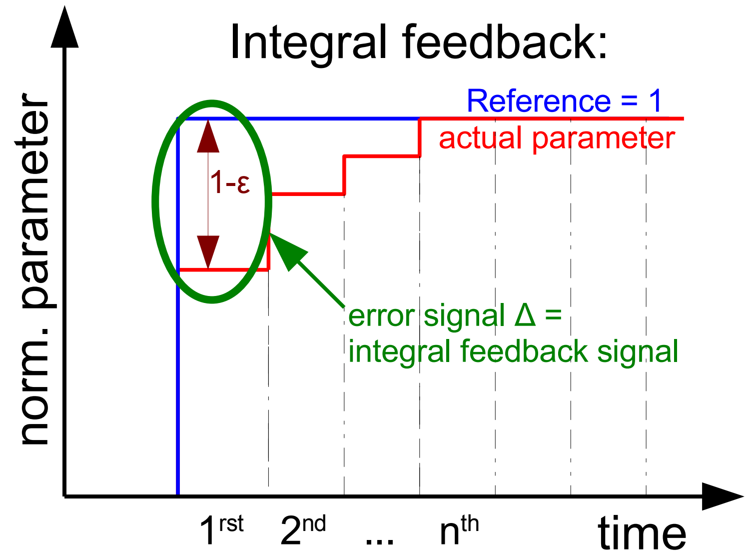


- From the steering point of view: → control schemes are similar

- Machine imperfections (beta-beat, hysteresis....), calibration errors and offsets can be translated into a steady-state ε_{ss} and scale error ε_{scale}:

$$\Delta x(s)=R_i(s)\cdot\delta_i \rightarrow \Delta x(s)=R_i(s)\cdot(\epsilon_{ss}+(1+\epsilon_{scale})\cdot\delta_i)$$





- Uncertainties and scale error of beam response function affects convergence speed (= feedback bandwidth) rather than achievable stability
- Choice of feedback vs. feed-forward
  - mainly depends on available robust beam parameter measurements

Control Paradigms III/III

- Good understanding of the beam measurement principle, corrector elements and beam physics is essential for the design of a robust feedback system!
  - instrument systematics and errors
  - overview of the final/whole feedback loop and requirements

- Two observations concerning the design of feedbacks:
  - common: optimise instrument's response before "closing the loop"
    - often: multiple low-pass filters to reduce measurement noise
  - feedback approach: optimise closed loop response = minimise phase lags
    - most FB systems have anyway low-pass characteristics
      → LP filters just increase phase lag and are thus (often) unnecessary
    - measurement does not need to be calibrated accurately (integrators)

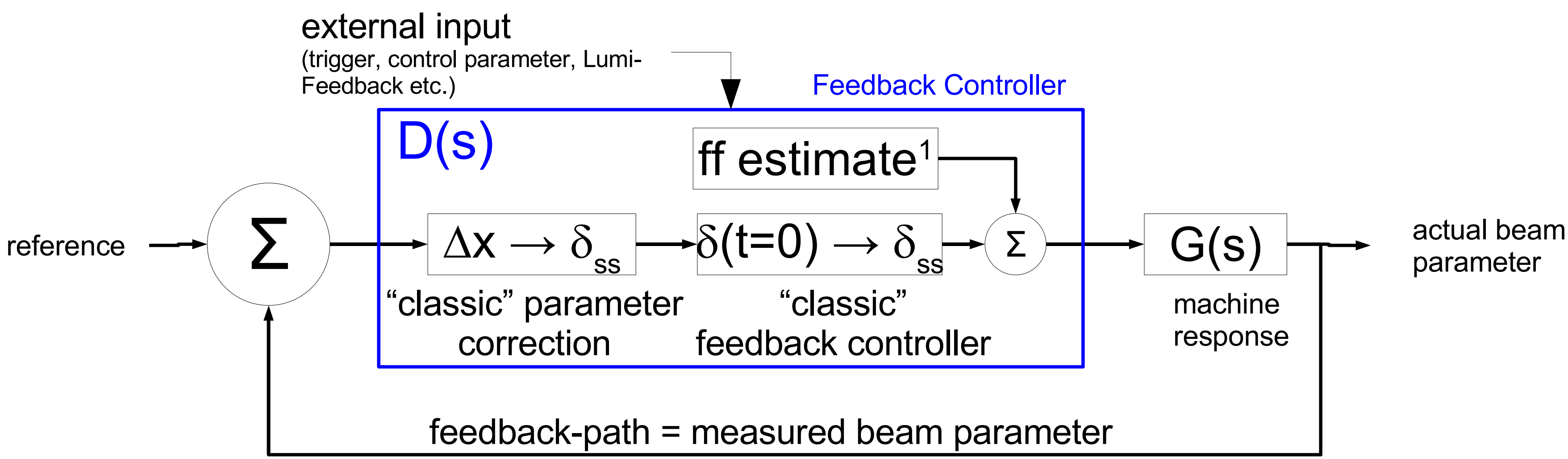
→ 'KISS' principle = keep it simple – keep it safe

Multiple-Input-Multiple-Output (MIMO) Process Control

- 'Divide and Conquer' feedback controller design approach:
  1. Compute steady-state corrector settings $\vec{\delta}_{ss} = (\delta_1, \ldots, \delta_n)$ based on measured parameter shift $\Delta x = (x_1, \ldots, x_n)$ that will move the beam to its reference position for $t \rightarrow \infty$.
  2. Compute a $\vec{\delta}(t)$ that will enhance the transition $\vec{\delta}(t=0) \rightarrow \vec{\delta}_{ss}$
  3. Feed-forward: anticipate and add deflections $\vec{\delta}_{ff}$ to compensate changes of well known and properly described sources

space domain (item 1)

time domain (items 2–3)



- (N.B. here G(s) contains the process and monitor response function)

Space- & Time Domain vs. Full MIMO only design

- 'Space domain' corresponds to a "traditional" parameter control
  - numerous strategies/algorithms available: SVD, MICADO/SIMPLEX, ...
  - easy cross-check with slow feedback control (aka. "measure & correct")
- easier/possible to compensate time-variable and non-linear processes
- robust/easier to adjust in case of FB element failures/errors
- enables staged commissioning or partial operation of FBs
  - from simple to complex (↔ "operational learning-process")
  - (re-)commissioning has to/will/can be done my non-FB experts

- Alternative: MIMO only approach (using Youla, Kucera, ...)
  - most real-world, non-linear and/or time-varying system cannot be inverted
  - mixes beam-physics (🙂) with accelerator control aspects (🙁)
  - employment guarantee: FB expert "mandatory" for follow-up modifications, tuning and operation of the feedback loops

- For a steady-state system, effects on orbit, Energy, Tune, Q' and C⁻ can essentially be cast into matrices:

$$\boxed{\Delta \vec{x}(t) = \underline{R} \cdot \vec{\delta}_{ss}} \quad with \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cdot \cos(\Delta \mu_{ij} - \pi Q)$$

matrix multiplication

  - similar equations can be established for other beam parameters:
    - LHC: $\underline{R}_{orbit} \in \mathbb{R}^{1056\times530}$ $\underline{R}_{Q} \in \mathbb{R}^{2\times16}$ $\underline{R}_{Q'} \in \mathbb{R}^{2\times32}$ $\underline{R}_{C^-} \in \mathbb{R}^{2\times10/12}$
  - matrices are beam observables and can be measured with beam!
- Task in Space domain – assuming steady-state errors:
  - beam parameter control consists essentially in inverting these matrices

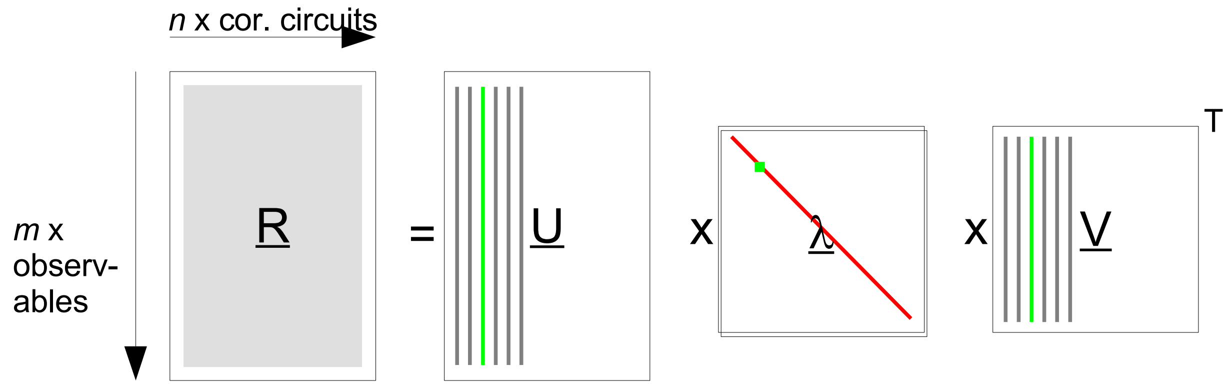
$$\boxed{\left\|\vec{x}_{ref} - \vec{x}_{actual}\right\|_2 = \left\|\underline{R} \cdot \vec{\delta}_{ss}\right\|_2 < \epsilon \;\rightarrow\; \vec{\delta}_{ss} = \tilde{R}^{-1} \Delta \vec{x}}$$

- Some potential complications:
  - 'Singularities' = over/under-constraint matrices = “more corrector circuits than beam observables”
  - noise, element failures, spurious measurement offsets, calibrations, ...

