Influence of Varying Tune Width on the Robustness of the LHC Tune PLL and its Application for Continuous Chromaticity Measurement

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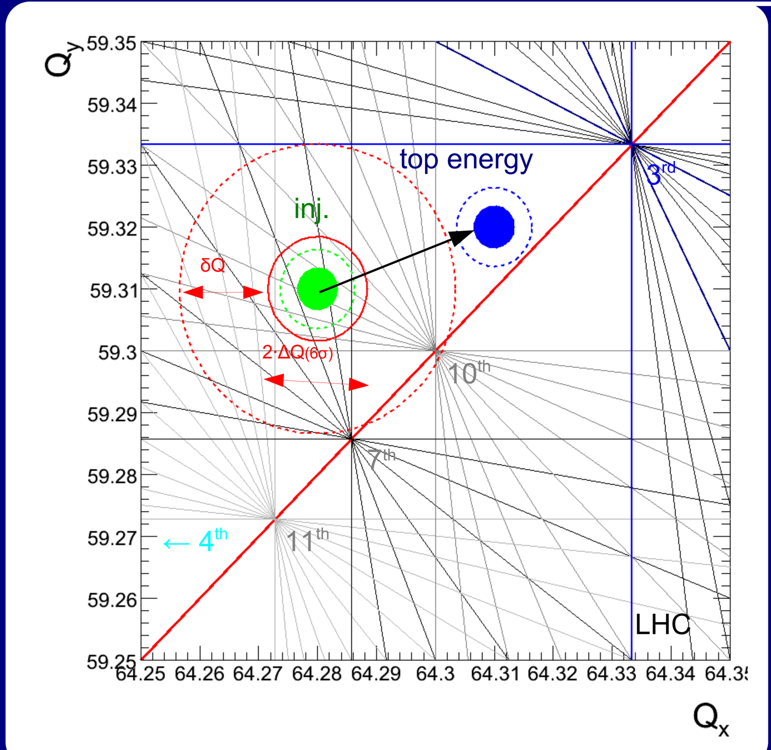
Abstract

Tune and chromaticity measurement is an integral part for safe and reliable LHC operation. Tight tolerances on the maximum transverse beam excursions allow oscillation amplitudes of less than 30 um. This leaves only a small margin for transverse beam and momentum excitations required for measuring tune and chromaticity.
This contribution discusses a robust tune phase-locked-loop (PLL) operation in the presence of non-linearities and varying chromaticity. The loop design was tested at the SPS, using the LHC PLL prototype system.
The system was also used to continuously measure tune width and chromaticity changes, using resonant transverse excitations of the tune side-slopes.

Parameter Stability Requirements



- Stability requirements on LHC tune and chromaticity are primarily driven by the ability to control particle loss. The lack of synchrotron radiation damping (hadrons) requires up to 12th order resonances to be avoided.
- LHC collimation imposes tight constraints on the orbit/betatron oscillations:

$$\Delta z \lesssim 35\,\mu m \quad \rightarrow \quad \frac{\Delta p}{p} < 10^{-4}$$

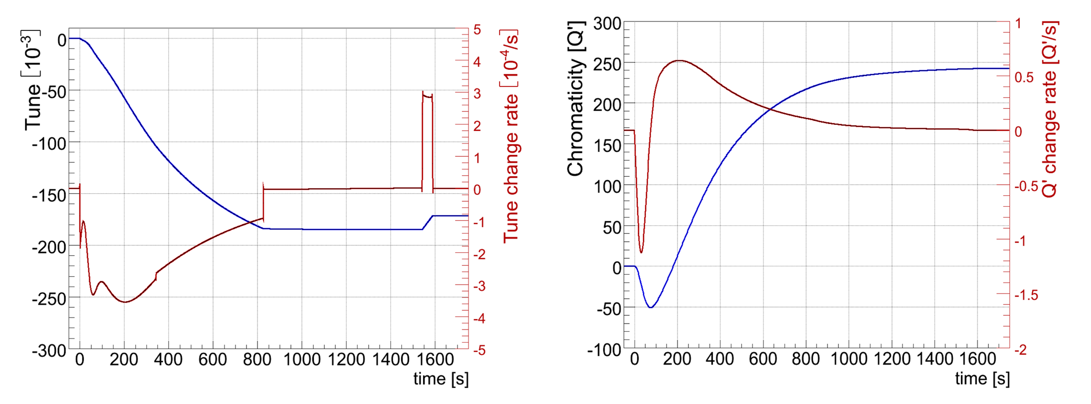
- → Classic Method: $\Delta p/p > 10^{-3}$ & $\Delta Q_{res} \approx 10^{-3}$ → $\Delta Q'_{res} \sim 1$
- → LHC: $\Delta p/p < 10^{-4}$ & $\Delta Q'_{res} \sim 1$ → $\Delta Q_{res} < 10^{-4}$

$$\Delta Q = Q' \frac{\Delta p}{p}$$

  - tough, still not established!
  - R&D on alternative methods ongoing: cont. head-tail, side-exciter...

