Feedback Architecture and Commissioning at the LHC

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Overview

Will cover:

- Feedback architecture and its 'test-bed'
- Some comments on their commissioning

Disclaimer:

- Already covered in earlier presentations:
  - Beam Instrumentation and their commissioning
- Will evolve most issues around orbit feedback system
  - largest multi-input-multi-output system, largest complexity
  - issues and control schemes are the same for tune (Q), chromaticity (Q'), coupling and energy feedback

Preliminary Remark:

- Traditional requirements on beam stability (in particular orbit)...

... to keep the beam in the pipe!

- LHC: Requirements/time-line of key beam parameters control depend on:
  1. Capability to control level/ tolerances of particle losses in the machine
     - Machine protection & Collimation
     - Quench prevention
  2. Commissioning and operational efficiency

Expected Dynamic Perturbations vs. Requirements

- Expected dynamic perturbations*
  - For details, please see additional slides

	Orbit [σ]	Tune $[0.5\cdot f_{rev}]$	Chroma. [units]	Energy [Δp/p]	Coupling [c_]
Exp. Perturbations:	~ 1-2 (30 mm)	0.025 (0.06)	~ 70 (140)	± 1.5e-4	~0.01 (0.1)
Pilot bunch	-	± 0.1	+ 10 ??	-	-
Stage I Requirements	± ~ 1	±0.015→0.003	> 0 ± 10	± 1e-4	« 0.03
Nominal	± 0.3 / 0.5	±0.003 / ±0.001	1-2 ± 1	± 1e-4	« 0.01

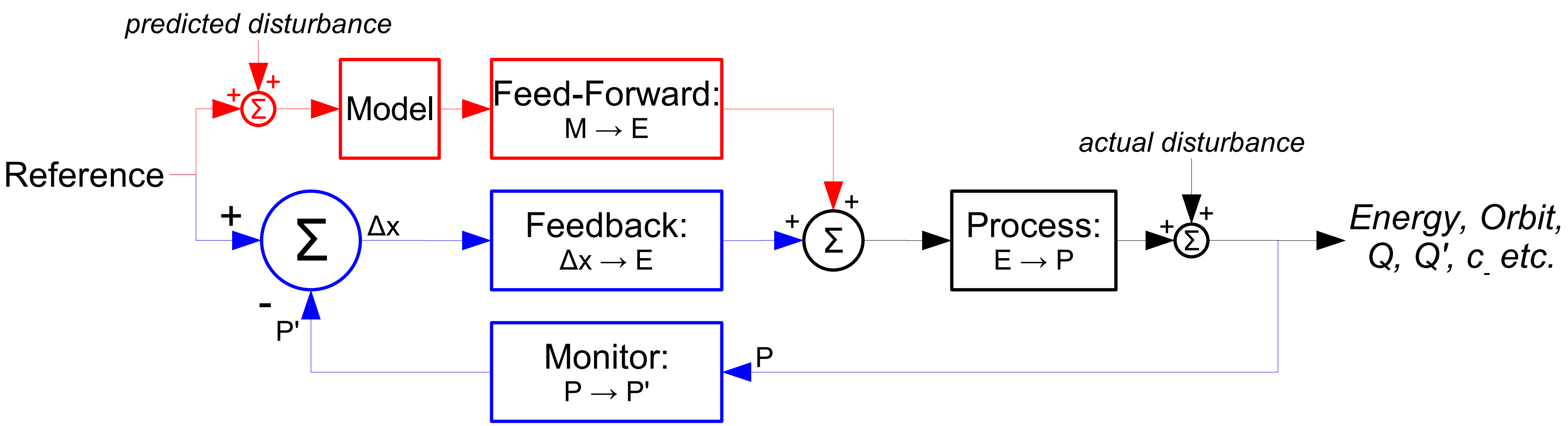
- Feedback priority list: Tune/Coupling → Chromaticity → Orbit → Energy
- Feedback list of “what's easiest to commission”:
  - 1rd: Orbit → functional BPM system → OK
  - 1½: Energy → consequence of 100k turn acquisition → OK
  - 2nd: Tune/Coupling → functional Q-meter (-PLL) → Day I-N
  - 3rd: Chromaticity → functional Q-meter and Δp/p modulation → ??
- Foresee time to commission feedbacks at an early stage
  - Most instruments are commissioned parasitically with first circulating beam
  - Feedbacks can significantly speed up commissioning if used at an early stage

* numbers in brackets are 'worst case'

Parameter control, either through...

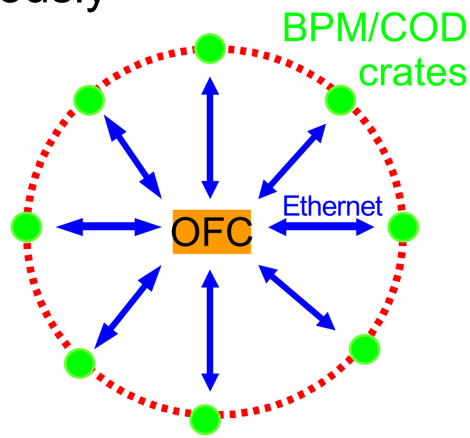
- Feed-Forward: (FF)
  - Steer parameter using precise process model and disturbance prediction
- Feedback: (FB)
  - Steering using rough process model and measurement of parameter
  - Two types: within-cycle (repetition Δt<<10 hours) or cycle-to-cycle (Δt>10 hours)

preferred choice!



- From the steering point of view: → All control schemes possible
- Choice of Feedback vs. Feed-forward
  - depends on available robust beam parameter measurements

- Small perturbations around the reference orbit will be continuously compensated using beam-based alignment through a central global orbit feedback with local constraints:



  - 1056 beam position monitors
    - BPM spacing: $\Delta\mu_{BPM}\approx 45°$ (oversampling → robustness!)
    - Measure in both planes: > 2112 readings!
  - One Central Orbit Feedback Controller (OFC)
    - Gathers all BPM measurements, computes and sends currents through Ethernet to the PC-Gateways to move beam to its reference position:
      - high numerical and network load on controller front-end computer
      - a rough machine model is sufficient for steering (insensitive to noise and errors)
      - most flexible (especially when correction scheme has to be changed quickly)
      - easier to commission and debug
  - 530 correction dipole magnets/plane (71% are of type MCBH/V, ±60A)
    - total 1060 individually powered magnets (60-120 A)
    - ~30 shared between B1&B2

- With more than 3100 involved devices the largest and most complex system

Summary: Total Number of (FB) Corrector Circuits

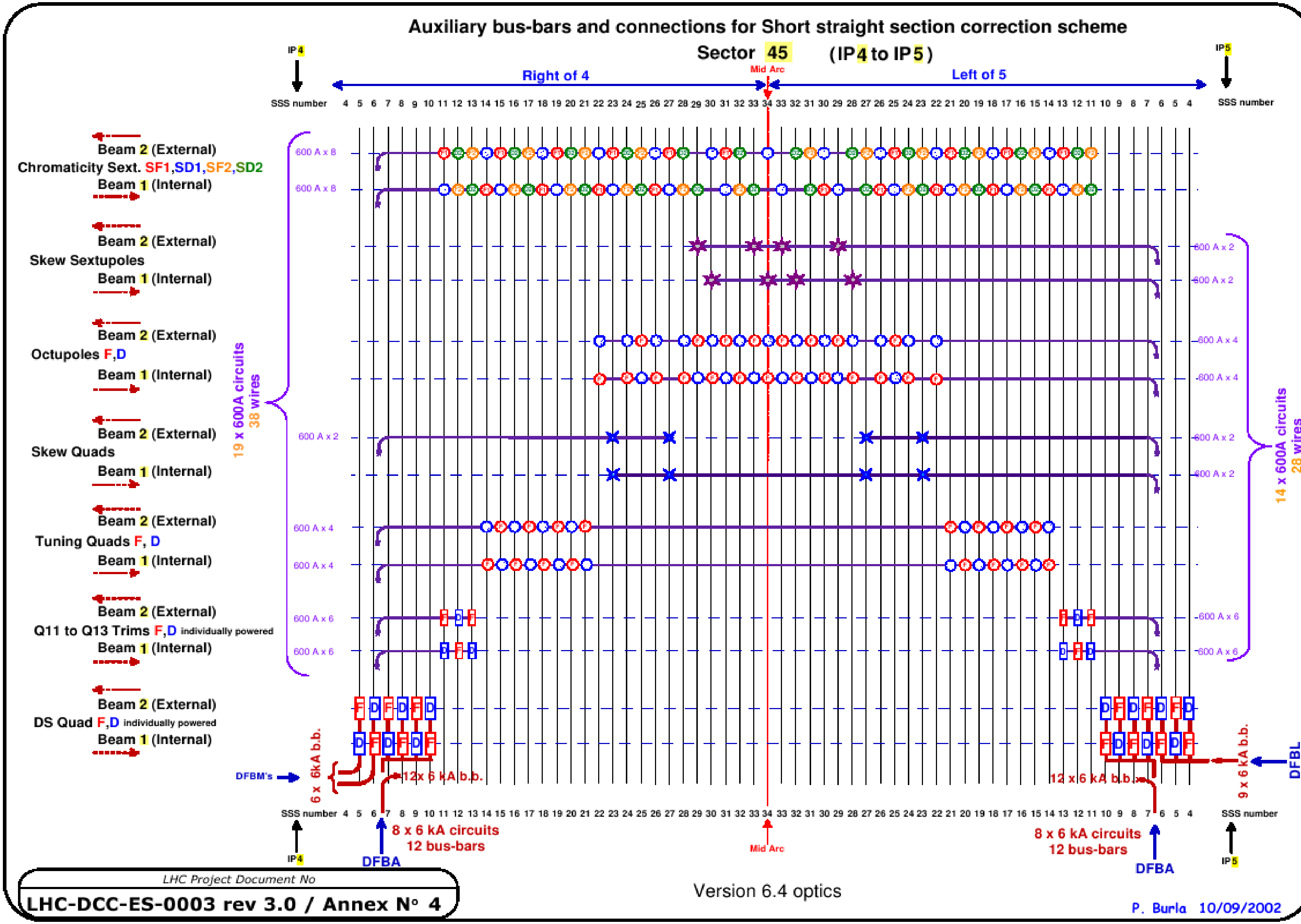
- Tune:
  - 16x ±600A circuits powered from even IPs (2, 4, 6, 8), 2 families
  - independent for Beam 1&2, but coupling between planes
  - can use them independently, possible use of DS Quadrupoles

- Chromaticity:
  - 32x ±600A circuits powered from even IPs, 4 families

- Coupling: four skew quadrupoles per arc, 1/2 families
  - Beam 1: 12x ±600A
  - Beam 2: 10x ±600A

- Total: 1130 of 1720 circuits/power converter → more than half the LHC is controlled by beam based feedback systems!