→ “The world goes SVD....”

Space-Domain: Singular Value Decomposition (SVD) on a slide

Linear algebra theorem*:



eigen-vector relation:

$$\lambda_i \vec{u}_i = \underline{R} \cdot \vec{v}_i$$

$$\lambda_i \vec{v}_i = \underline{R}^T \cdot \vec{u}_i$$

- though decomposition is numerically more complex final correction is a simple vector-matrix multiplication:

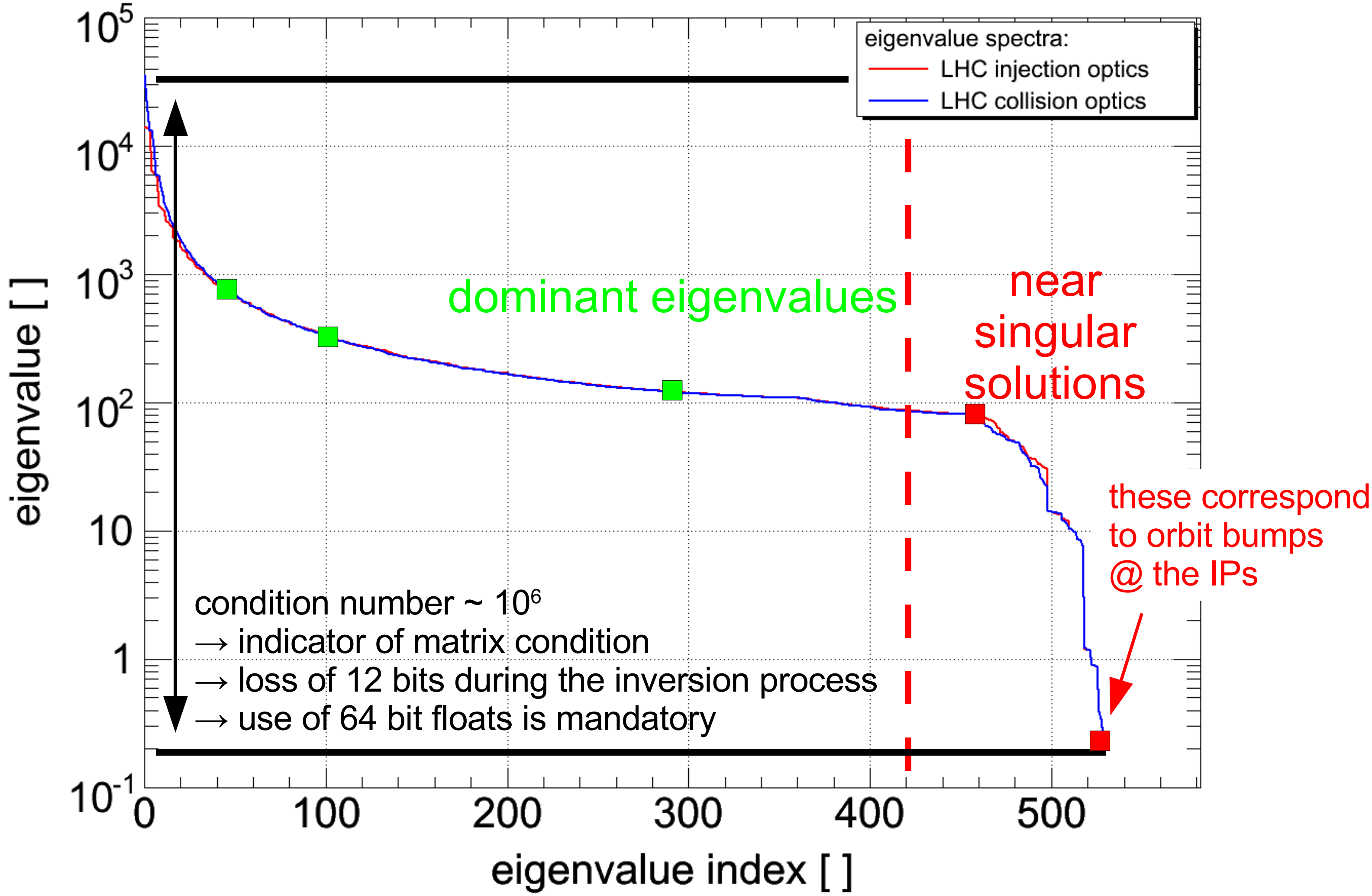
$$\vec{\delta}_{ss} = \tilde{R}^{-1} \cdot \Delta \vec{x} \quad with \quad \tilde{R}^{-1} = \underline{V} \cdot \underline{\lambda}^{-1} \cdot \underline{U}^T \quad \Leftrightarrow \quad \vec{\delta}_{ss} = \sum_{i=0}^{n} \frac{a_i}{\lambda_i} \vec{v}_i \quad with \quad a_i = \vec{u}_i^T \Delta \vec{x}$$

- numerical robust, minimises parameter deviations Δx <u>and</u> circuit strengths δ
- Easy removal of singularities, (nearly) singular eigen-solutions have $\lambda_i \sim 0$
  - to remove those solution: if $\lambda_i \approx 0 \rightarrow$ '$1/\lambda_i := 0$'
  - discarded eigenvalues corresponds to solution pattern unaffected by the FB

*G. Golub and C. Reinsch, "*Handbook for automatic computation II, Linear Algebra*", Springer, NY, 1971

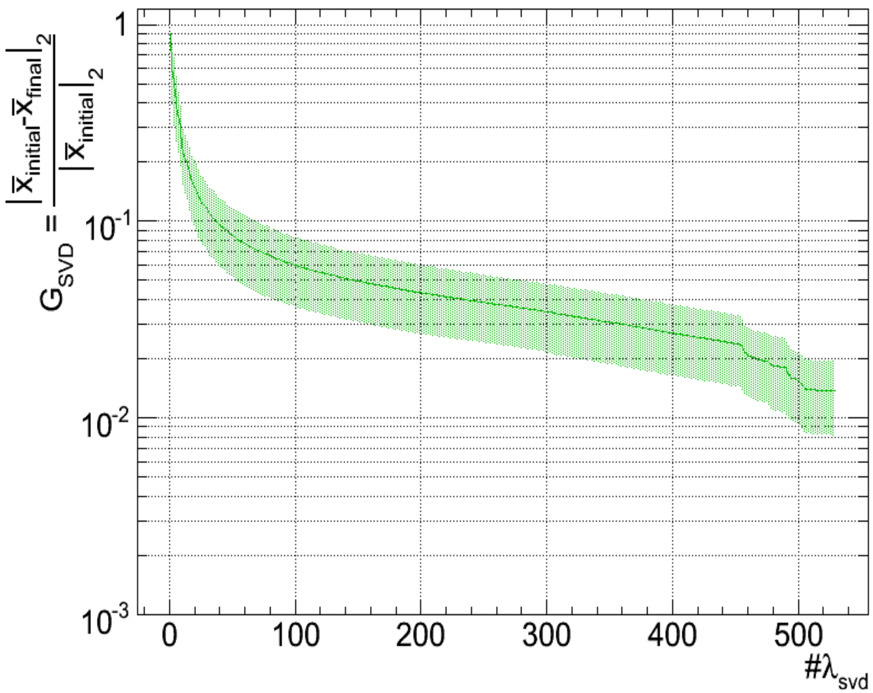
SVD example: LHC eigenvalue spectrum

Eigenvalue spectra for vertical LHC response matrix using all BPMs and CODs:

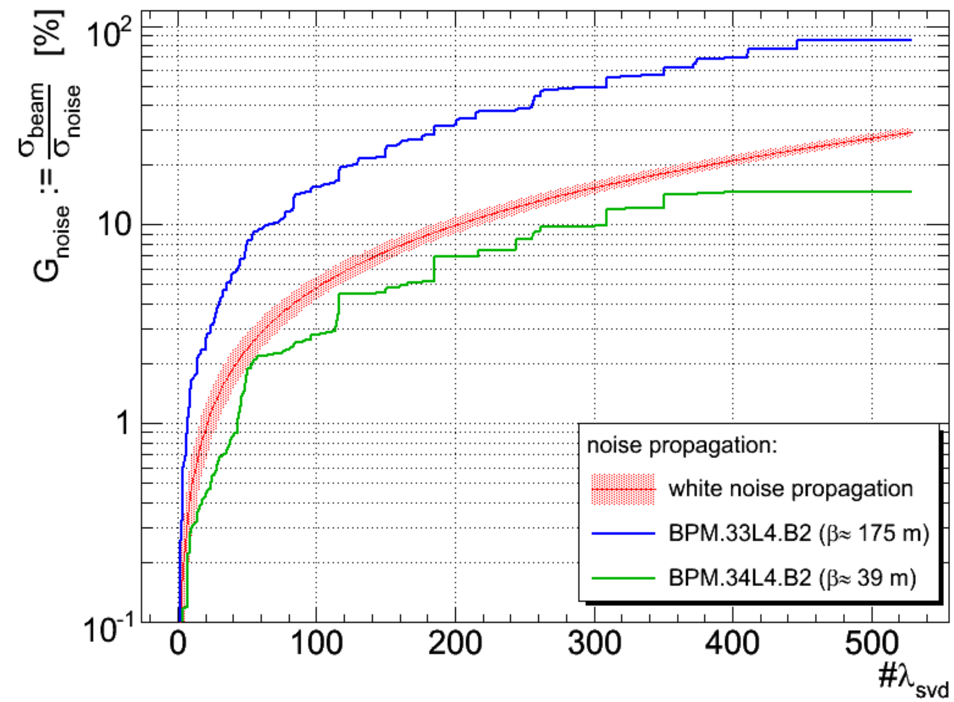


Space-Domain:
Example SVD based orbit correction

- Orbit attenuation



- Sensitivity to BPM noise



- Number of for the inversion used eigenvalues steers accuracy versus robustness of correction algorithm
- Likewise applies for Tune, Chromaticity and Coupling correction
  – However: Only two out of '*n*' eigenvalues are non-singular