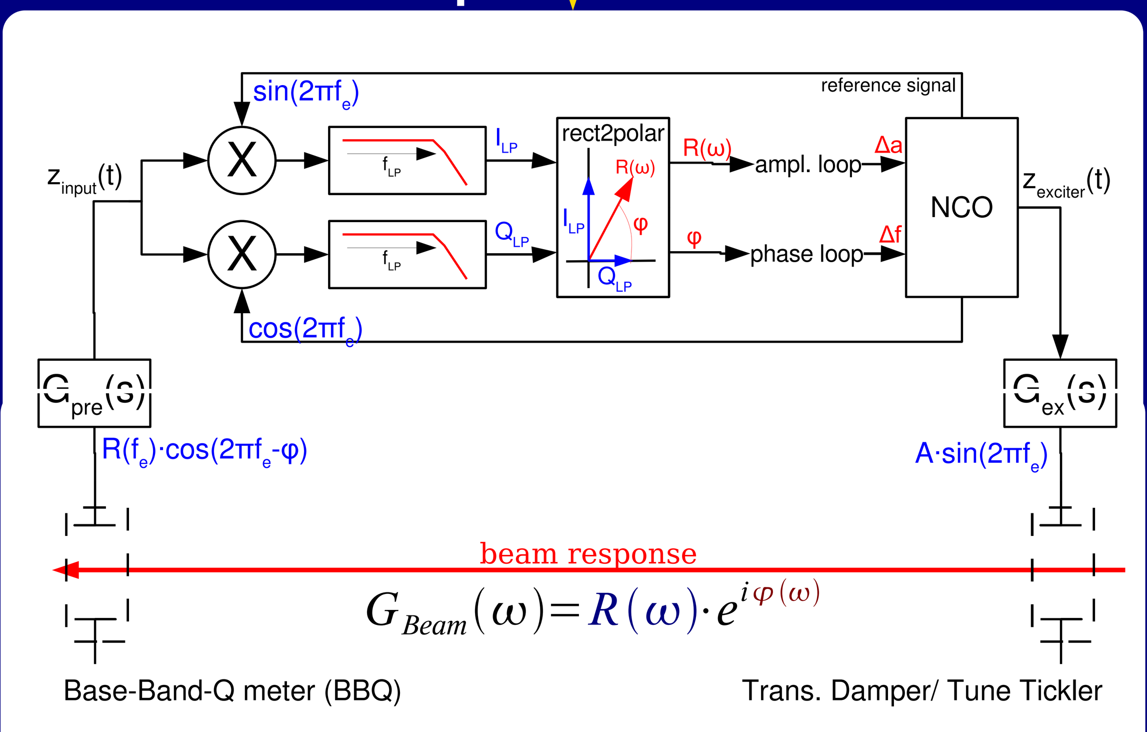
Expected Perturbations vs. Requirements:

- Sources: power supply drifts and ripples, hysteresis, ramp tracking errors, beam-beam, e-cloud, decay & snap-back, persistent currents, ...
  - → Ramp: max. $\Delta Q' \approx 300$ units @ $\Delta Q'/\Delta t \approx 1.2$ units/s



	Orbit [σ]	Tune [0.5·f_rev]	Chroma. [units]	Energy [Δp/p]	Coupling [c_]
Perturbations:	~ 1-2 (30 mm)	0.025 (0.06)	~ 70 (300)	± 1.5e-4	~0.01 (0.1)
Max. Drift Rate:	~ 25 μm/s	< 10^{-3} /s	< 1.3 s		
Pilot	-	± 0.1	+ 10 ??	-	-
Commissioning	± ~ 1	±0.015→0.003	> 0 ± 10	± 1e-4	« 0.03
Nominal	± 0.15	±0.003 / ±0.001	2 ± 1	± 1e-4	« 0.01

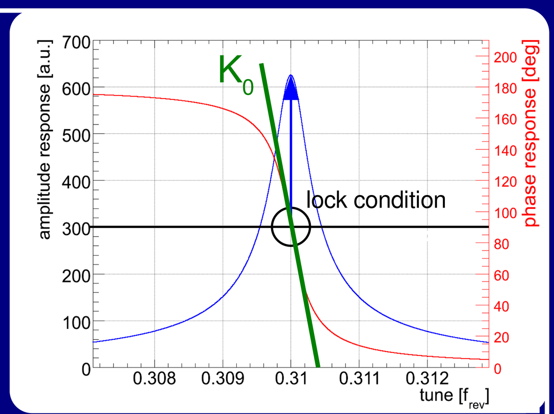
Phase-Locked-Loop:



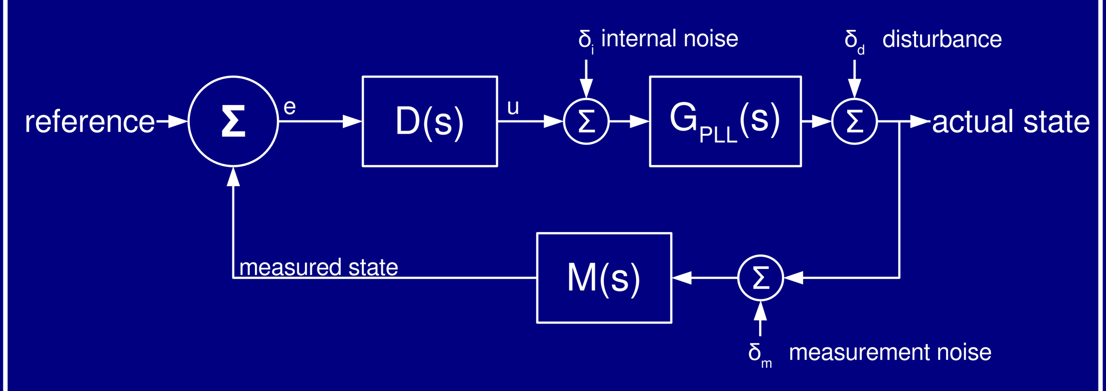
- FPGA based decoupled loop implementation:
  - → phase-locked-loop → tune
    - controller task: adjust exciter frequency to match resonance condition $\Delta\varphi = \varphi - 90° = 0°$
  - → excitation amplitude loop → limits max. excitation
- Further compensation for other non-beam related phase responses:
  - → constant lag (data processing, cables),
  - → analogue pre-filters, beam exciter response...

PLL Controller Design Schematic:

- Control loop dynamics split into two parts:
  1. PLL low-pass filter:
     → $\tau = \frac{1}{f_{BW}}$
  2. Beam response:
     → open loop gain: K_0 = const. (first order)



- Open-Loop Transfer Function: $G_{PLL}(s) \approx \frac{K_0}{\tau s + 1}$
  (close-to-lock-condition approximation)



- Youla's Affine Parameterisation → PI controller:

$$D(s) = K_P + K_i \frac{1}{s} \quad with \quad K_p = K_0 \frac{\tau}{\alpha} \ \wedge \ K_i = K_0 \frac{1}{\alpha}$$

  - optimal controller gains are coupled
  - K_0 fixed by effective tune width → see right
  - free parameter: closed-loop bandwidth $1/\alpha$

Youla's Affine Parameterisation:

- For any open-loop stable system, all stable closed loop controllers D(s) can be written as:

$$D(s) = \frac{Q(s)}{1 - Q(s)G(s)} \qquad (1)$$

- Reduced closed-Loop response functions:

classic:

$$T_0(s) := \frac{y}{r} = \frac{DG}{1+DG}$$
$$S_{d0}(s) := \frac{y}{\delta_d} = \frac{1}{1+DG}$$
$$S_{i0}(s) := \frac{y}{\delta_i} = \frac{G}{1+DG}$$
$$S_{u0}(s) := \frac{u}{\delta_d} = \frac{D}{1+DG}$$

→ reduced:

$$T_0(s) = QG$$
$$S_{d0}(s) = 1 - QG$$
$$S_{i0}(s) = (1 - QG)G$$
$$S_{u0}(s) = Q$$

- Using the following common ansatz (here: PLL example):

$$Q(s) = F_Q(s) G^i(s) = \frac{1}{\alpha s + 1} \cdot \frac{\tau s + 1}{K_0}$$

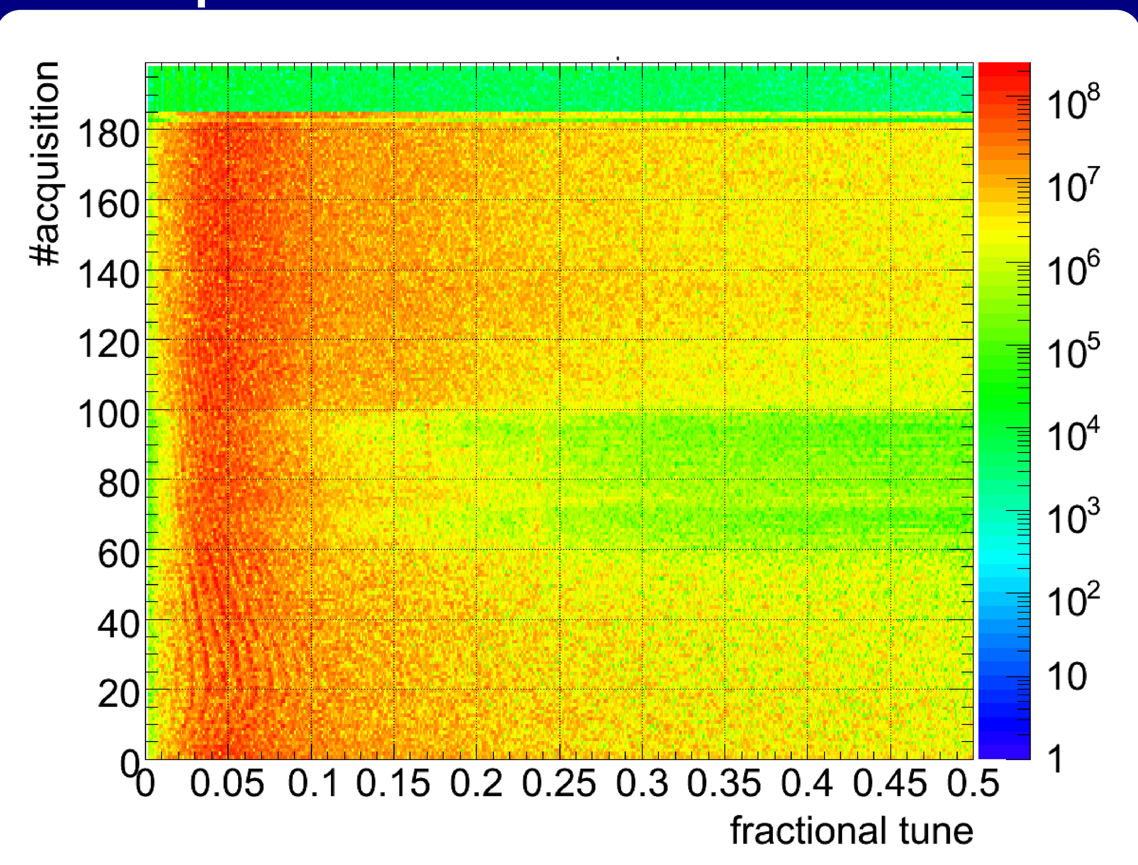
  - – $F_Q(s)$: models the desired closed-loop response
  - – $G^i(s)$: pseudo-inverse of the nominal plant G(s)
    - in case of non-lin. or unstable zeros e.g.:

$$G(s) = G_{lin}(s) \cdot G_{NL}(s) \cdot e^{-\lambda s} \ldots \quad \rightarrow \quad G^i(s) = (G_{lin})^{-1}$$

    - (yields Smith predictor, anti-windup schemes, etc.)
  - MP system: $\rightarrow T_0(s) = F_Q(s) = \frac{1}{\alpha s + 1}$

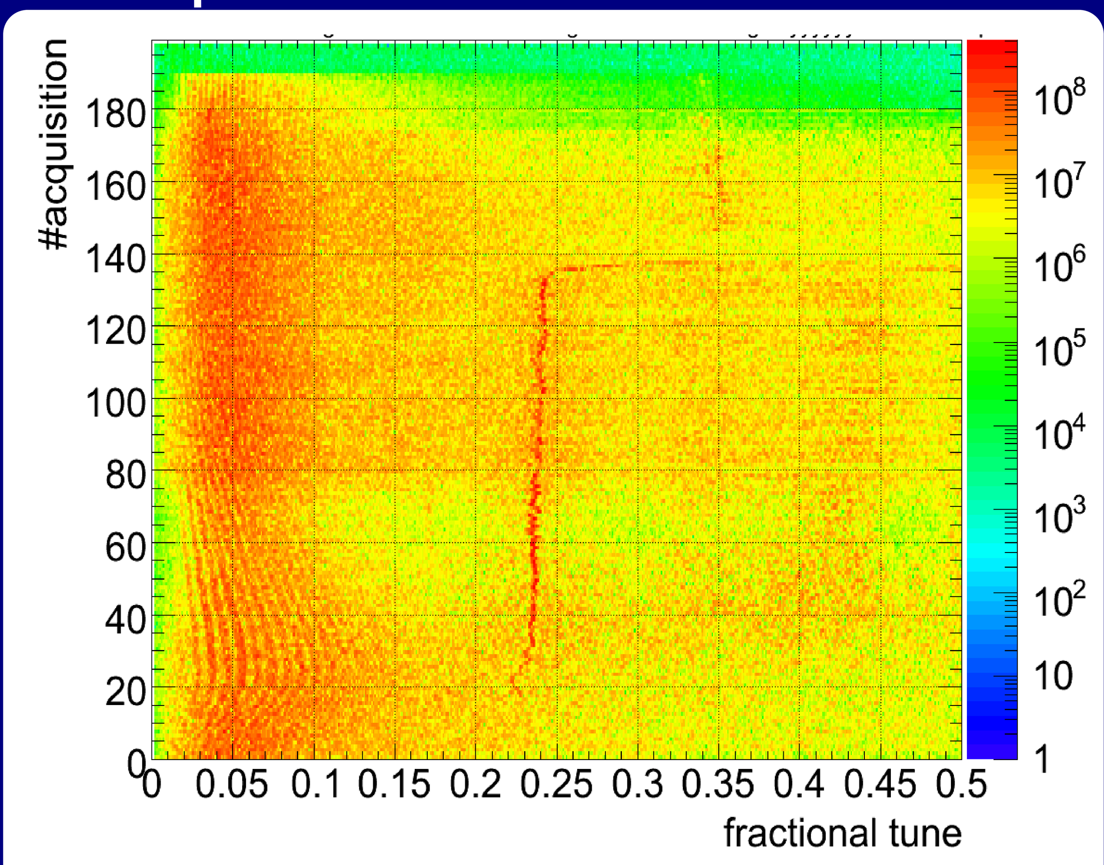
[1] D. C. Youla et al., *"Modern Wiener-Hopf Design of Optimal Controllers"*, IEEE Trans. on Automatic Control, 1976, vol. 21-1,pp. 3-13 & 319-338
[2] R.J. Steinhagen, *"Feedbacks on Tune and Chromaticity"*, DIPAC'07, 2007