Powering Layout of the SSS Correction Scheme



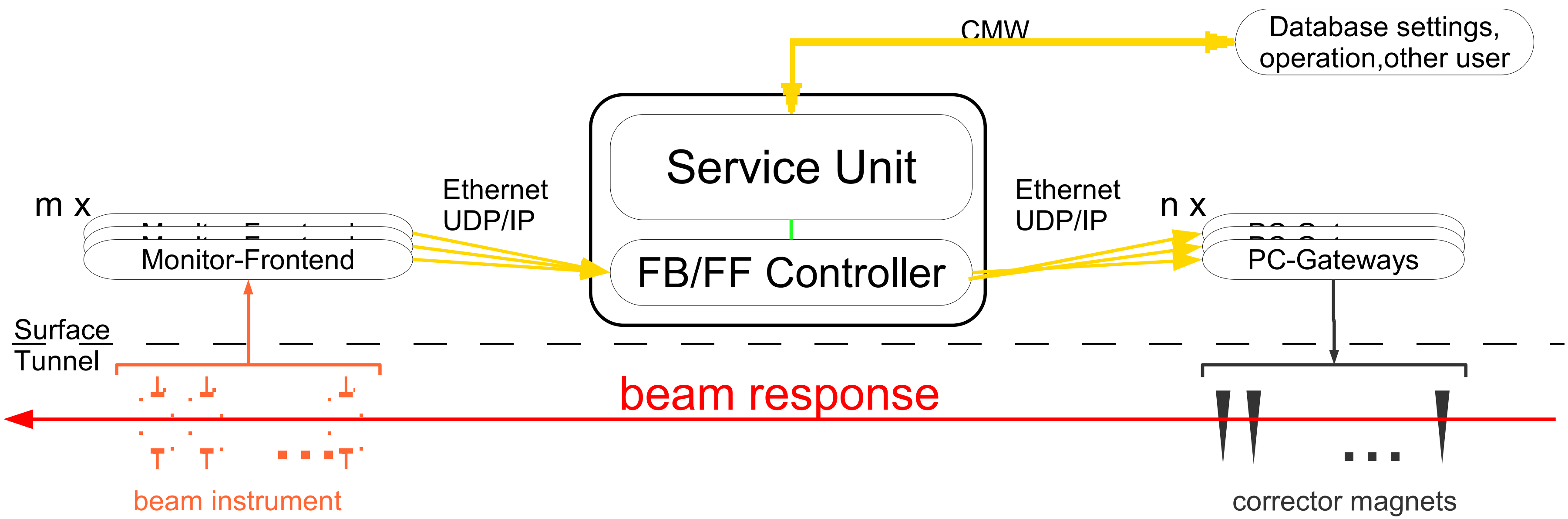
Common Feedback/Feed-forward Control Layout

LHC feedback control scheme implementation split into two sub-systems:

– Service Unit: Interface to users/software control system

– Feedback Controller: actual parameter/feedback control logic

- Simple streaming task for all feed-forwards/feedbacks:
  $(\text{Monitor} \rightarrow \text{Network})_{FB} \rightarrow$ Data-processing → Network → PC-Gateways
- Can run auto-triggered (no timing necessarily required)
- Hardware and functional specifications already available

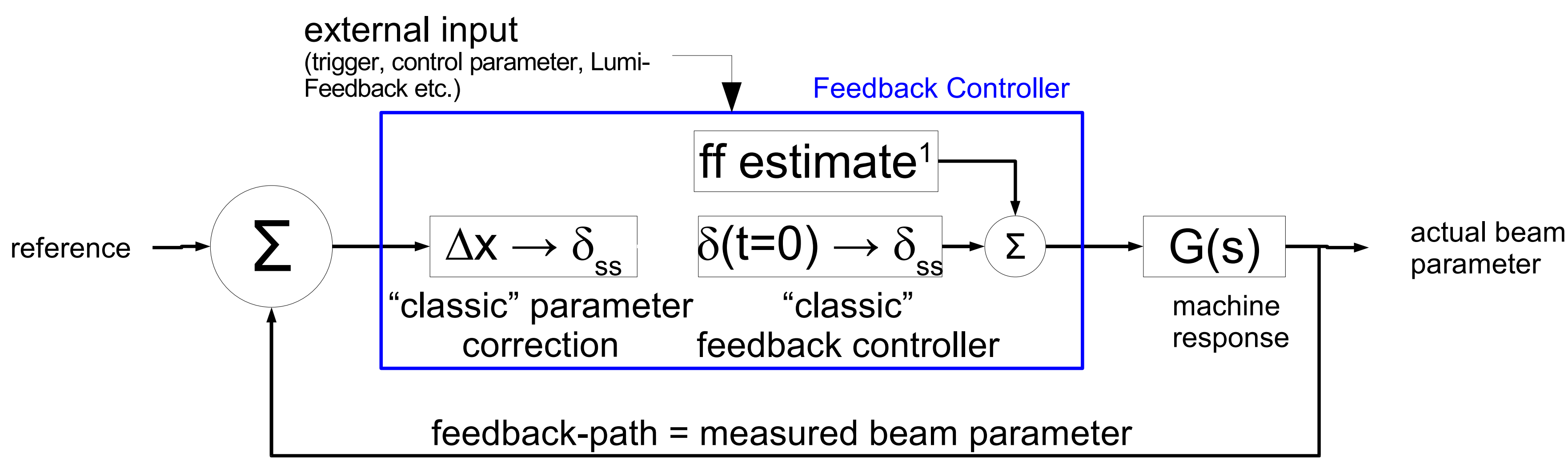


Orbit Feedback/Feed-Forward Control Scheme

- The feedback controller consists of three stages:
  1. Compute steady-state corrector settings $\vec{\delta}_{ss} = (\delta_1, \ldots, \delta_n)$ based on measured parameter shift $\Delta x=(x_1,\ldots, x_n)$ that will move the beam to its reference position for $t\rightarrow\infty$.
  2. Compute a $\vec{\delta}(t)$ that will enhance the transition $\vec{\delta}(t=0)\rightarrow\vec{\delta}_{ss}$
  3. Feed-forward: anticipate and add deflections $\vec{\delta}_{ff}$ to compensate changes of well known and properly described[1] sources:

space domain (stage 1)

time domain (stages 2–3)



[1] properly described = accurate & fast real-time model of the source

Space-Domain: No "black feedback magic"

- Effects on orbit, Energy, Tune, Q' and C^- can essentially be cast into matrices:

$$\Delta \vec{x}(t) = \underline{R} \cdot \vec{\delta}_{ss} \quad with \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cdot \cos(\Delta \mu_{ij} - \pi Q)$$

matrix multiplication

  - similar for other parameters but different dimension
  - their control consists essentially in inverting these matrices
    - no special arrangement/decoupling of circuits necessary!

$$\underline{R}_{orbit} \in \mathbb{R}^{1056 \times 530} \qquad \underline{R}_{Q} \in \mathbb{R}^{2 \times 16} \qquad \underline{R}_{Q'} \in \mathbb{R}^{2 \times 32} \qquad \underline{R}_{C^-} \in \mathbb{R}^{2 \times 10/12}$$

- Some potential complications:
  - Singularities = over/under-constraint matrices, noise, element failures, spurious BPM offsets, calibrations, ...
  - Time dependence of total control loop
  - Controls: How to receive, process, send data ...

Task in space domain:

Solve linear equation system and/or find (pseudo-) inverse matrix R^{-1}

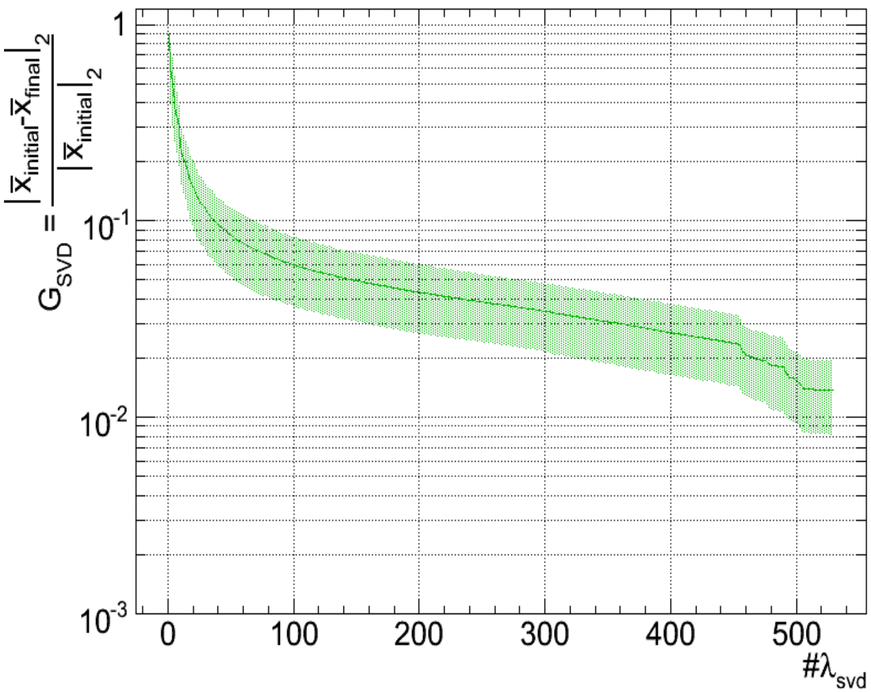
$$\left\|\vec{x}_{ref}-\vec{x}_{actual}\right\|_2=\left\|\underline{R}\cdot\vec{\delta}_{ss}\right\|_2<\epsilon \quad\rightarrow\quad \vec{\delta}_{ss}=\tilde{R}^{-1}\Delta\vec{x}$$