- Controller design often regarded as specialists' topic only - <u>wrong</u>!
- Youla showed[1] that all stable closed loop controllers D(s) can be written as:

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)} \qquad (1)$$

- Example: first order system

$$G(s)=\frac{K_0}{\tau s+1} \quad \text{with } \tau \text{ being the circuit time constant} \qquad (2)$$

- Using for example the following ansatz:

$$Q(s)=F_Q(s)G^i(s)=\frac{1}{\alpha s+1}\cdot\frac{\tau s+1}{K_0} \qquad (3)$$

  - $F_Q(s)$ models the desired closed-loop response $\rightarrow T_0(s)=\frac{1}{\alpha s+1}$
  - $G^i(s)$ being the pseudo-inverse of the nominal plant G(s)
- (1)+(2)+(3) yields the following PI controller:

$$D(s)=K_P+K_i\frac{1}{s} \quad \text{with} \quad K_p=K_0\frac{\tau}{\alpha} \wedge K_i=K_0\frac{1}{\alpha}$$

[1]D. C. Youla et al., *"Modern Wiener-Hopf Design of Optimal Controllers"*, IEEE Trans. on Automatic Control,1976, vol. 21-1,pp. 3-13 & 319-338

$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)}$$

- Optimal control [or design] ...

  "... deals with the problem of finding a control law for a given system such that a given optimality criterion is achieved. A control problem includes a cost functional that is a function of state and control variables."

  – Common criteria: closed loop stability, minimum bandwidth, minimisation of action integral, power dissipation, ...

- classic closed loop: $T_0(s) = \frac{D(s)G(s)}{1 + D(s)G(s)}$ → "this tells me???"

- Using Youla's method: "design closed loop in a open loop style":

  – effective closed loop TF: $T_0(s) = Q(s)G(s) = F_Q(s)$

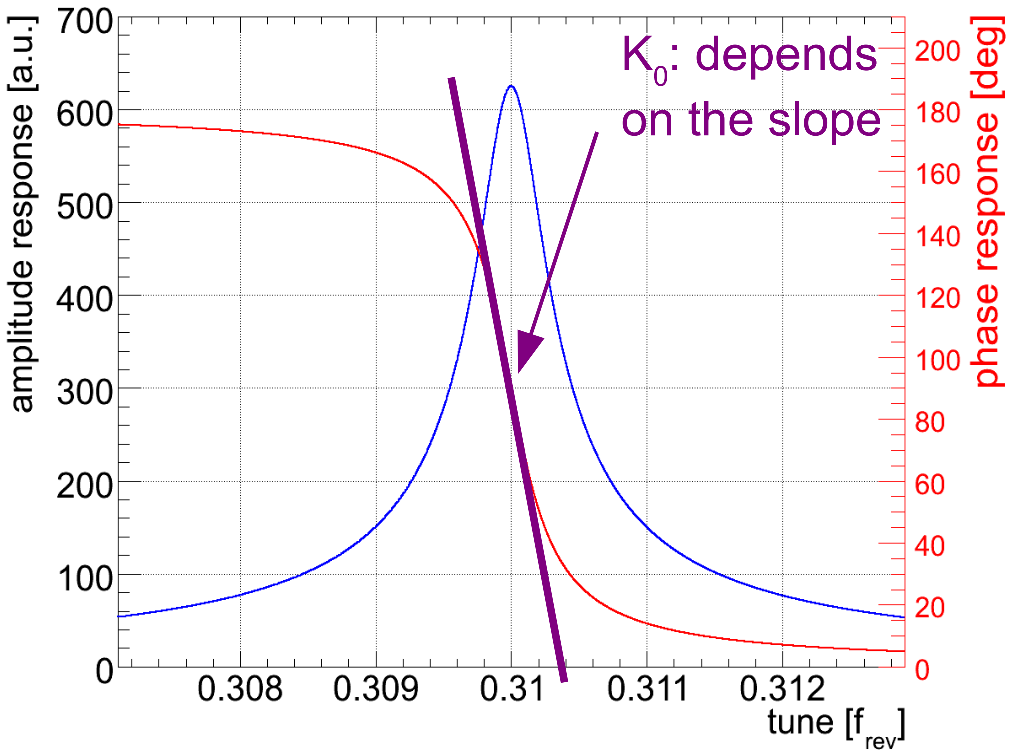
  – Response and optimality can directly be deduced by construction of $F_Q(s)$

  – usually: keep feedback controller simple and require that the desired closed-loop transfer function $F_Q(s)$ is e.g. of first or second order

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)}$$

- The PLL loop dynamics and its design split into two parts:
  - PLL low-pass filter: → $\tau=\frac{1}{f_{BW}}$
  - Beam response:
    - first order: K_0 = const.

$$G_{PLL}(s)=\frac{K_0}{\tau s+1} \quad with \quad \tau=\frac{1}{f_{bw}}$$



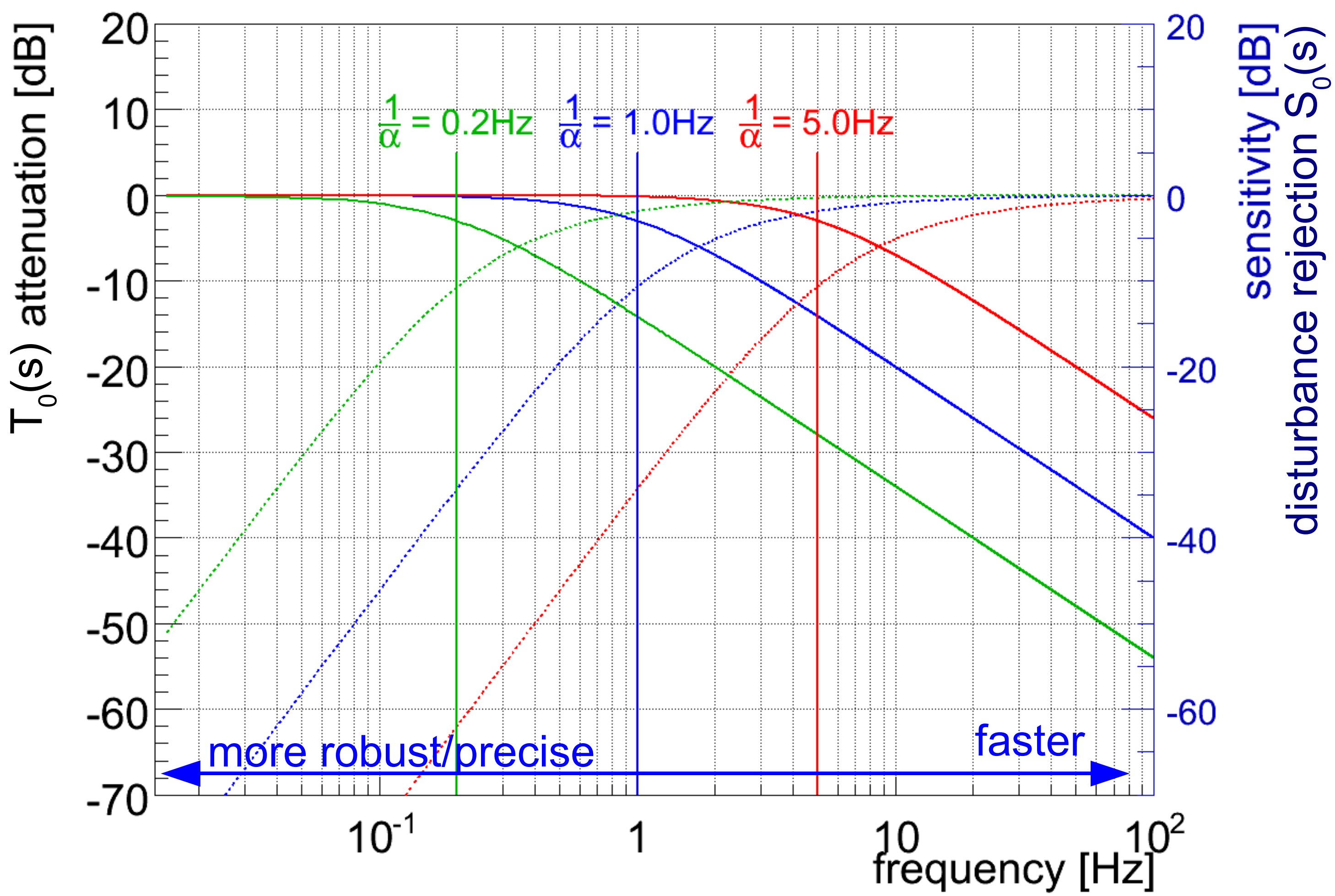
  - Youla's method: optimal control → classic PI controller
    - α is the (only) "free" parameter

$$D(s)=K_P+K_i\frac{1}{s} \quad with \quad K_p=K_0\frac{\tau}{\alpha} \wedge K_i=K_0\frac{1}{\alpha}$$