FFT Acquisition without Tune PLL



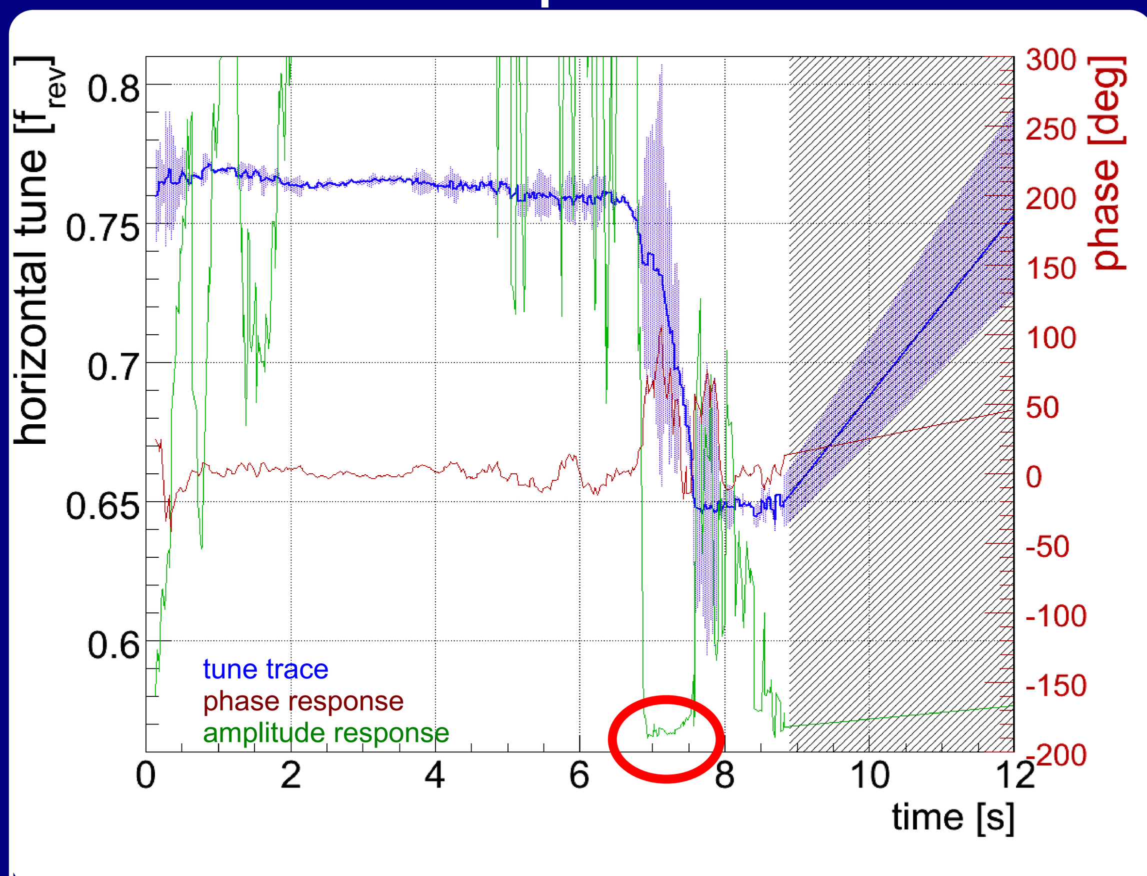
- → Same system implements continuous FFT acq. used to monitor driven and/or residual oscillation.
- → Beam synchronised excitation/acquisition:
  - 256 → 2^{18} turns (LHC: 22 ms → ~ 23 s)
- → For details, please see poster FRPMN073.

FFT Acquisition with Tune PLL



- Tune resolution:
  - → FFT based (1024 turns): $\Delta Q_{res} \approx 10^{-3} \ldots 10^{-4}$
  - → PLL based: $\Delta Q_{res} \ll 10^{-3} \ldots 10^{-6}$
    - depends on required tracking bandwidth
    - excitation below the 1 μm level
      → negligible/no emittance blow-up ($\sigma_{beam} \approx$ 1mm)

PLL Tune Trace Example:



- SPS 25ns fixed target beam:
  - → 26GeV → 450GeV, ~ 3e12 protons/beam
  - → Horizontal tune: $Q_h \approx 26.76 \rightarrow 26.66$ (3rd order slow extraction)
- Phase and non-vanishing amplitude indicates lock during ramp.
- Tracked tune change: $\Delta Q \approx 0.1$ within ~ 200-300 ms
  - → faster than maximum expected LHC tune drift!