- Singular Value Decomposition (SVD) is the preferred orbit feedback workhorse:
  - standard and proven eigenvalue approach
  - insensitive to COD/BPM faults and their configuration (e.g. spacing)
  - minimises parameter deviations and COD strengths
  - numerical robust:
    - guaranteed solution even if orbit response matrix is (nearly) singular
      (e.g. two CODs have similar orbit response ↔ two rows are (nearly) the same)
    - easy to identify and eliminate singular solutions
  - high complexity:
    - Gauss(MICADO): $O= \frac{1}{2} mn^2 + 1/6\ n^3$
    - SVD: $O= 2mn^2+4n^3$

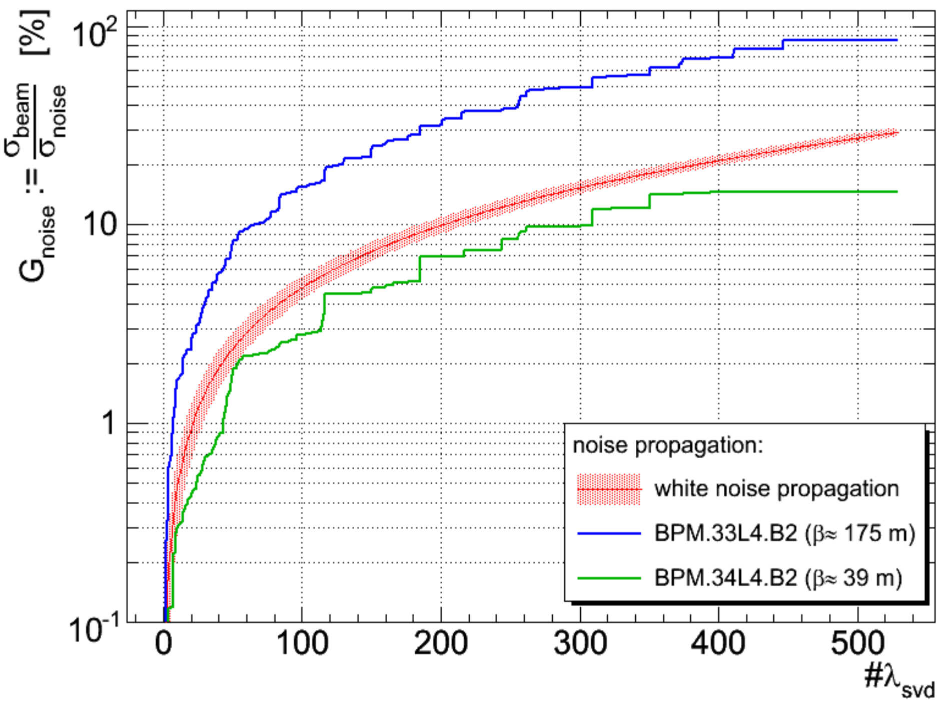
m=n: SVD is 9 times more expensive, even on high-end CPUs full initial decomposition may take several seconds (LHC: ~15 s/plane), but once decomposed and inverted: simple matrix multiplication ($O(n^2)$) complexity, LHC orbit correction <15ms!)

Example SVD based orbit correction

- Orbit attenuation



- Sensitivity to BPM noise



- Number of for the inversion used eigenvalues steers accuracy versus robustness of correction algorithm
- Likewise applies Tune, Chromaticity and Coupling correction
  – However: Only two out of '*n*' eigenvalues are non-singular

- Similar to PLL, power converter response can be approximated by low-pass:

$$G(s)=\frac{K_0}{\tau s+1} \quad with\, e.g. \quad \tau \approx 0.5\ldots 1\, s\,(\Leftrightarrow f=1\ldots 2\, Hz) \quad (1)$$

- Youla's affine parameterisation[1] for stable plants:

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)} \quad (2)$$

- Using the following ansatz

$$Q(s)=F_Q(s)G^i(s)=\frac{1}{\alpha s+1}\cdot\frac{\tau s+1}{K_0} \quad (3)$$

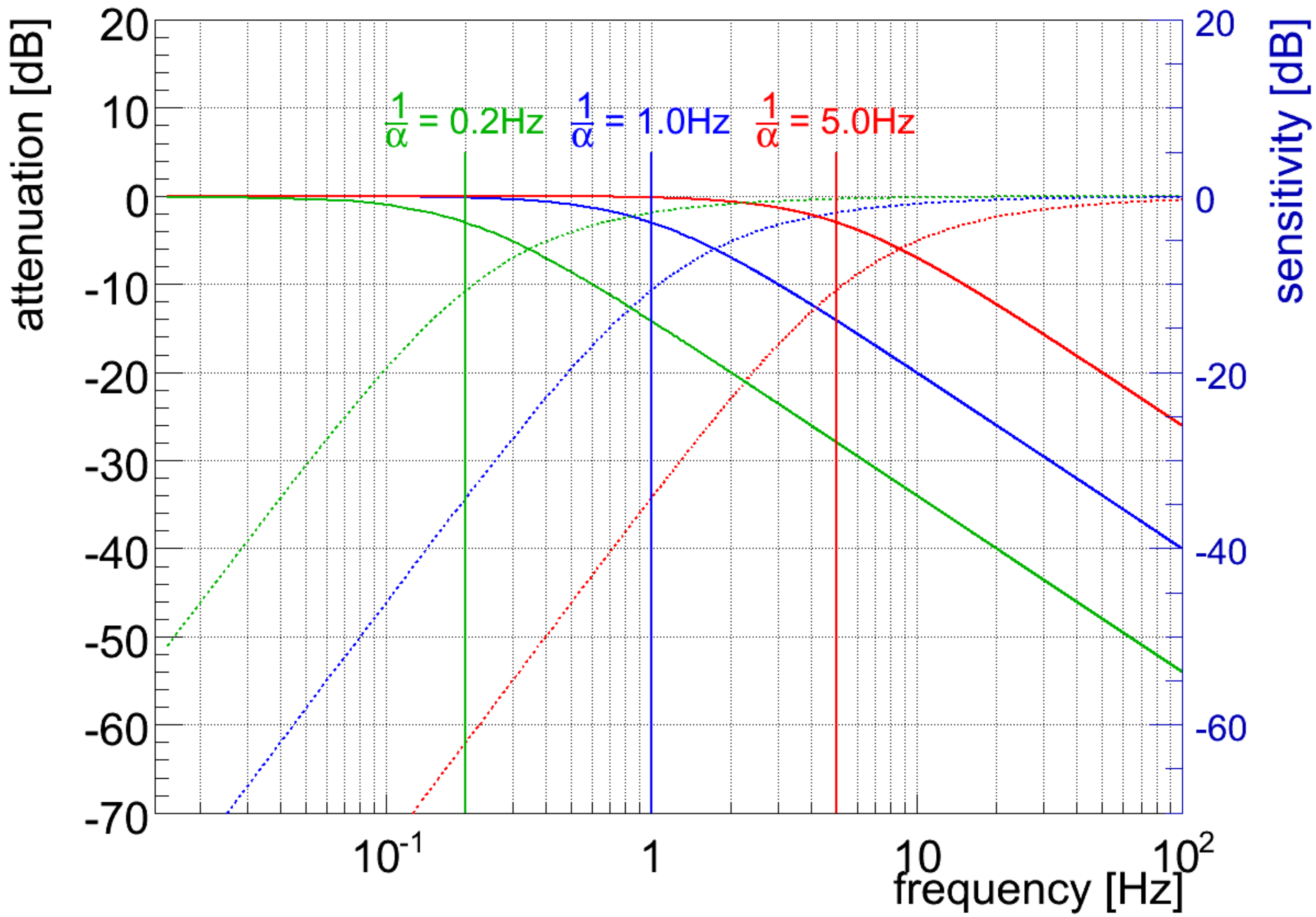
- (1)+(2)+(3) yields:

$$D(s)=K_P+K_i\frac{1}{s} \quad with \quad K_p=K_0\frac{\tau}{\alpha} \wedge K_i=K_0\frac{1}{\alpha}$$