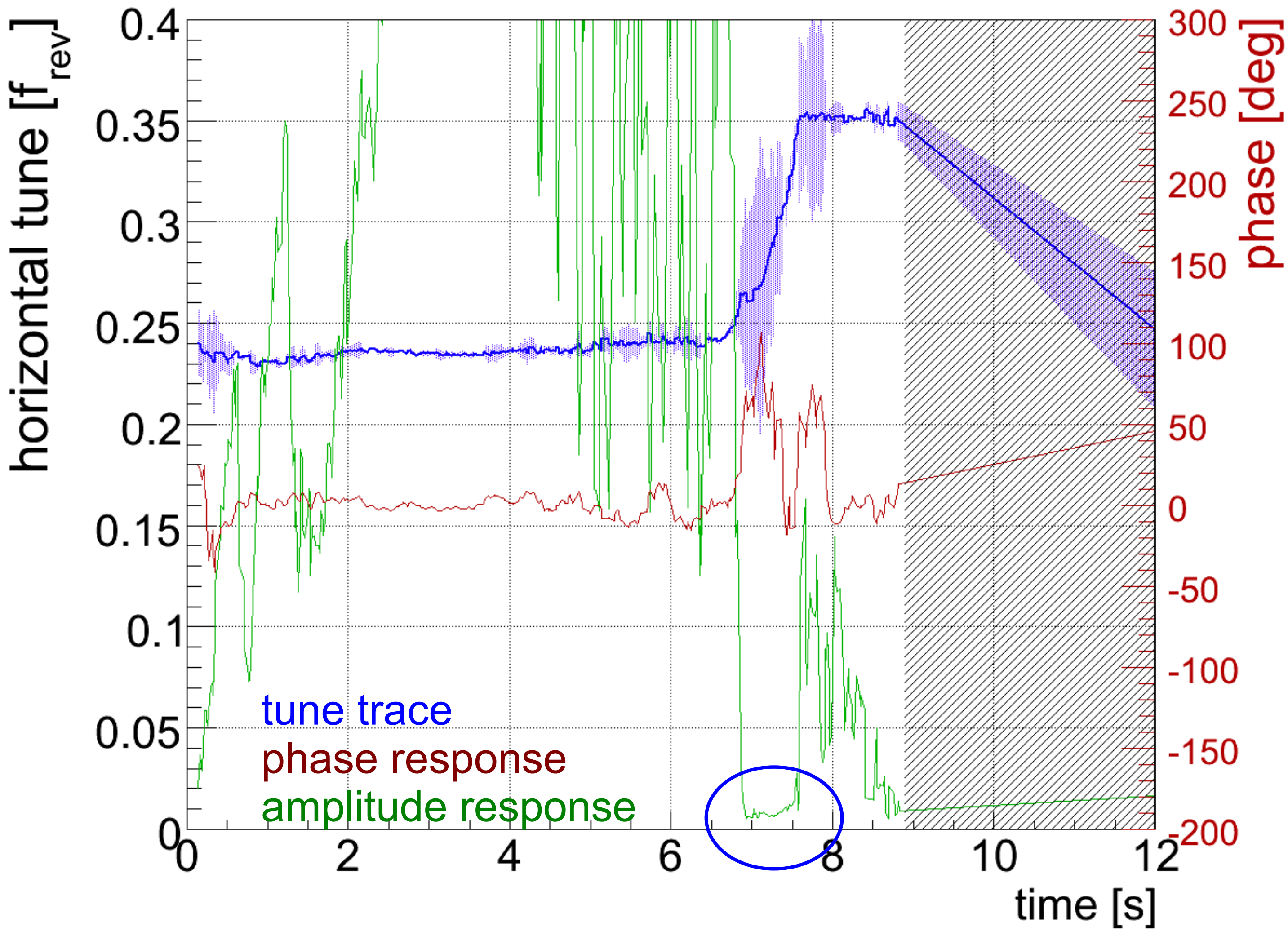
- α > τ...∞ facilitates the trade-off between speed and robustness

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)}$$

  - operator has to deal with one parameter → enables simple adaptive gain-scheduling based on the operational scenario!



Example: LHC PLL Tune Tracking at the SPS Real-Beam Data



- Phase error and non-vanishing amplitude indicate lock during ramp
- $\Delta Q/\Delta t|_{max} \approx 0.3/s$ ~ two orders of mag. faster than required for LHC

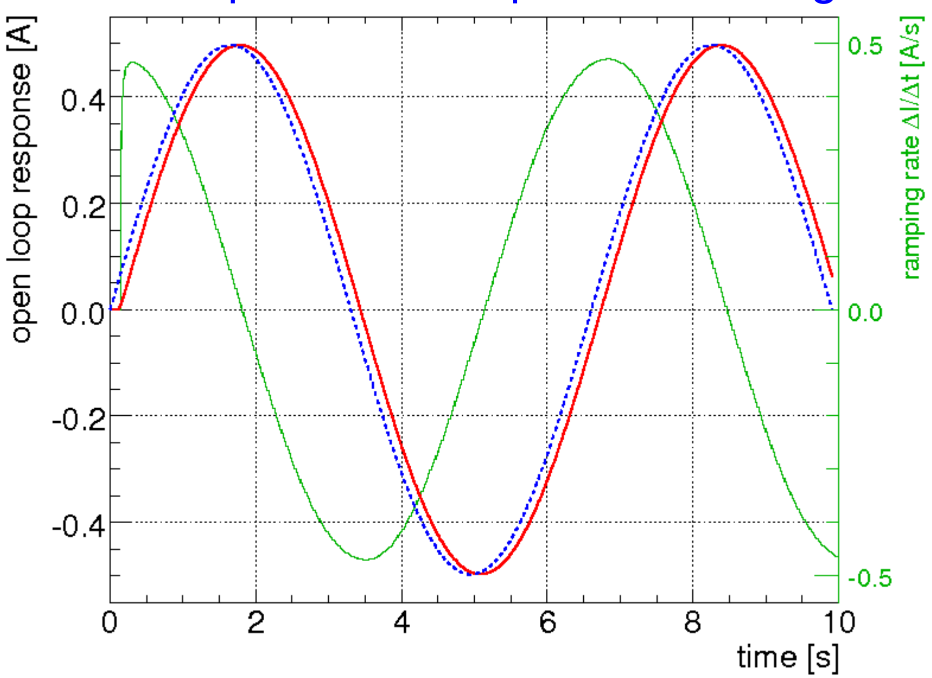
  $f_{rev} \approx 43$ kHz

details:
→ Andrea's presentation

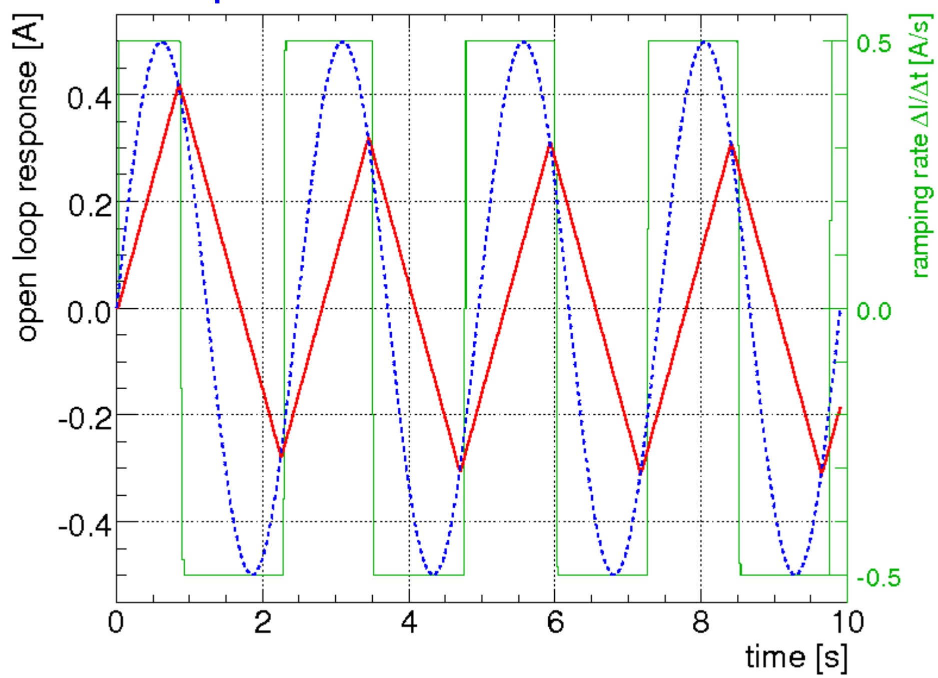
Two common non-linear effects in accelerators:

- Delays: computation, data transmission, dead-time, etc.
- Rate-Limiter: limited slew rate of corrector circuits (due to voltage limitations)
  - e.g. LHC: ±60A converter: $\Delta I/\Delta t|_{max} < 0.5$ A/s