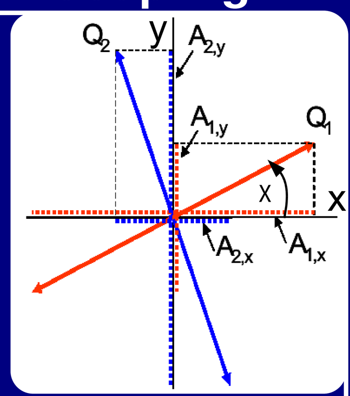
J.R. Steinhagen, 2007

Feedback Robustness ...and Betatron Coupling:

Strictly: PLL measures tune eigenmodes

- β- coupling may rotate these w.r.t. unperturbed tunes (q_x, q_y, $\Delta=|q_y-q_y|$):

$$Q_{1,2}=\frac{1}{2}\left(q_x+q_y\pm\sqrt{\Delta^2+|C^-|^2}\right)$$

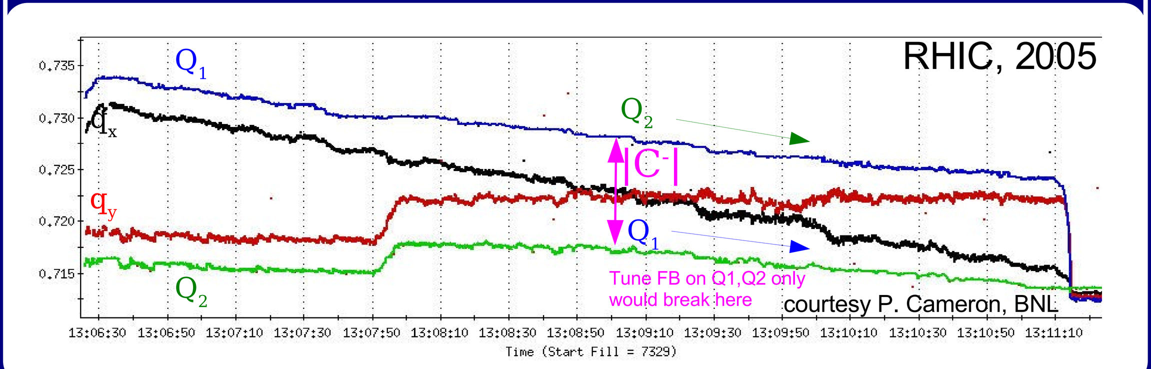


- Possible improvements:
  - optimise tune working point (inc. tune-split),
  - vertical orbit stabilisation in lattice sextupoles (→ orbit FB!),
  - active compensation and correction of coupling:

$$r_1=\frac{A_{1,y}}{A_{1,x}} \wedge r_2=\frac{A_{2,x}}{A_{2,y}}$$

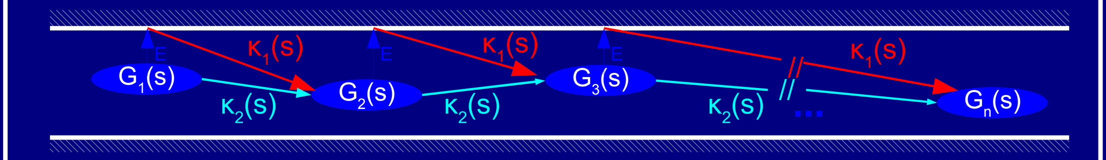
$$\Rightarrow |C^-|=|Q_1-Q_2|\cdot\frac{2\sqrt{r_1 r_2}}{(1+r_1 r_2)} \wedge \Delta=|Q_1-Q_2|\cdot\frac{(1-r_1 r_2)}{(1+r_1 r_2)}$$

  - use q_x, q_y → quadrupole strengths (Tune FB)
  - use: $|C^-|$, χ → skew-quadrupole strengths (Coupling FB)

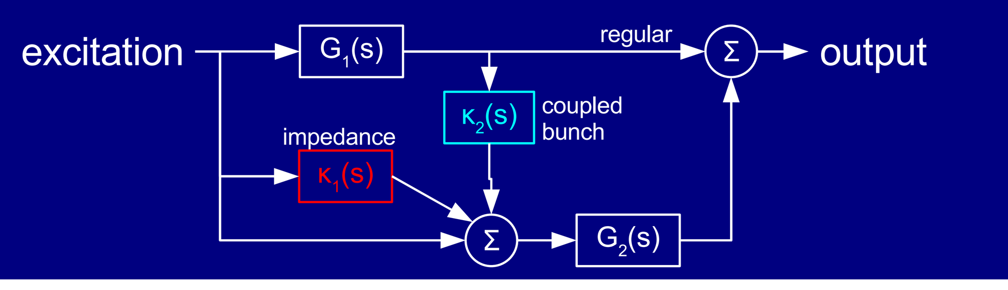


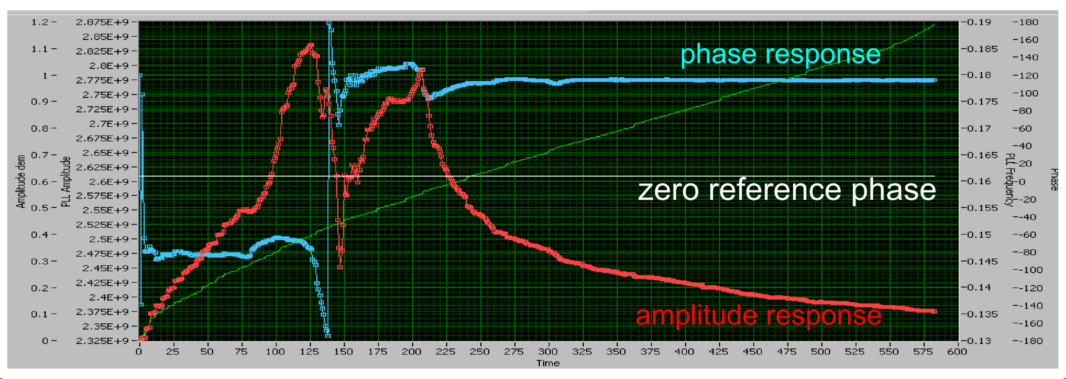
...in the Presence of Coupled Bunch Instabilities