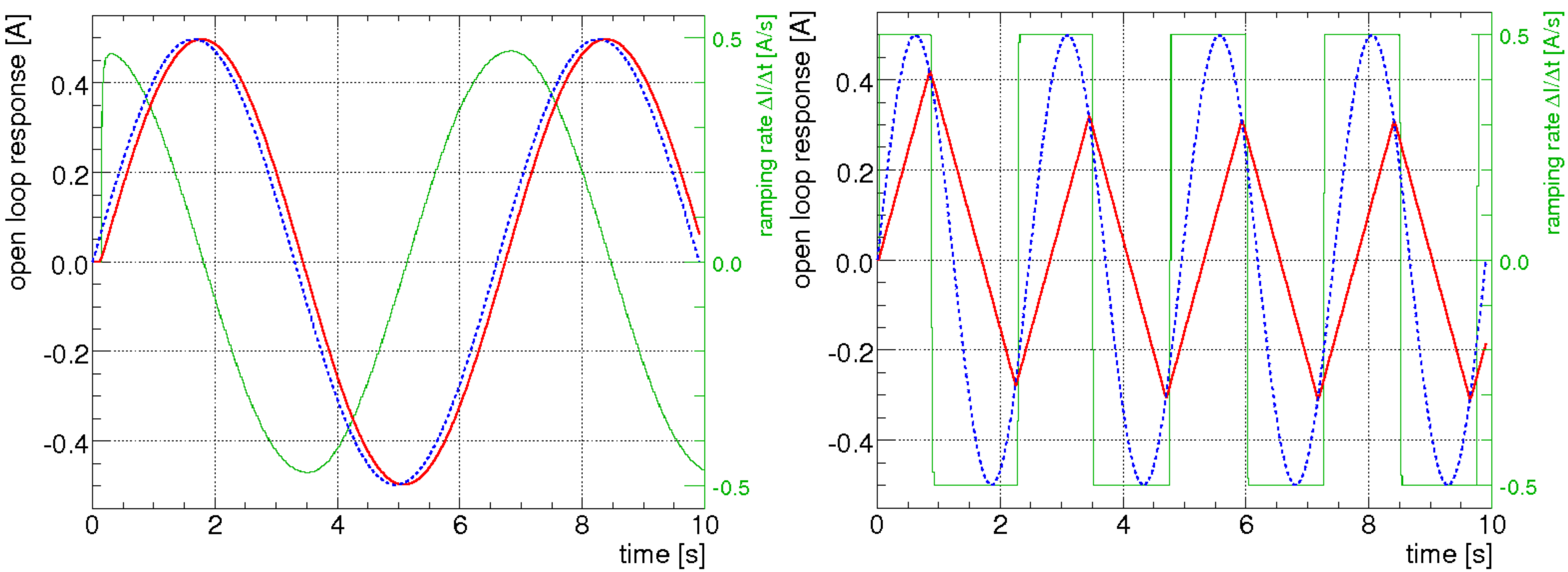
- α > τ…∞ moderates closed loop response between (trade-off):
  - fast and less accurate tracking vs. slow and more accurate tracking

[1] D. C. Youla et al., *"Modern Wiener-Hopf Design of Optimal Controllers"*, IEEE Trans. on Automatic Control,1976, vol. 21-1,pp. 3-13 & 319-338

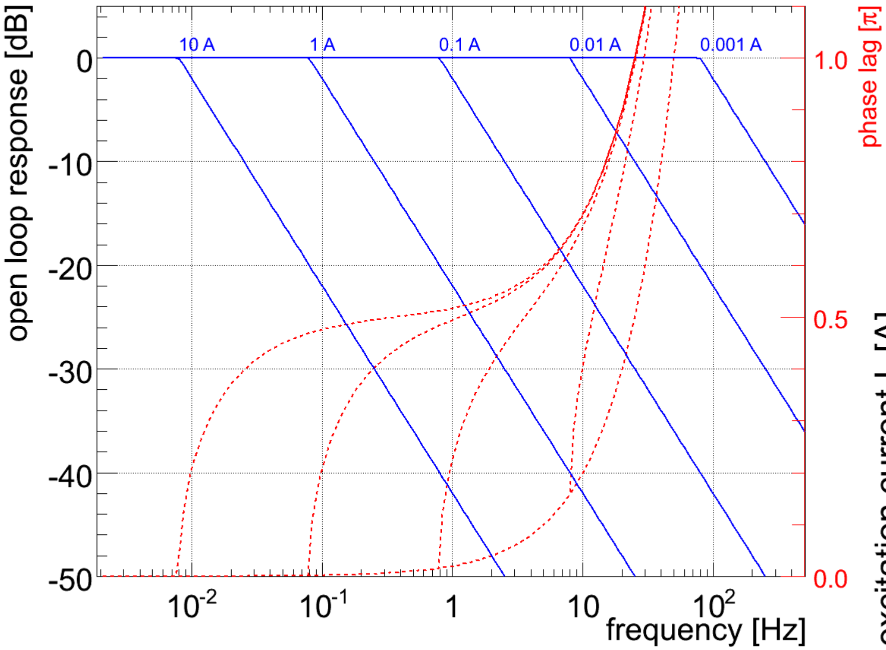
- α facilitates the trade-off between speed and robustness
  - operator/gain-scheduled has to deal with only one parameter



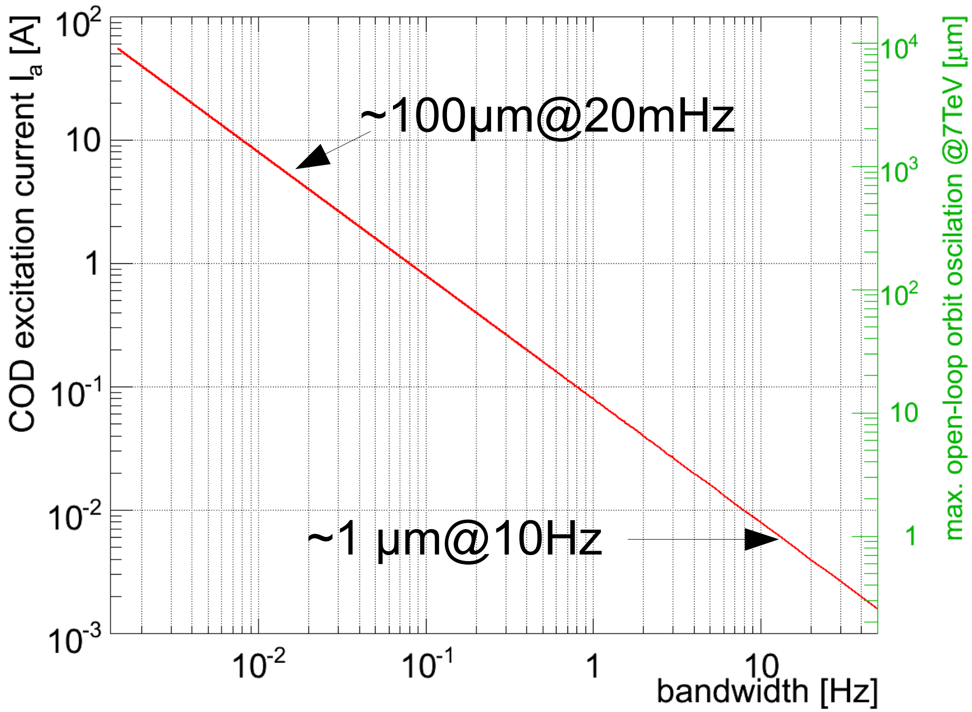
- Two main dynamic contributions
  - Delays: computation, data transmission, etc.
  - Slew rate of the corrector circuits (voltage limitation):
    - ±60A converter: $\Delta I/\Delta t|_{max} < 0.5$ A/s
    - ±600A converter: $\Delta I/\Delta t|_{max} < 10$ A/s



- The open-loop corrector circuit bandwidth depends on the excitation current:
  - non-linear phase once rate limiter is in action



$\Delta I = 0.1A \leftrightarrow \Delta x \approx 16\ \mu m @ \beta = 180m$



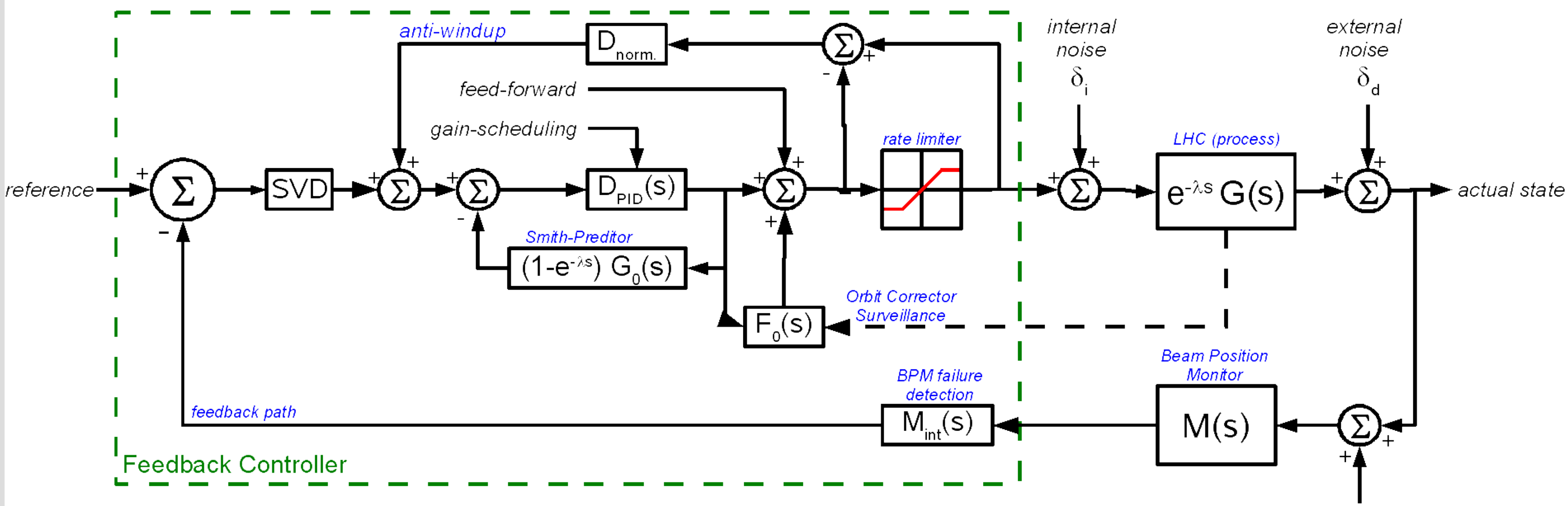
- Consider ~16μm@1Hz as effective bandwidth @ 7TeV
- Injection: ~15 times faster!