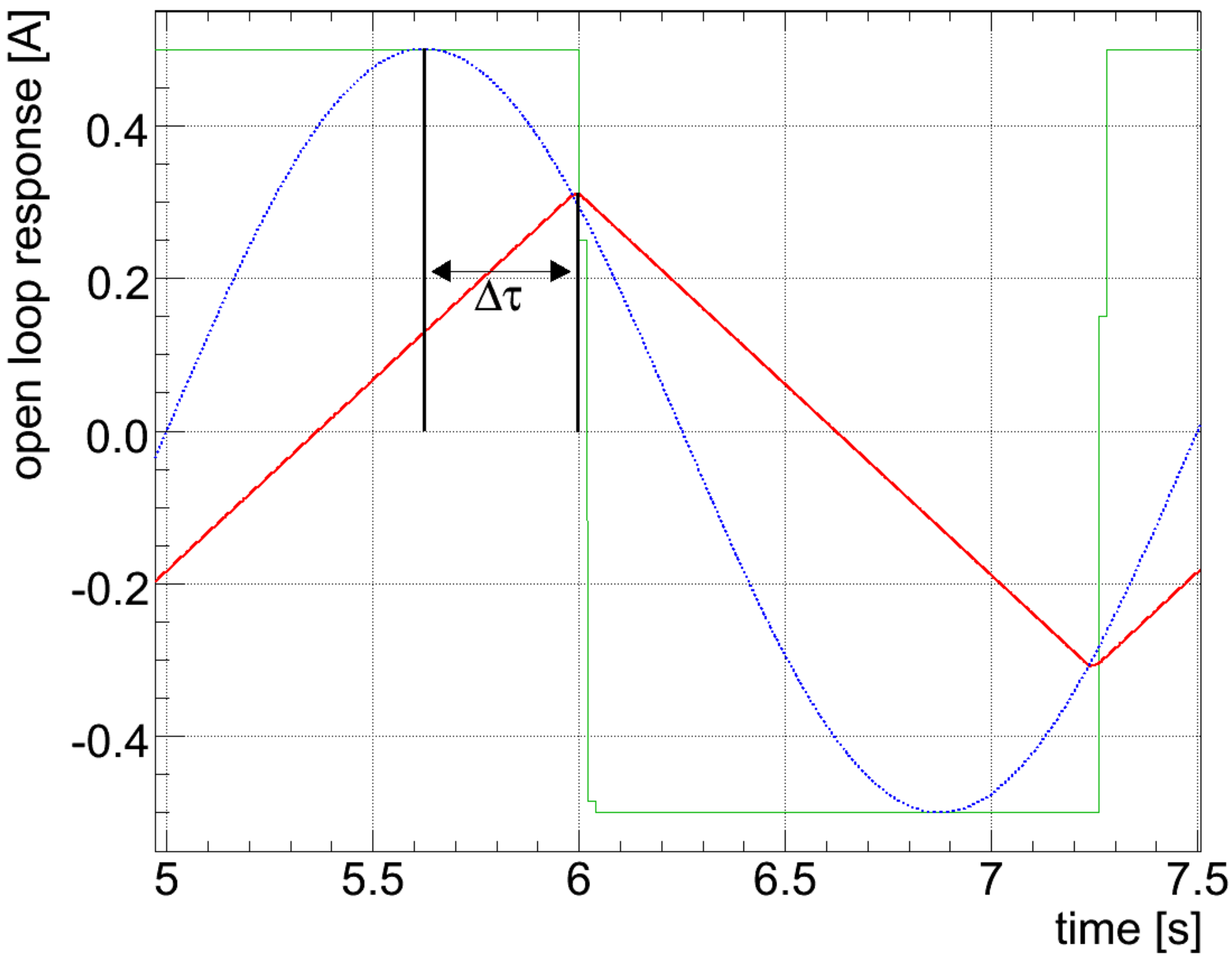
slow perturbation: perfect tracking



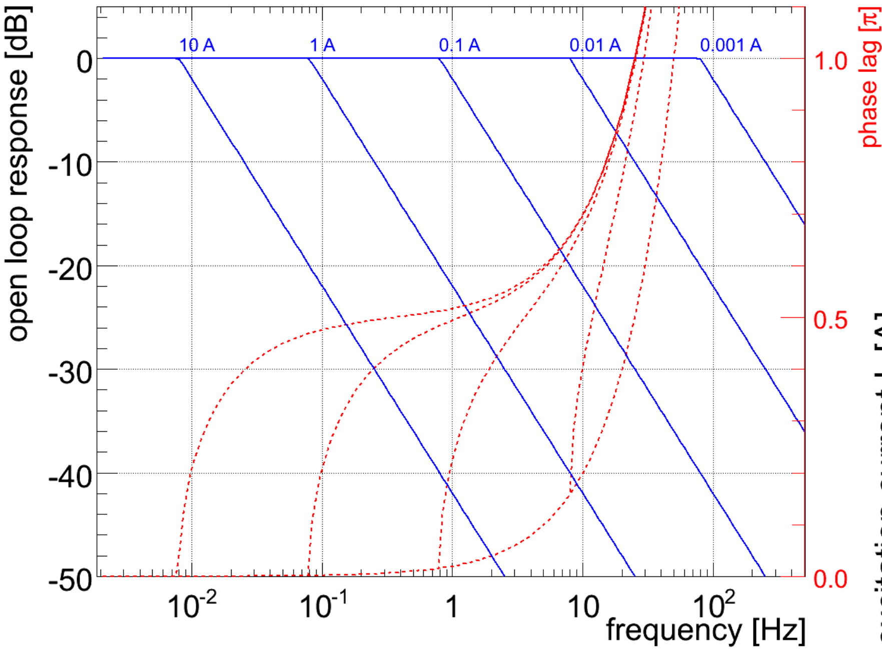
fast perturbation: saw-tooth



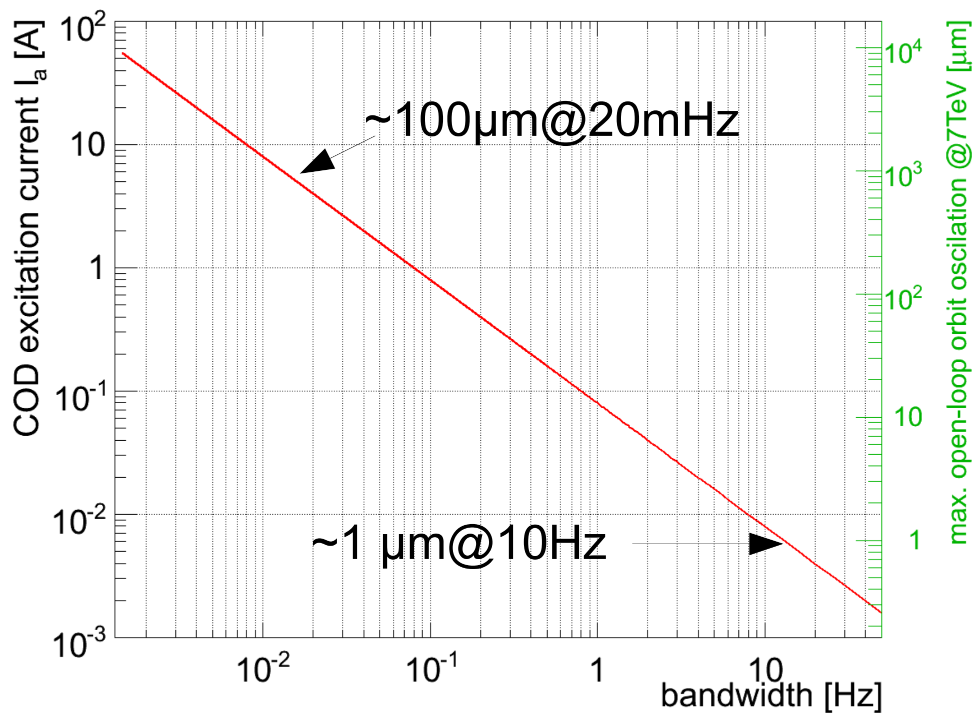
- Rate-limiter in a nut-shell:
  - additional time-delay Δτ that depends on the signal/noise amplitude
  - (secondary: introduces harmonic distortions)



- Open-loop circuit bandwidth depends on the excitation amplitude:
  - + non-linear phase once rate-limiter is in action...



ΔI=0.1A ↔ Δx≈16 μm@β=180m



- Consider ~16μm@1Hz as effective bandwidth @ 7TeV

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)}$$

- ... cannot a priori be compensated.
  - however, their deteriorating effect on the loop response can be mitigated by taking them into account during the controller design.
- Example: process can be split into stable and instable 'zeros'/components

$$G(s)=\frac{A_0(s)A_u(s)}{B(s)}=G_0(s)\cdot G_{NL}(s) \quad e.g. \quad G(s)=G_0(s)\cdot \underbrace{e^{-\lambda s}}_{\lambda\text{: delay}}$$

- Using the modified ansatz (F_Q(s): desired closed-loop transfer function):

$$Q(s)=F_Q(s)\cdot G^i(s)=F_Q(s)\cdot G_0^{-1}(s)$$

- yields the following closed loop transfer function

$$\rightarrow \quad T(s) \ = Q(s)G(s)=F_Q(s)\cdot G_{NL}(s) \underbrace{=}_{here:} F_Q(s)\cdot e^{-\lambda s}$$

  - Controller design F_Q(s) carried out as for the linear plant
  - Yields known classic predictor schemes:
    - delay → Smith Predictor
    - rate-limit → Anti-Windup Predictor