- High-sensitivity tune pick-up (BBQ) enables PLL to operate within the transverse feedback "noise"
  - Pro: minimises inter-loop coupling effects
  - Con: no benefit from transverse damper suppression of e-cloud, impedance, beam-beam, etc. driven coupled bunch modes → bunch selector



- Two bunch model:



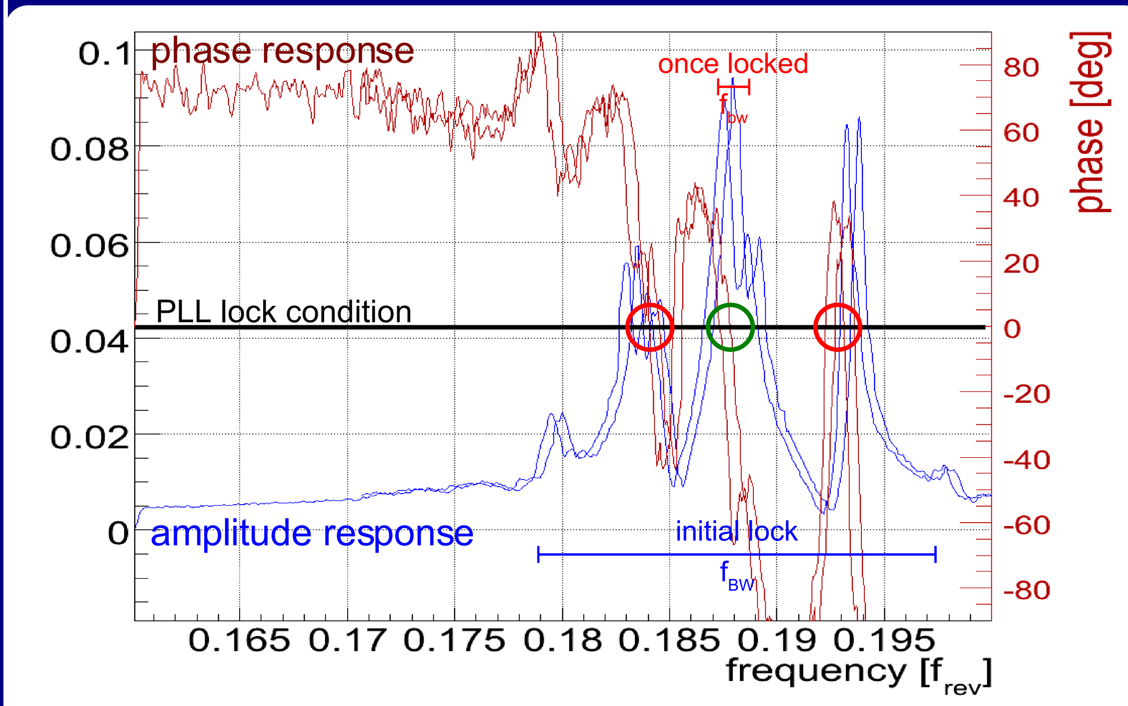


...in the Presence of Synchrotron Side-bands

Hilbert Transform relates amplitude to phase (MP System):

$$\varphi(\omega)=Hilbert\left[ln|R(\omega)|\right]$$

→ PLL locks on the within the bandwidth largest peak



Option I: gain scheduling

initial lock: open bandwidth to cover more than one side band (noise/chirp)

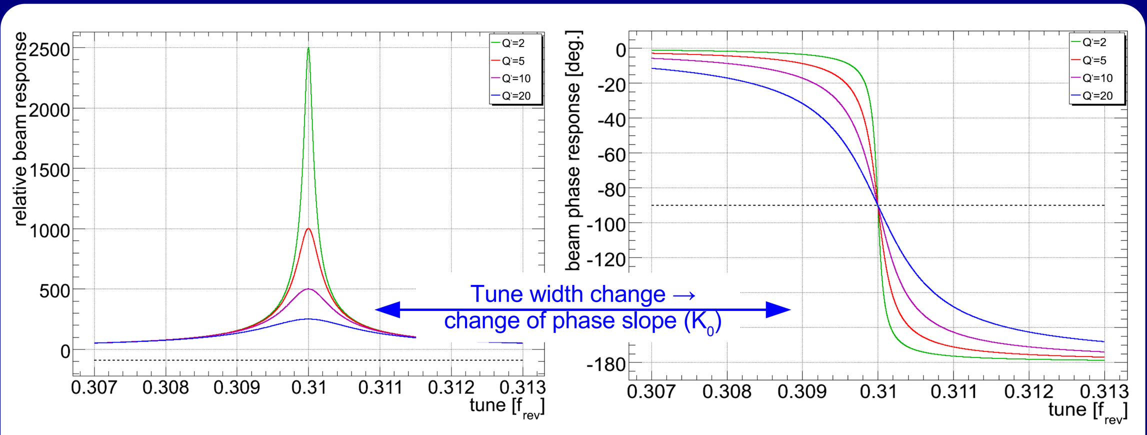
- side-bands "cancel out", strongest resonance prevails