Including Non-Linearities in the Controller Design III/III

- If G(s) contains non-stable zeros e.g. delay λ & non-linearities $G_{NL}(s)$

$$G(s)=\frac{e^{-\lambda s}}{\tau s+1}\cdot G_{NL}(s)$$

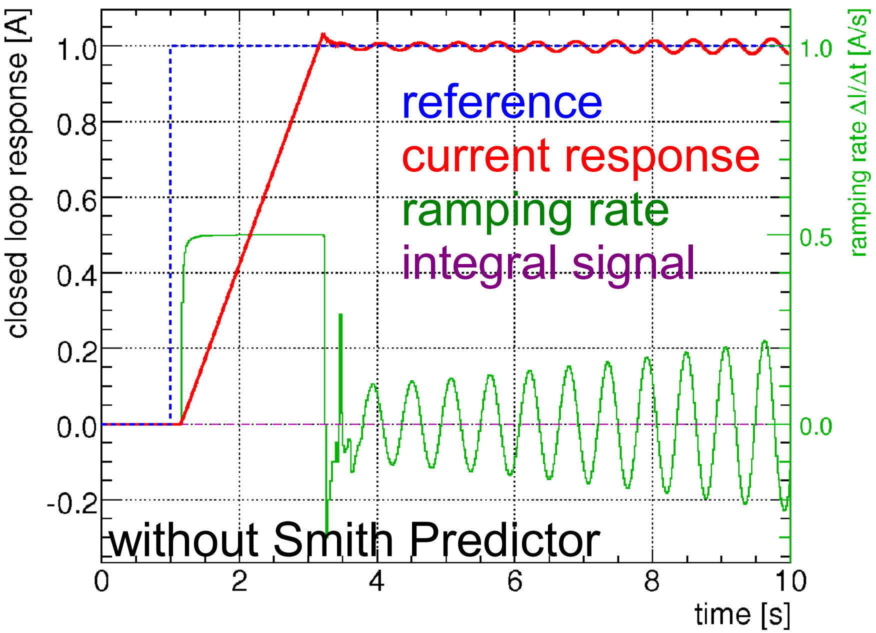
- with τ the power converter time constant, then: $G^i(s)=\frac{\tau s+1}{1}$
- Using (1) and (4) yields $T_0(s)=F_Q(s)\cdot e^{-\lambda s}G_{NL}(s)$
- Inserting in (1) effortlessly yields Smith-Predictor and Anti-Windup schemes:

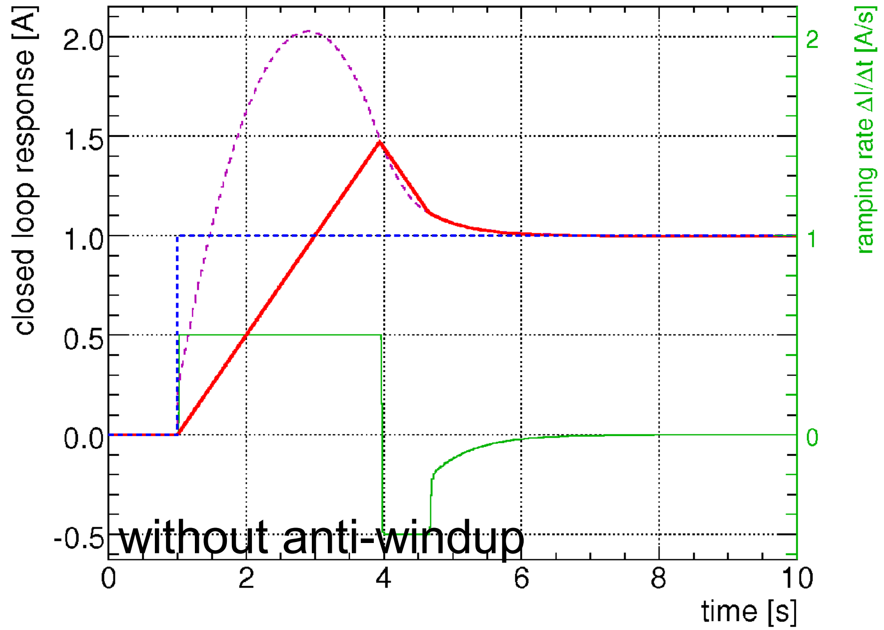


$D_{PID}(s)$ controller gains are independent of the compensator!!

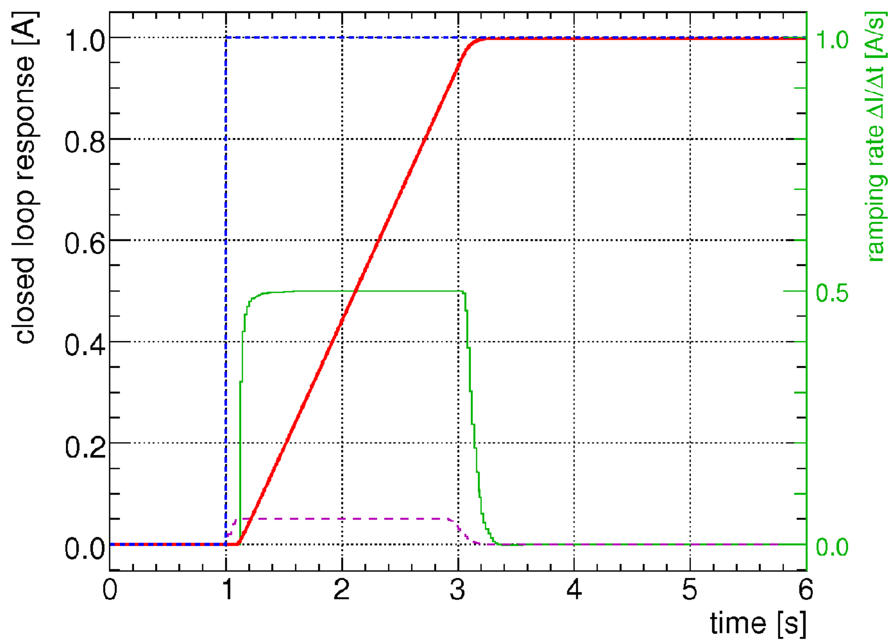
can be adjusted based on the operational scenario

Some Results: Smith-Predictor and Anti-Windup



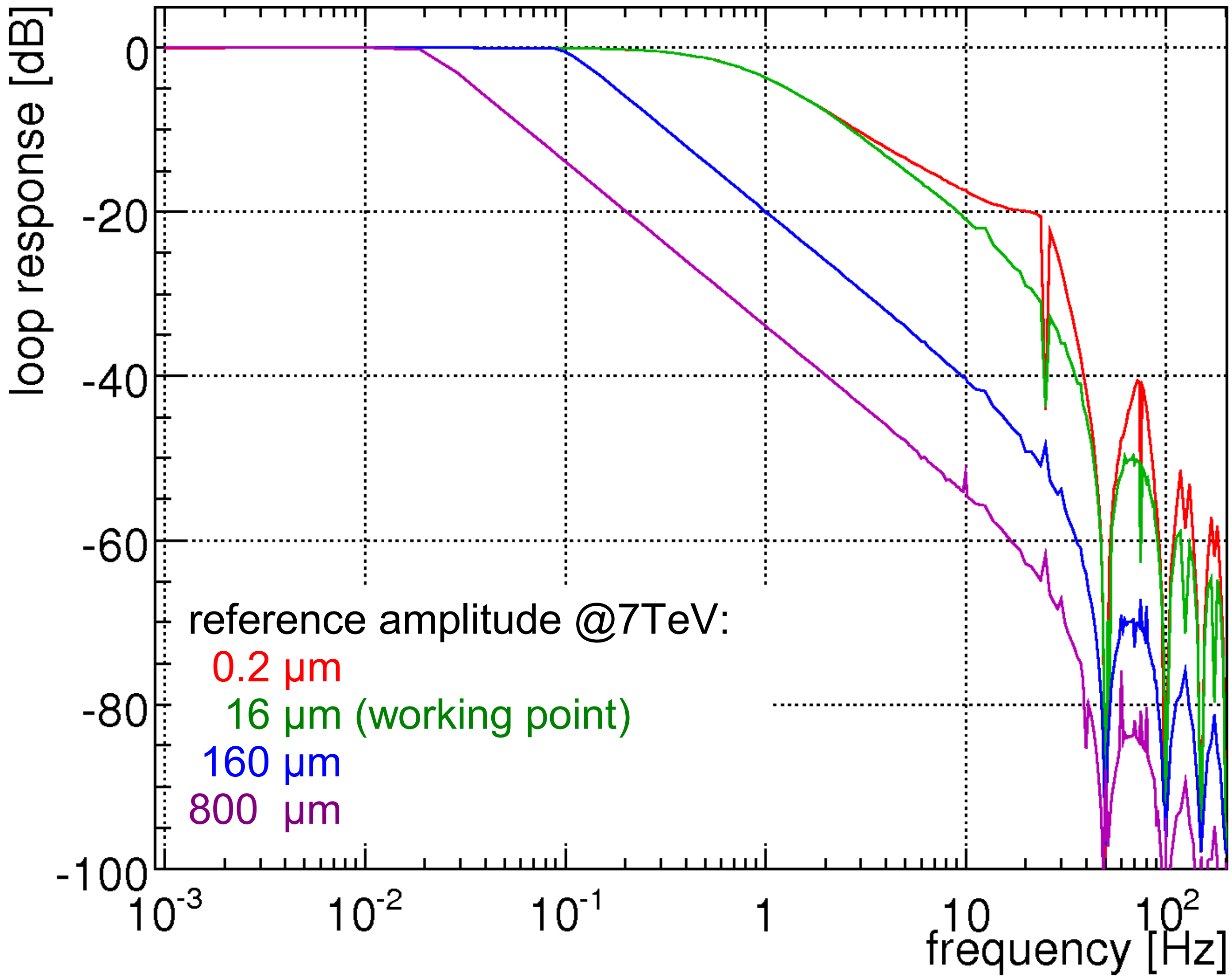


with full delay and windup compensator scheme:



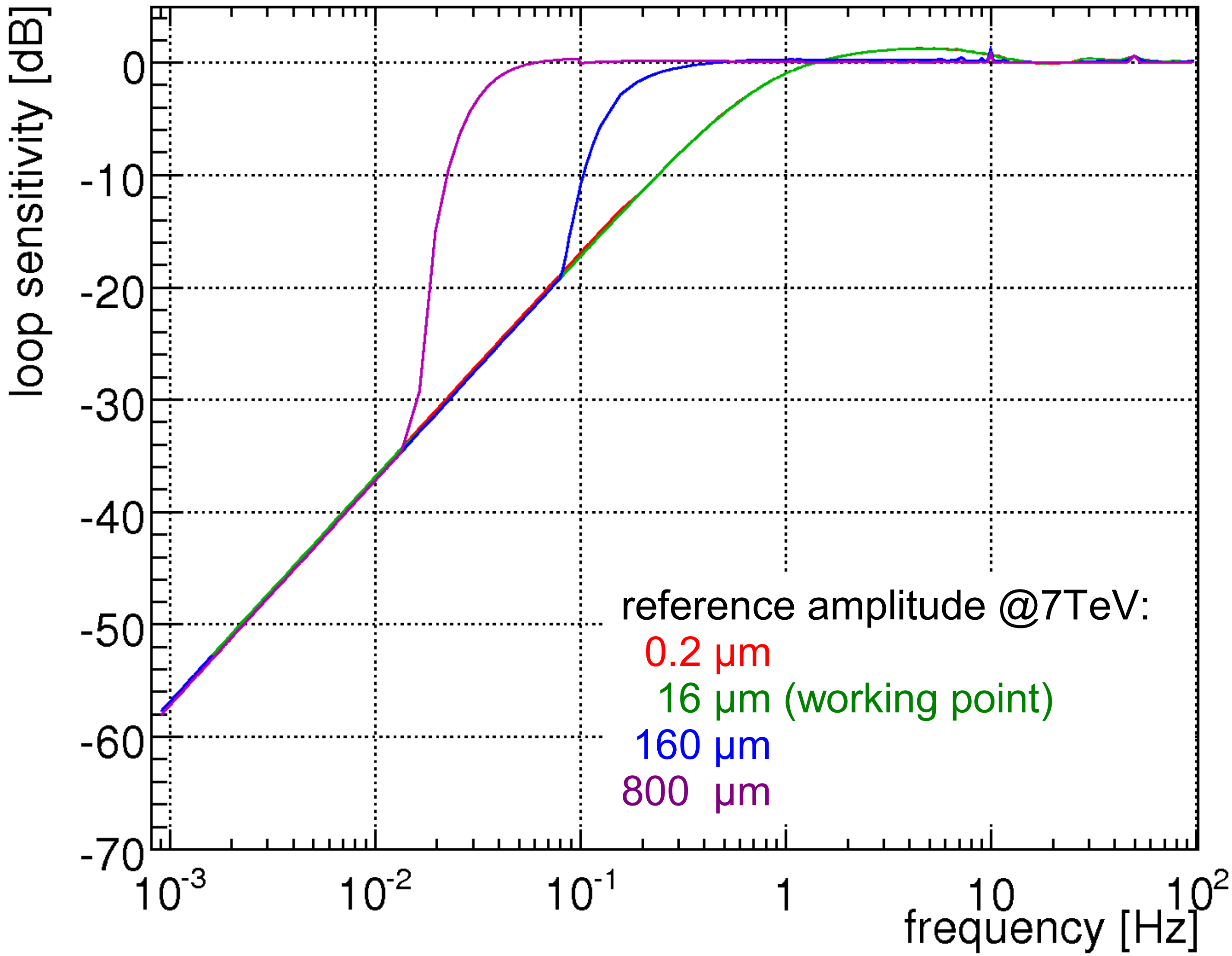
Nominal Feedback Response T_0

- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)



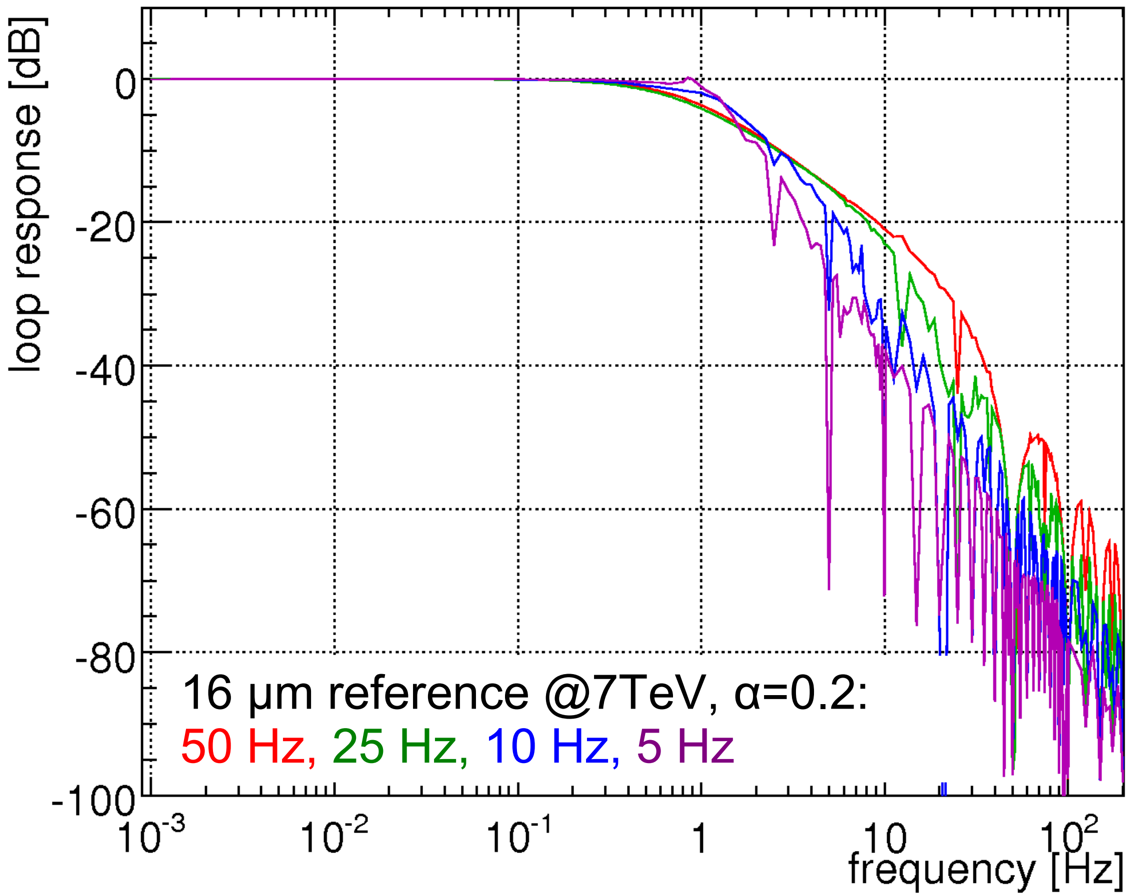
Nominal Feedback Disturbance Rejection S_{d0}

- Full LHC orbit simulation @1KHz sampling, (BPM sampling: 25Hz)



Loop Bandwidth versus Sampling frequency

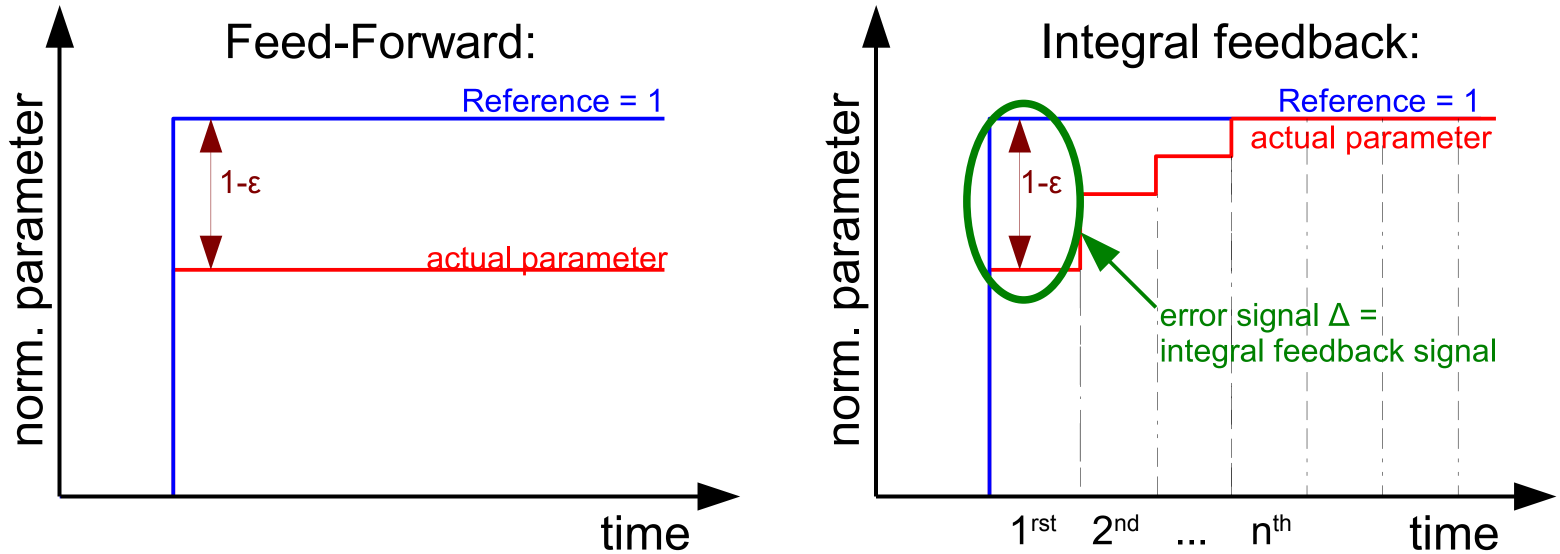
- ... sample the position (Q, ...) at 10Hz to achieve a closed loop 1Hz bandwidth



  - ... a theoretic limit assuming a perfect system!
  - common: sampling frequency > 25 ...40 desired closed-loop bandwidth

- Machine imperfections (beta-beat, hysteresis....), calibration errors and offsets can be translated into a steady-state ε_{ss} and scale error ε_{scale}:

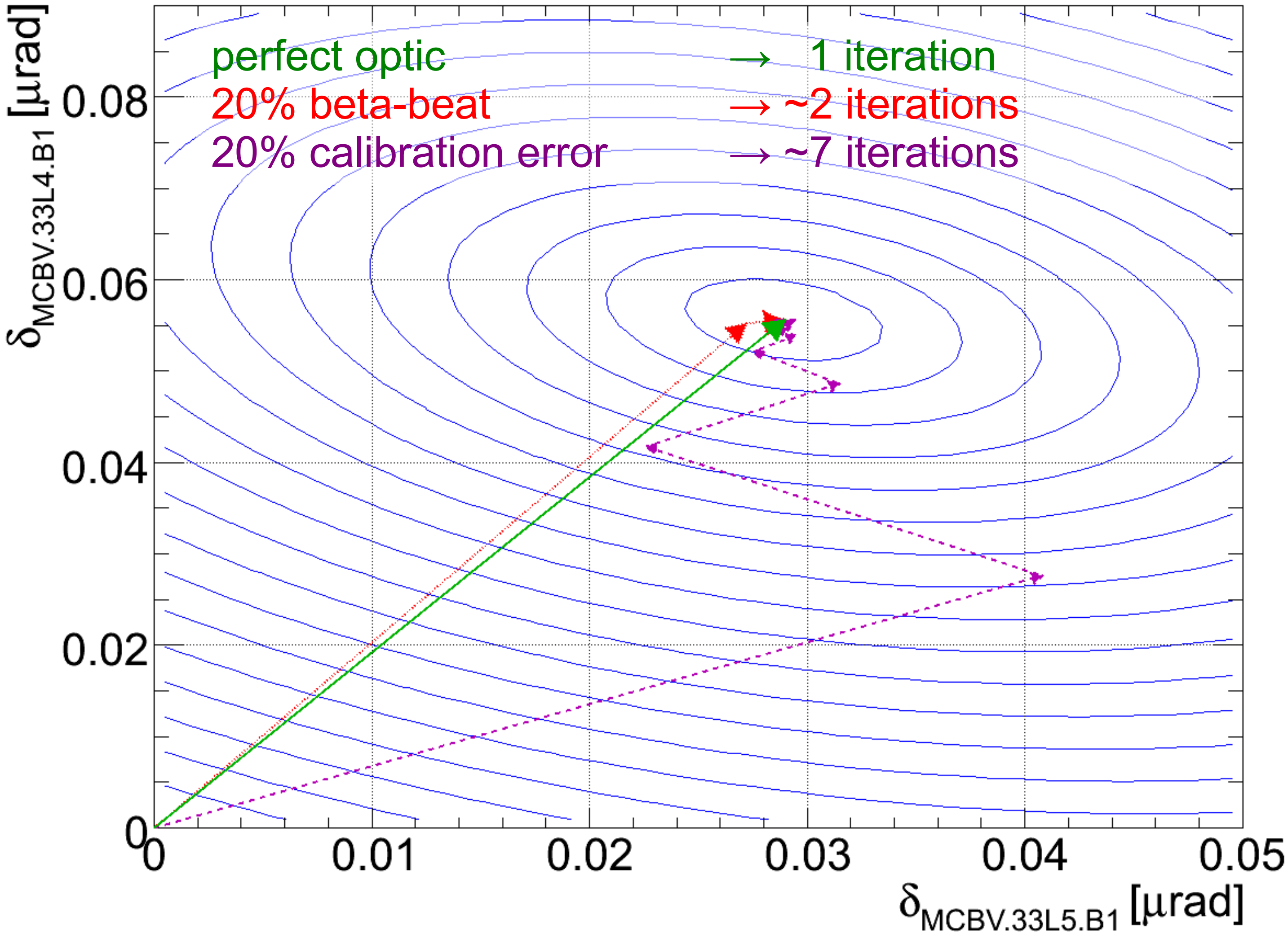
$$\Delta x(s)=R_i(s)\cdot\delta_i \rightarrow \Delta x(s)=R_i(s)\cdot(\epsilon_{ss}+(1+\epsilon_{scale})\cdot\delta_i)$$



- Uncertainties and scale error of beam response function affects rather the convergence speed (= feedback bandwidth) than achievable stability
- Stability limit: BPM noise and external perturbations w.r.t. FB bandwidth

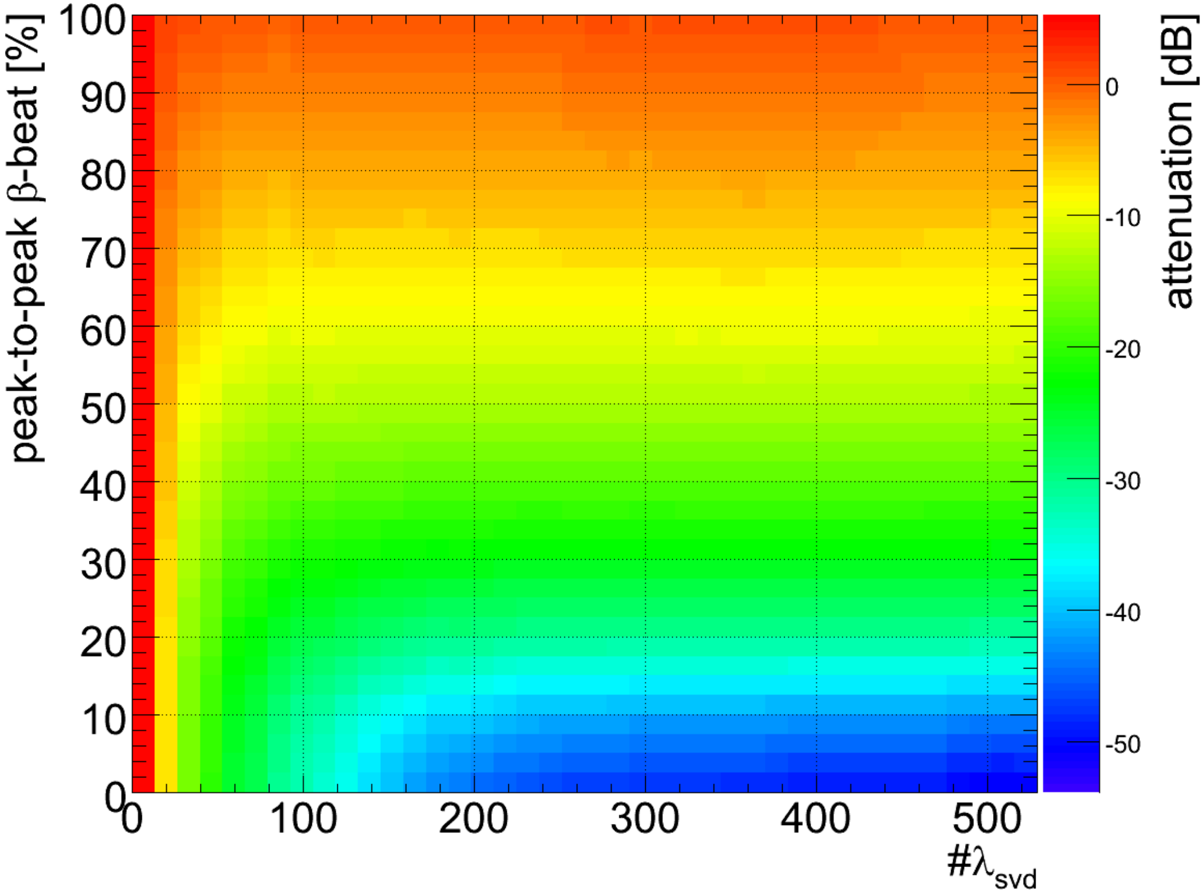
Optics and Calibration Uncertainties

- Imperfect optic and calibration error can deteriorate the convergence speed on the level of the SVD based correction:
- Example: 2-dim orbit error surface projection



Example: Sensitivity to beta-beat

- Low sensitivity to optics uncertainties = high disturbance rejection:
  - LHC simulation: Inj. Optics B1&B2 corrected



$\#\lambda_{svd}$ controls correction precision

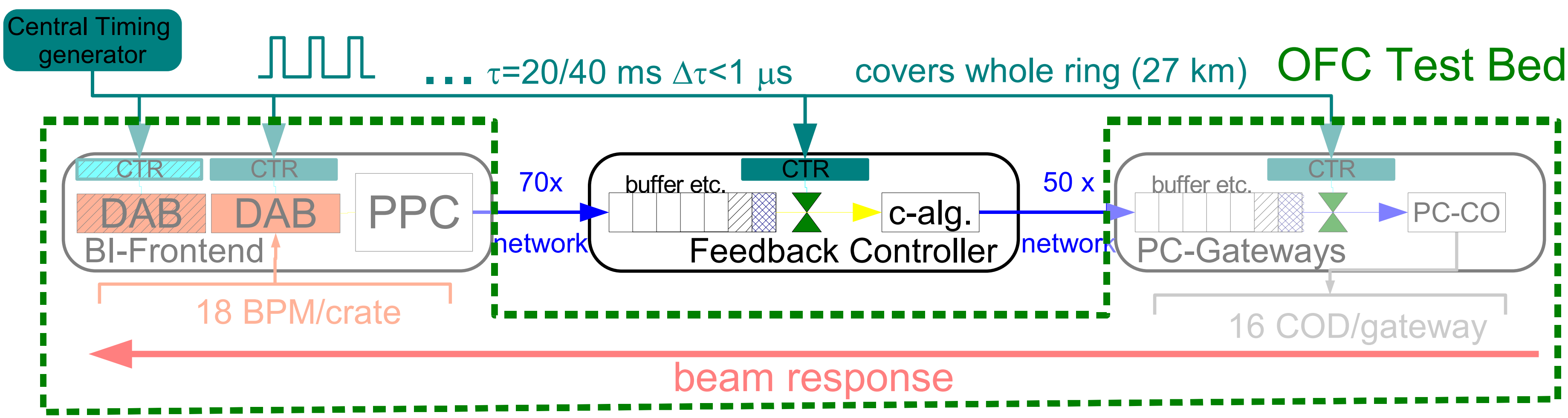
attenuation =

$$20\cdot\log\left|\frac{orbit\ r.m.s.\ after}{orbit\ r.m.s.\ before}\right|_{ref}$$

- Robust Control: OFB can cope with up to about 100% β-beat!
  - Robustness comes at a price of a (significantly) reduced bandwidth!