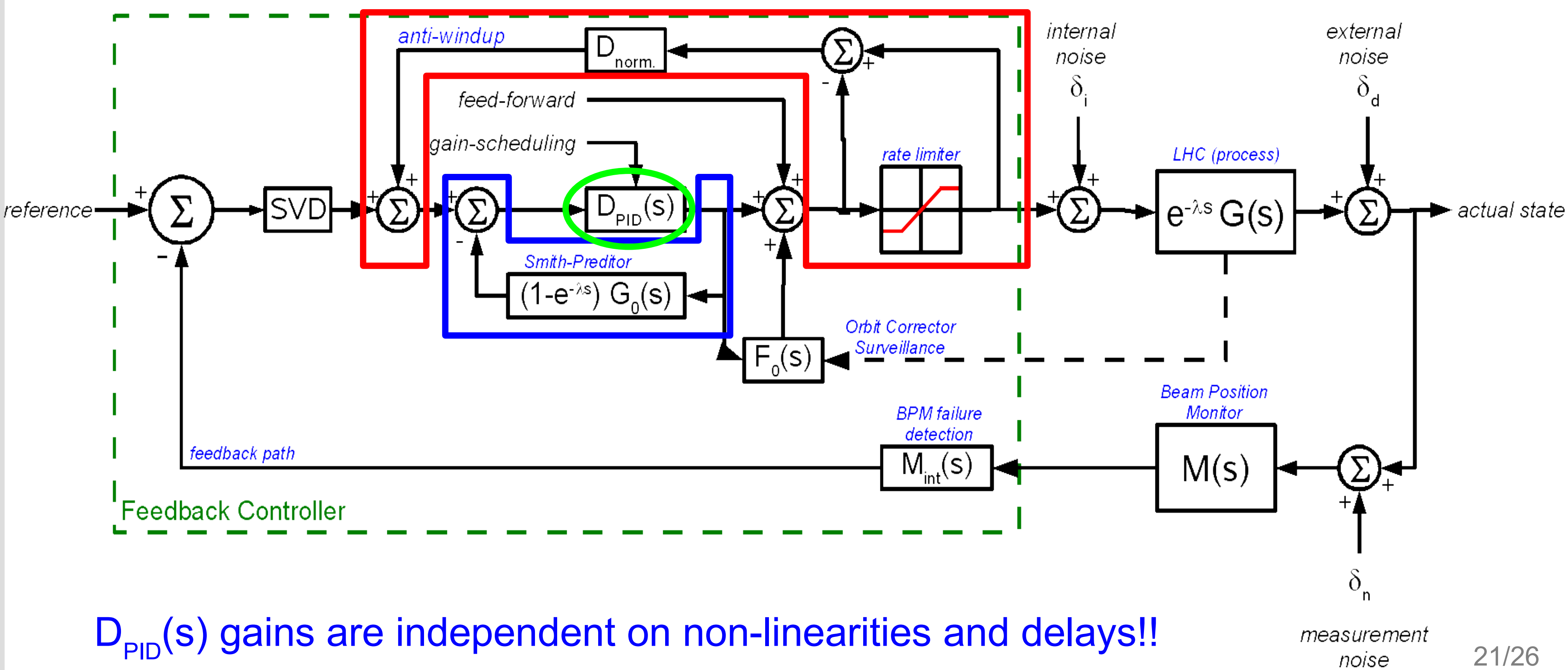
Example: LHC Feedbacks & Delays + Rate-limiter

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)}$$

- If G(s) contains e.g. delay λ & non-linearities $G_{NL}(s)$

$$G(s)=\frac{e^{-\lambda s}}{\tau s+1}G_{NL}(s)$$

- with τ the power converter time constant and $$G^i(s)=\frac{\tau s+1}{1}$$
- yields Smith-Predictor and Anti-Windup paths: $$T(s)=F_Q(s)\cdot e^{-\lambda s}G_{NL}(s)$$



$D_{PID}(s)$ gains are independent on non-linearities and delays!!

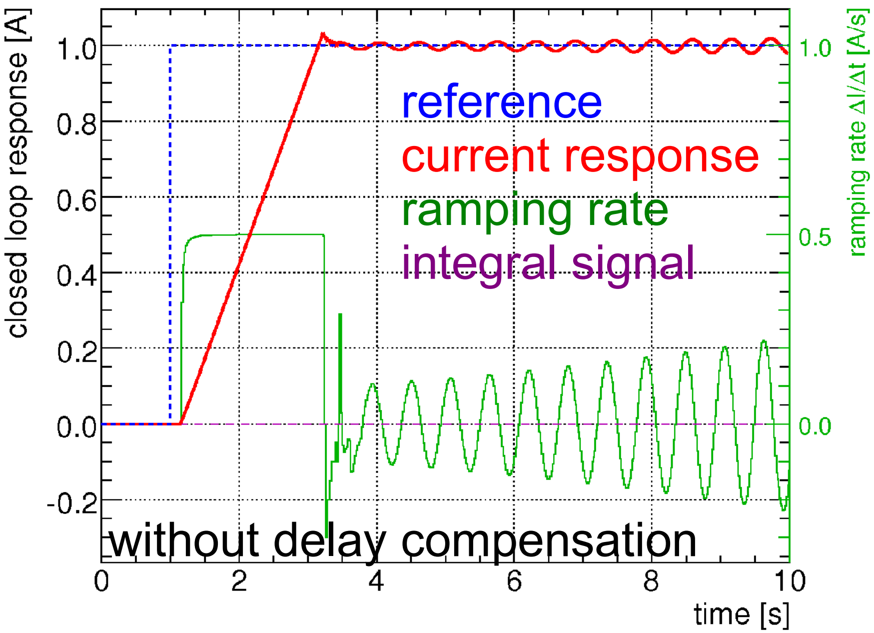
Time Domain:
Example: LHC Feedbacks Controller – CODE Snippet

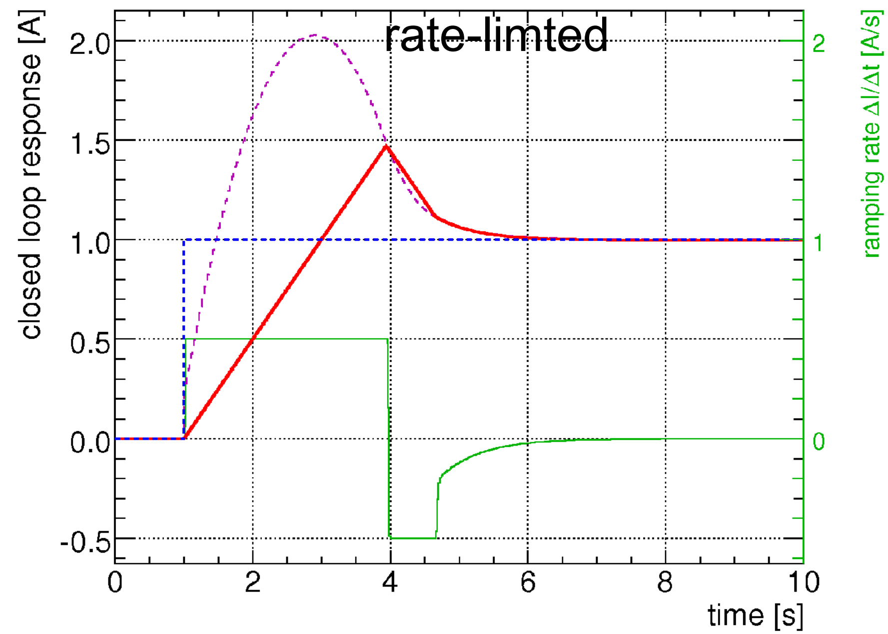
- Final implementation does not have to be complicated,
  Alternative C-style representation:
  - Colour coding:
    - classic linear controller
    - Anti-Windup branch
    - Smith-Predictor branch

- Total number of lines: ~ 50
  - + a few lines for:
    - settings management
    - scheduling/automatisation
    - exception handling
    - ...

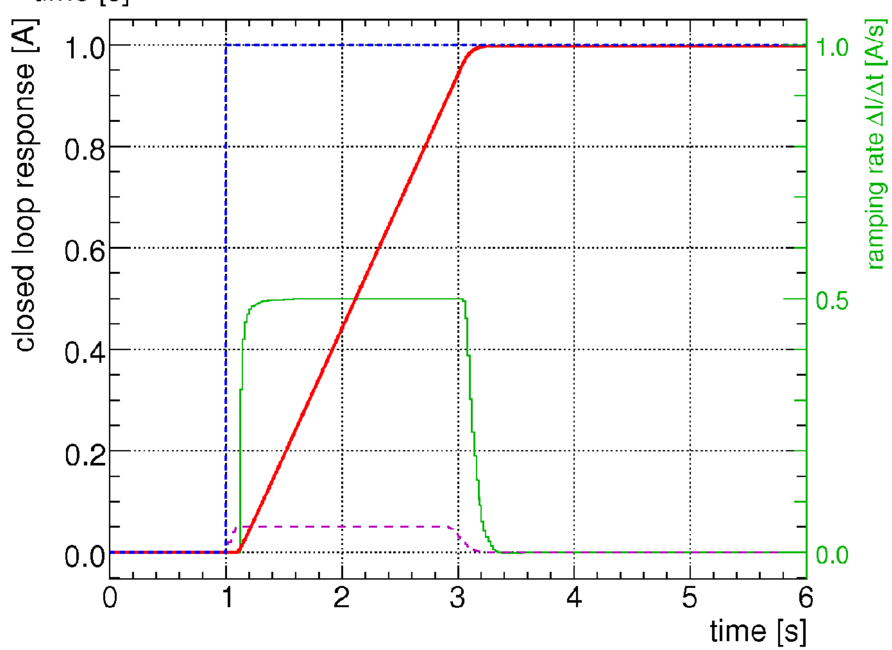
```
Ecurrent=0;
DeltaI = request-fmeasurement;

// subtract Smith Predictor inhibit signal
if (Smit_Predictor==kTRUE) DeltaI -= SmithP;
dcurrent[index]=DeltaI;

// PID controller
// analogue to digital gains calibration (essentially sampling)
Double_t C = MAX_SAMPLING*sampling;

// proportional control
Ecurrent += +Kp*dcurrent[index];
Ecurrent += -Kp*dcurrent[index-1];

// integral control
Ecurrent += +Ki*C*dcurrent[index];

// derivative control
Ecurrent += +Kd/C*dcurrent[index];
Ecurrent += -Kd/C*2*dcurrent[index-1];
Ecurrent += +Kd/C*dcurrent[index-2];


// if DeltaI is large -> boost response to the rate limit
if (non_linear_control==kTRUE)
if (fabs(non_linear_parameter*dcurrent[index]*sampling_f)>=(MAX_RAMP)) {
   Ecurrent=Ecurrent/fabs(Ecurrent)*(MAX_RAMP/sampling_f);
   non_linear=Ecurrent/fabs(Ecurrent)*MAX_RAMP*scale;
} else non_linear=0;


// MIMO system: rate limit has to be scaled the other circuits
// slowest determines the speed of the fastest circuits
// (for coherence reasons)
if (anti_windup==kTRUE)
if (fabs(Ecurrent)>(MAX_RAMP/sampling_f)) {
   if (Ecurrent+=0) Ecurrent=Ecurrent/fabs(Ecurrent)*(MAX_RAMP/sampling_f);
}

setcurrent+=Ecurrent;
// alternatively: if feedback signal from the PC are available
// setcurrent=current[index]+Ecurrent;
if (open_loop==kTRUE) setcurrent=request;
index++;

// Smith Predictor renew for next cycle
SmithP1=Smith_Predictor1(x,setcurrent); // undelayed plant
SmithP2=Smith_Predictor2(x,setcurrent); // delayed plant
SmithP = (SmithP1-SmithP2);             // SP inhibit signal
```