once locked: reduce bandwidth for better stability/resolution

Option II: larger excitation bandwidth

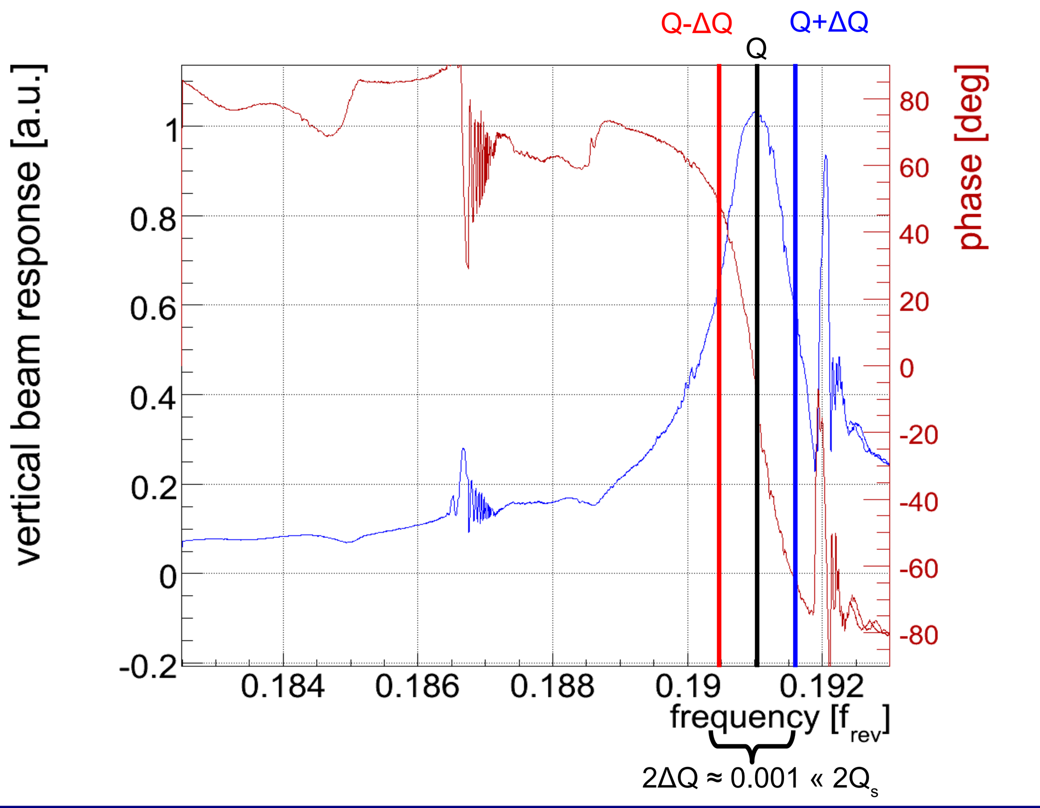
multiple exciter or broadband excitation (e.g. Tevatron/FNAL)

Tune Feedback Robustness ...and Tune-width:



- Design and optimal PLL parameters (tracking speed, etc.) depend - beside measurement noise – on the effective ΔQ.
- Intrinsic trade-off:
  - Optimal PI for large tune width ΔQ ↔ sensitivity to noise (unstable loop) for small ΔQ
  - Optimal PI for small tune width ΔQ ↔ slow tracking speed for large ΔQ
- Possible mitigation through 'gain scheduling'

ΔQ Measurement using PLL Side Exciter:

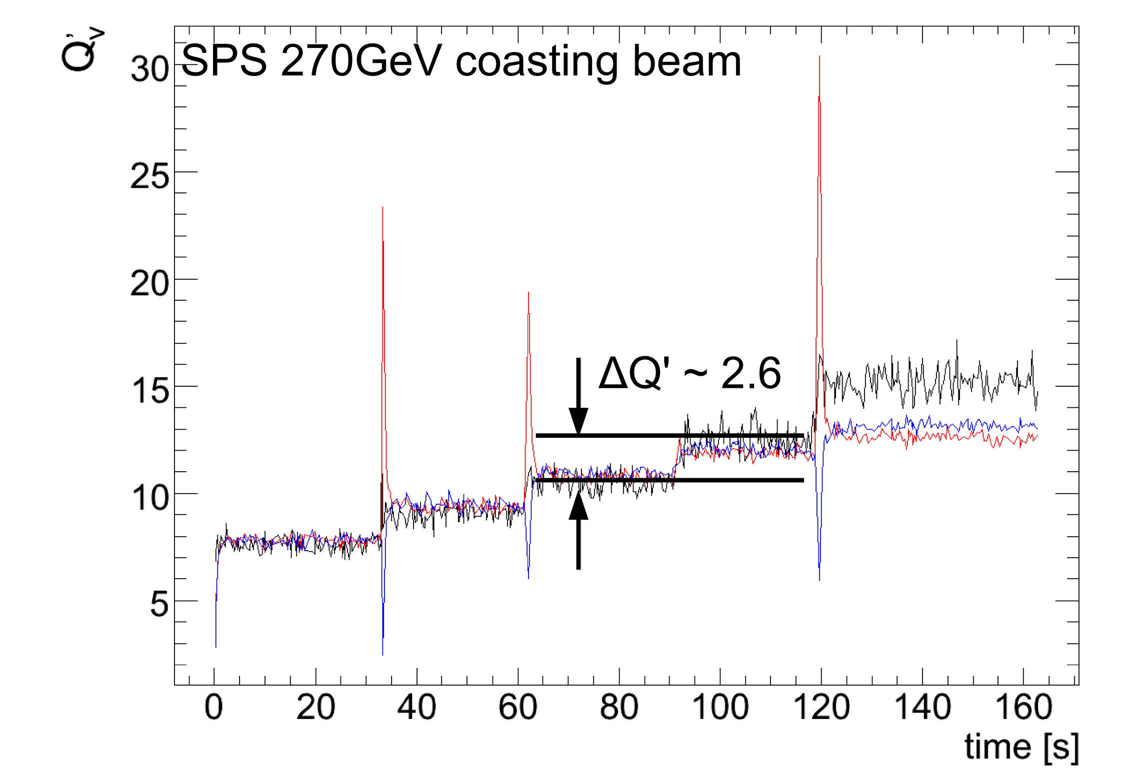


- Resonant phase change ↔ tune width change
  - → "free" real-time tune footprint measurement
  - → measurable dependence of ΔQ ~ Q'

driven resonance:

$$\tan(\varphi)\approx\frac{\Delta Q\cdot\omega_Q\omega_D}{\omega_Q^2-\omega_D^2}$$

Tune-width Dependence on Chromaticity:



- Side-exciter based tune-width changes linearly with Q'
  - No additional momentum modulation
  - Transients due to Q-PLL lock lag ($\Delta Q/\Delta t > \Delta Q\cdot f_{BW}$)
  - Abs. scale requires calibration to classic Q' meas.
  - Non-linear effects require further assessment
    - octupolar and h.-o. terms, impedance,
    - transverse and longitudinal feedback contribution

Interpretation of BTF Measurements:

- Classic kicked head-tail ansatz:
  - single particle phase advance per turn:

$$\dot{\theta}=\omega Q\approx 2\pi\cdot\left[Q_0+Q'\cdot\frac{\Delta p}{p}\right] \quad (1)$$