Motivation for Delay and Rate-Limiter Compensation
Example: LHC orbit (Q,Q',C^-, ...) feedback control





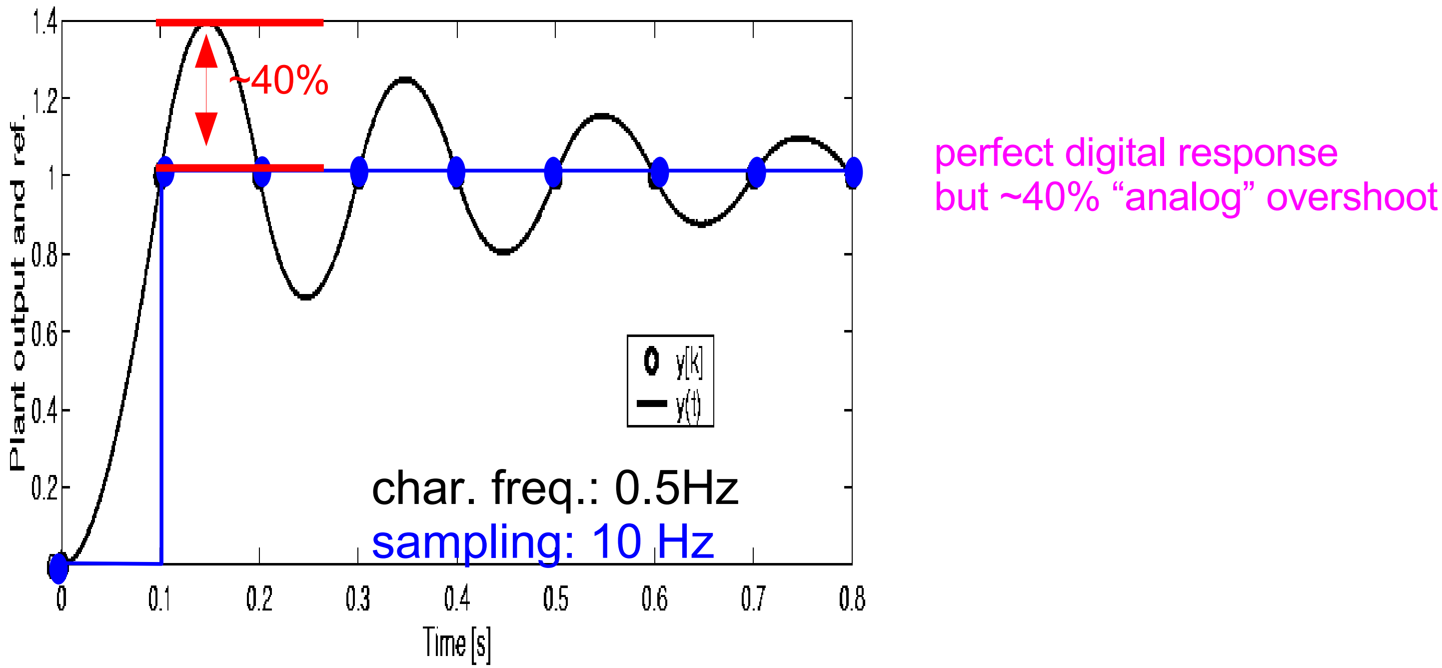
with full delay and windup compensator scheme:



Classic argument: Analogue vs. Digital Design

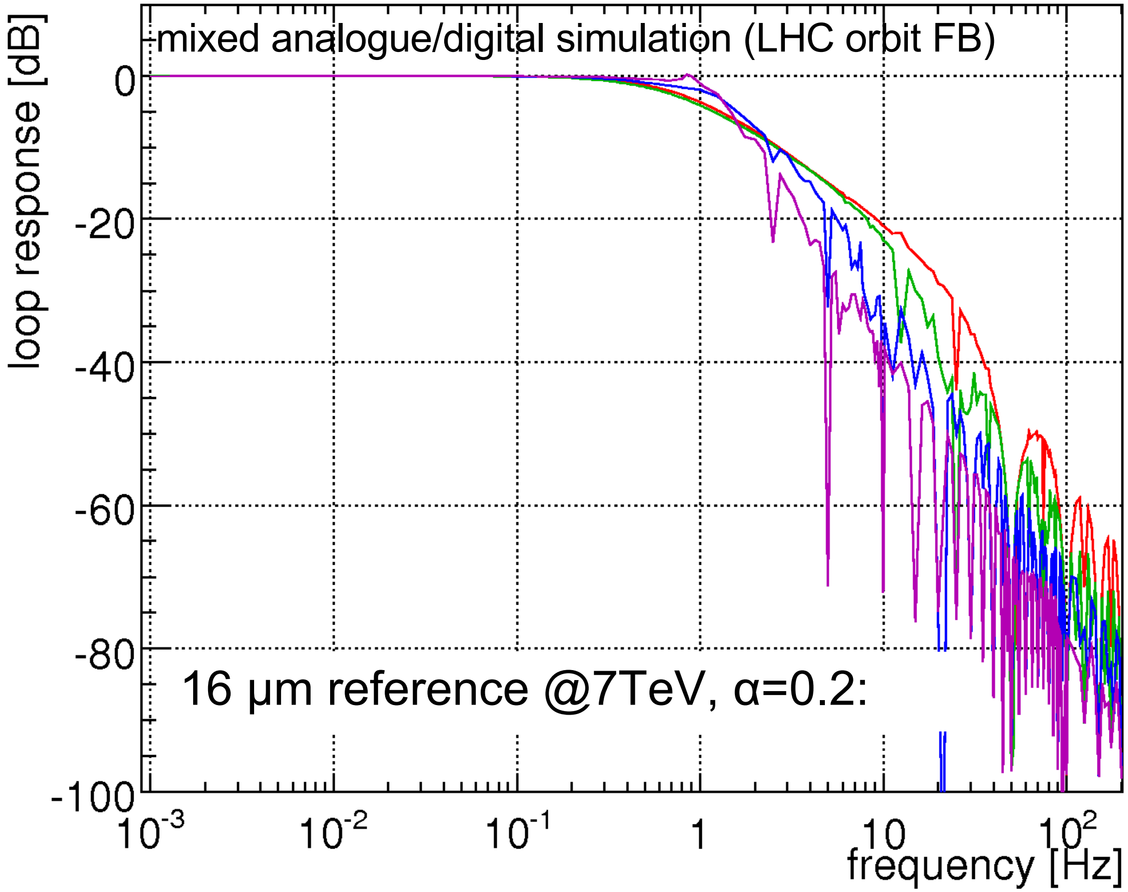
- Among many arguments:
  - Pro analogue: most process to be controlled are analogue
  - Pro digital: most controller are nowadays digital
    - “Con-example”: digital only controller design (inter-sample response)

time domain:



- Mitigation: iterative design approach between analogue and digital domain
  - sampling of simulation needs to be significantly larger than FB sampling
  - can be time consuming (especially for large MIMO systems)
  - beware of numerical instabilities and artefacts

- ... 10Hz sampling to achieve a closed loop 1Hz bandwidth:



  – ... a theoretic limit assuming a perfect system (no noise, model errors)!
  – common sense/advise: f_s > 25 ...40 x desired closed-loop bandwidth f_{BW}

Summary

- Feedbacks are only as good as the measurements they are based upon!
  - Systematic and thorough analysis of involved beam instrumentation and corrector circuits is essential!

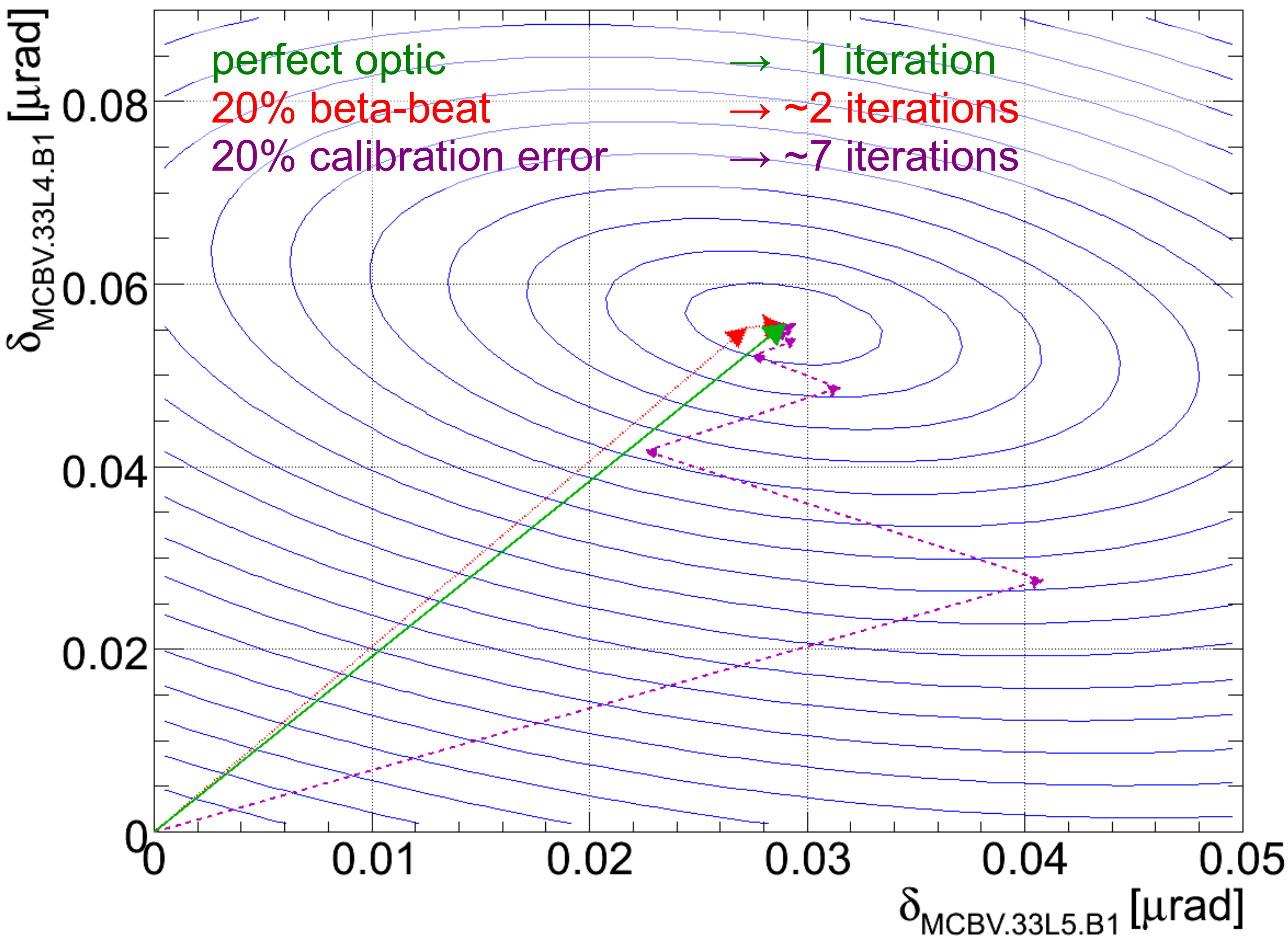
- Shown how large MIMO systems and their control can be decomposed into: 'Space-' (classic parameter correction, usually SVD based and 'Time-Domain'.

- Youla's affine parameterisation facilitates optimal adaptive non-linear control
  - enables simple gain-scheduling based on operational scenario

- Beware of sampling!

Reserve Slides

- Imperfect optic and calibration error can deteriorate the convergence speed on the level of the SVD based correction:
- Example: 2-dim orbit error surface projection



Optimal Controller Design
Youla's affine parameterisation - 2nd Order Example

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)}$$

- 2nd Example: classic 2nd order process:

$$G(s)=\frac{K_0\omega_0^2}{s^2+2\zeta\omega_0\, s+\omega_0^2}$$

K_0: open loop gain, ω_0: characteristic frequency
ζ_0: attenuation

- Using standard ansatz:

$$Q(s)=F_Q(s)\cdot G^i(s)=\frac{\omega_{cl}^2}{s^2+2\zeta_{cl}\omega_{cl}\, s+\omega_{cl}^2}\cdot G^i(s)$$

- yields classic PID controller (optimal gains):

$$D(s) = K_p+K_i\cdot\frac{1}{s}+K_d\cdot\frac{s}{\tau_d s+1}$$

with:

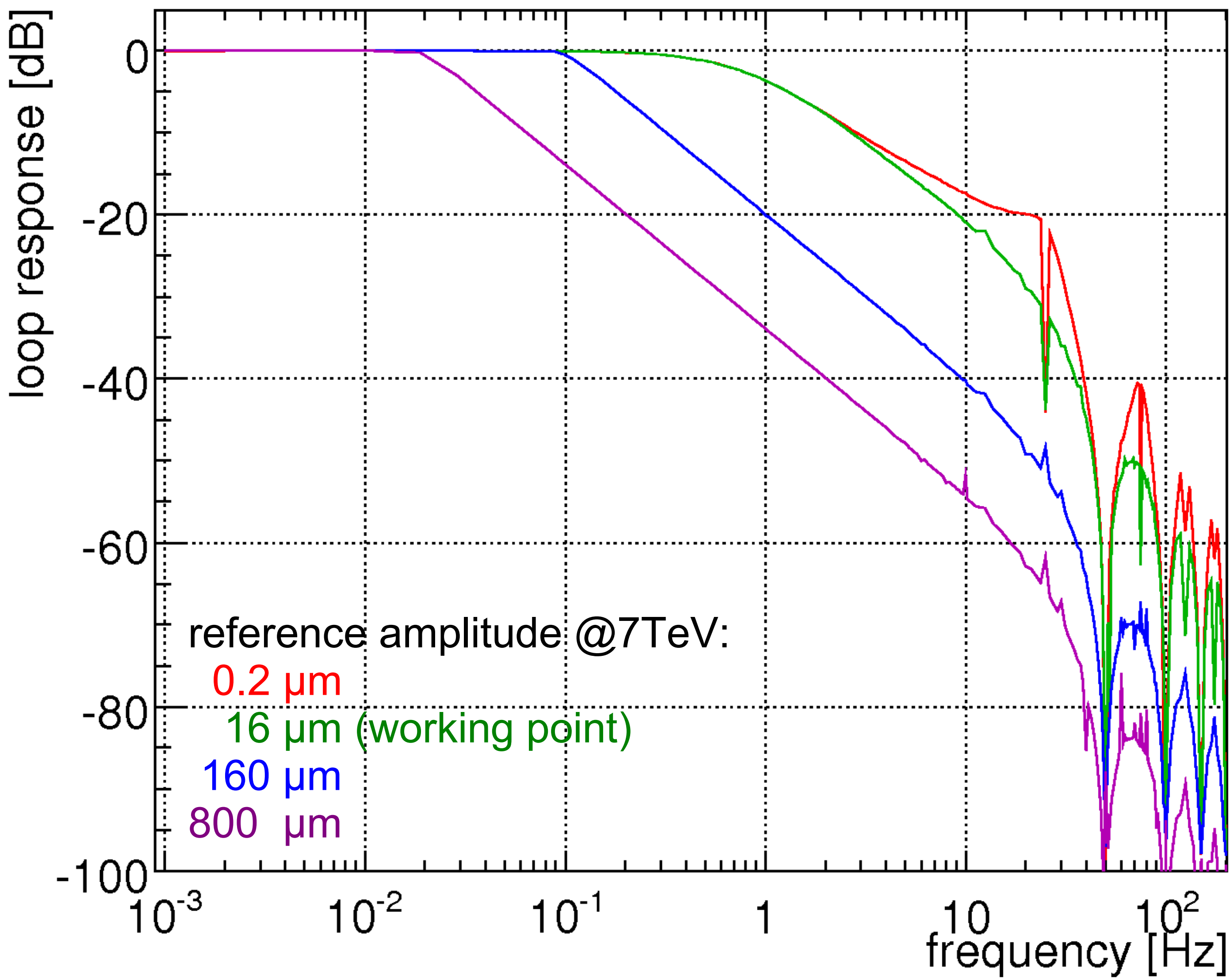
$$K_p = \frac{4\zeta_{cl}\zeta_0\omega_0\omega_{cl}-\omega_0^2}{4\mathrm{K}_0\zeta_{cl}^2} \qquad K_i = \frac{\omega_0^2\omega_{cl}}{2\mathrm{K}_0\zeta_{cl}}$$

$$K_d = \frac{4\zeta^2\omega_{cl}^2-4\zeta_0\omega_0\zeta_{cl}+\omega_0^2}{8\mathrm{K}_0\zeta_{cl}^3\omega_{cl}} \qquad \tau_d = \frac{1}{2\zeta_{cl}\omega_{cl}}$$

  - further simplification: require critical damping → $\zeta_{cl}:=1$
    - ω_{cl} ~ 'open loop bandwidth' is the remaining free parameter

Nominal Feedback Response T_0

- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)



Nominal Feedback Disturbance Rejection S_{d0}

- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)