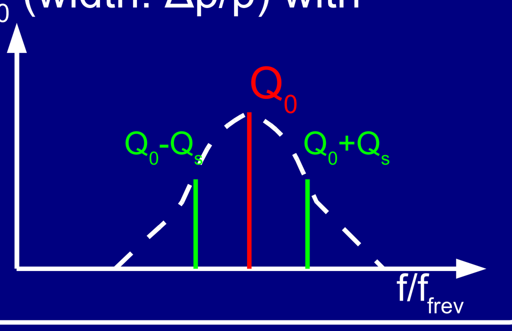
  - multiple turns (synchrotron oscillation):

$$\frac{\Delta p}{p}\rightarrow\left(\frac{\Delta p}{p}\right)_i(t)=\delta_{i,max}\cdot sin(2\pi Q_s t+\phi_i)$$

  - yields:

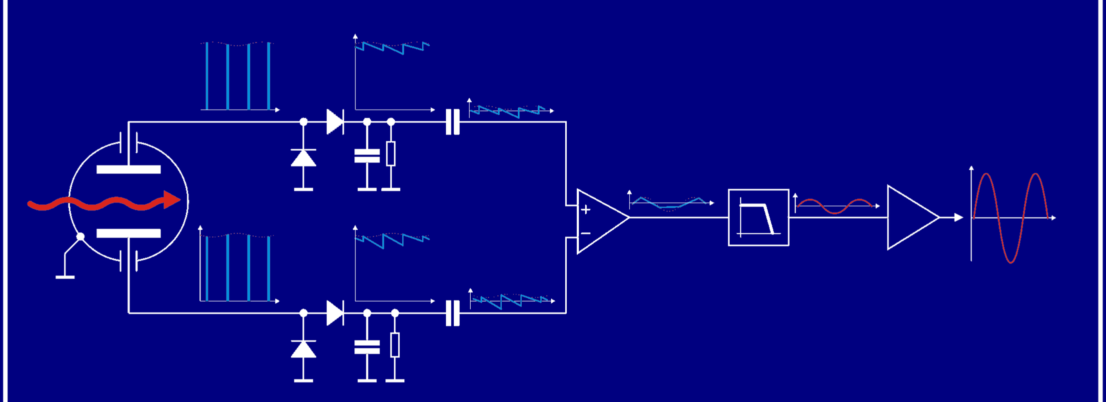
$$z(n)\sim sin(2\pi Q_0\cdot n+Q'\delta_i\cdot sin(\omega_s n+\phi)+h.o.)$$

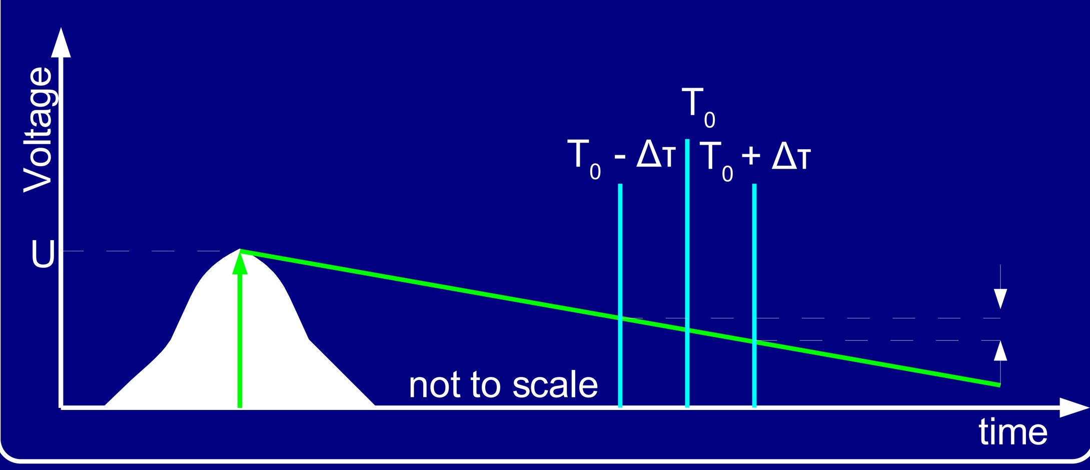
- observable: Betatron tune Q_0 (width: Δp/p) with synchrotron side-bands
- Does not explain measured beam transfer functions!



Residual Synchrotron Oscillation Dependence:

- Diode-based Base-Band-Q metre principle:



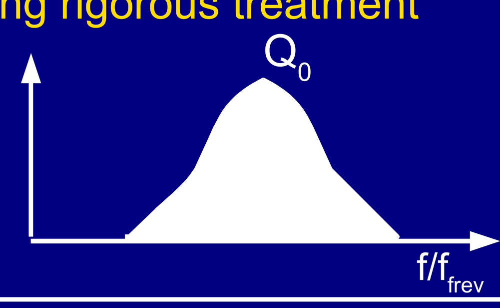


- Transverse single particle oscillation frequency varies with longitudinal bunch arrival time
- BBQ is a peak detector:
  - demodulates bunch arrival = synchrotron oscill.
  - measured driven beam response resembles the one of an un-captured beam:

$$\left(\frac{\Delta p}{p}\right)_{i,meas}\sim \underbrace{sin(2\pi Q_s t+\phi_i)}_{=const}\rightarrow\left(\frac{\Delta p}{p}\right)_{i,meas}=const$$

$$\rightarrow \quad \dot{\theta}\approx 2\pi\cdot\left[Q_0+Q'\cdot\frac{\Delta p}{p}\right]=const$$

  - Measured response is the convolution of above phase with the bunch momentum distribution
  - Further studies and a ongoing rigorous treatment required!



Summary:

- The prototype test of the BBQ based tune PLL were very successful! Mutually exclusive modes of PLL operation:
  - either: track tune changes with $\Delta Q/\Delta t \approx 0.1/s$
  - or: achievable tune resolution $\Delta Q_{res}\approx 10^{-4} \ldots 10^{-5}$

  } Youla's affine parameterisation facilitates the trade-off selection and simplifies the required dynamic adjustments due to varying tune width.
- BBQ based PLL showed to be very robust as long as bunch-to-bunch coupling was small will be addressed through selecting only single bunch
- Question is not: "Can we measure chromaticity?", but "Can we measure Q' with a given precision and minimal excitation?"
  - Requires studies of systematics with "slow" coasting beam to prove feasibility of LHC Q' baseline (ΔQ'=1 & Δp/p«10-4)

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