Commissioning the Orbit Feedback Controller – Test Bed

- Test bed complementary to Feedback Controllers:
  - Simulates the open loop and orbit response of COD→BEAM→BPM
    - Decay/Snap-back, ramp, squeeze, ground motion simulations, ...
    - Keeps/can test real-time constraints up to 1 kHz
  - Same data delivery mechanism and timing as the front-ends
    - transparent for the FB controller
    - same code for real and simulated machine:
      - possible and meaningful "offline" debugging for the FB controller



Commissioning of Feedbacks without Beam

- Most feedbacks checks can be and are done during hardware commissioning:
  - Interfaces and communication from BI and to PO front-ends
  - Synchronisation of BPM acquisition
    (using e.g. the BPM's 'calibration' mode)
  - Synchronisation of PO-Gateways
    (using the provided 50 Hz status feedback channel)
  - Interfaces to databases

- Using the 'test-bed' we can do the further tests without beam:
  - PID/Smith-Predictor/anti-windup at nominal/ultimate feedback frequency
  - Test automated countermeasures against failing BPMs or circuits
  - other parts of the feedback architecture:
    controls, non-beam-physics issues

Commissioning of Feedbacks with Beam

- Things that have to and can only be checked with beam:
  - Beam instrumentation: polarities, planes, mapping
  - Corrector circuits: polarities, planes, mapping (longitudinal and beam1/beam2)
  - Transfer function and rough test of calibrations
  - Circulating beam
  - Static coupling is under control

  } partially done while threading the first beam!

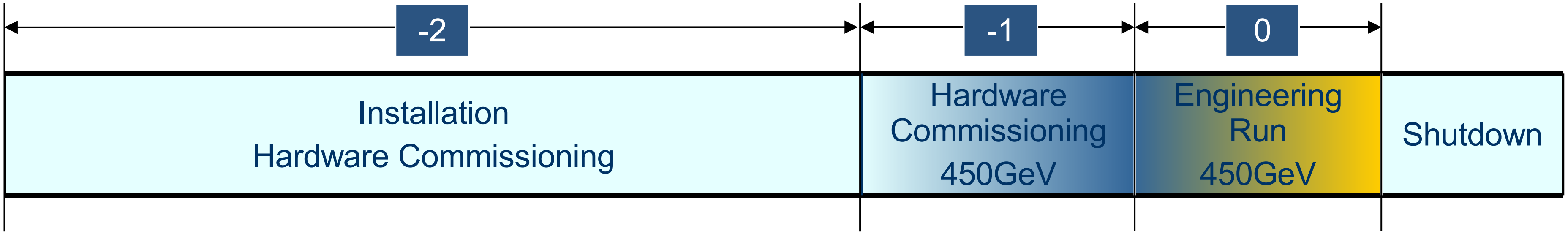
- It is possible to run feedbacks already after above procedures:
  - e.g. auto-triggered at 0.1 – 1 Hz
  - lower closed loop bandwidth (through parameter α)

Commissioning of Feedbacks: nominal performance

- Already after rough calibration of feedback controller/instruments/circuits:
  - BPM orbit resolution: pilot $\Delta x_{turn} \approx 200\ \mu m \rightarrow$ orbit: $\Delta x_{res} \approx 13\text{-}20\ \mu m$
    - Energy: $\Delta p/p_{res} \approx 10^{-6}$
  - Tune resolution (pilot): $\Delta Q_{res} \approx 10^{-3}...10^{-4}$
  - Chromaticity: $\Delta Q'_{res} \approx 10 \rightarrow \Delta Q'_{res} \approx 1$ (tough with nominal beam!)
    - have to prove the feasibility of the measurement
  - Actual stability depends on whether we (want to) steer to these limits

- Nominal feedback performance requires calibration of instrumentation/circuits well below the 20% level
  - one simple instrument → "easy" → required time: 14 s (best case), ~ one hours without automation
  - 1100++ simple instruments → "less easy"
  - requires fully automated procedures scripts (in development)
  - estimated time (if fully automated):
    - 4 hours without margin (pure excitation/measurement time)
    - 8-16 hours = 1-2 shifts including some operational margin

Commissioning of Transverse Feedback Sketch



- Phase “-2”
  - Software interfaces and mapping
  - low-level tests of acquisition electronics
  - addressing of corrector circuits
  - feedback loop logic tests

  } most of the tedious work can be done without beam

- Phase “-1”:
  - while threading the beam: rough polarity/mapping of BPMs and corrector circuits, followed by more detailed test of (omitted) circuits
  - Priority: Orbit/Energy → Tune/Coupling → Chromaticity (relevant only if ramping)
  - Should take advantage to commission all feedbacks at 450 GeV
- Phase 0: reaching “nominal” performance ...
  - refined lattice checks
  - instrumentation and circuit calibration below the 20% level

Conclusions

- Feedback architecture, strategies and algorithms are well established
  - The same feedback architecture for orbit, tune/coupling, chromaticity...
  - Orbit FB: stability better than about 200 µm should not pose a problem
  - Tune FB: $\Delta Q<0.003$ seems possible
  - Chromaticity FB: $\Delta Q_{res} \approx 10$ or even $\Delta Q_{res} \approx 1$
    - test of feasibility needed!

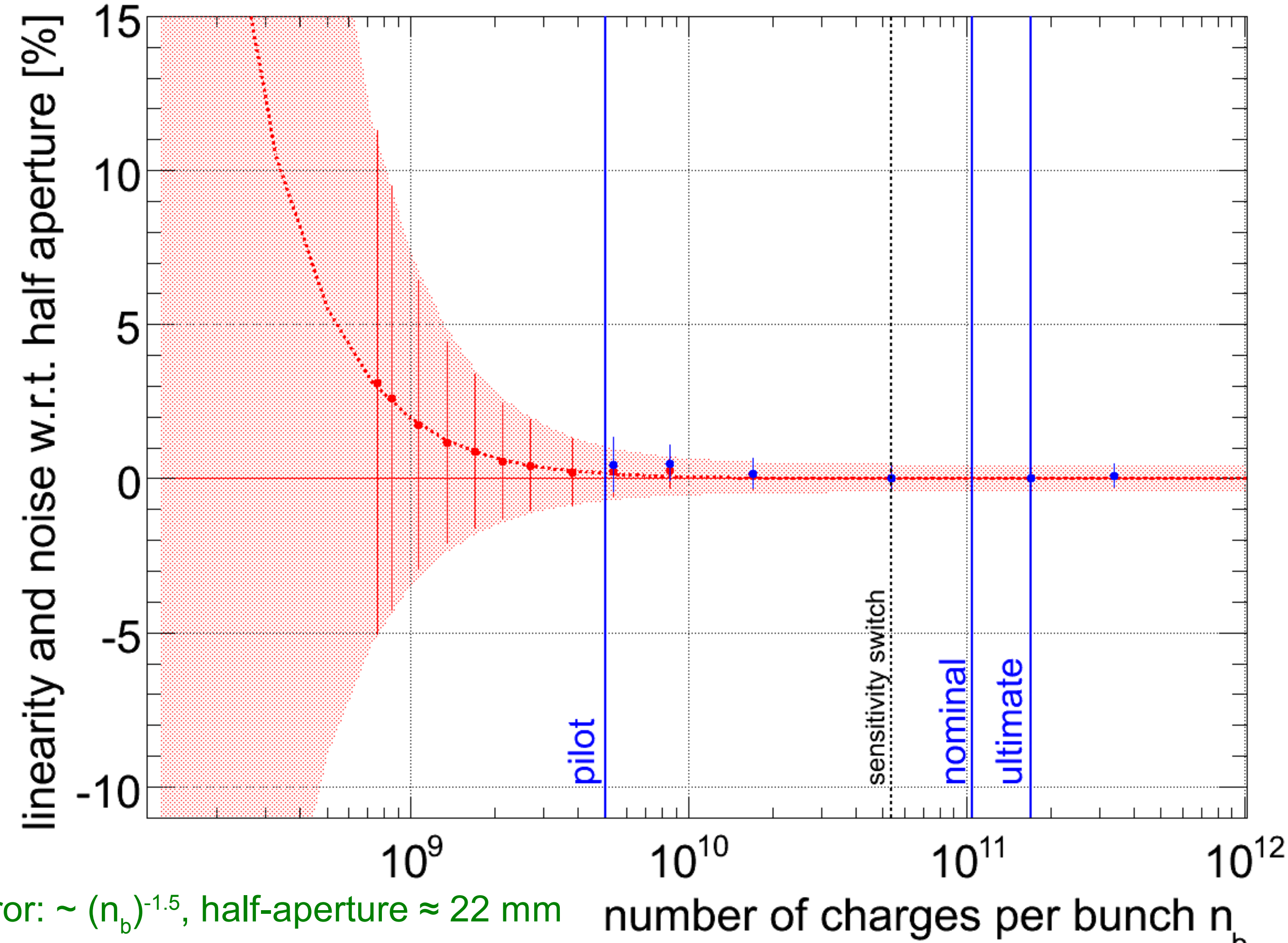
- Commissioning of feedbacks:
  - Most of the requirements for a minimum workable feedback systems are already fulfilled after threading and establishing circulating beam.
  - Redo the optics measurements and calibration with higher accuracies for nominal performance.

- Feedbacks are most useful when used at an early stage
  - Possibility to use feedback signals as feed-forward for next cycles

Reserve Slides

- 43x43 operation: max. intensity $4 \cdot 10^{10}$ protons/bunch

→ No gain-switching: BPMs will always operate at 'high' sensitivity



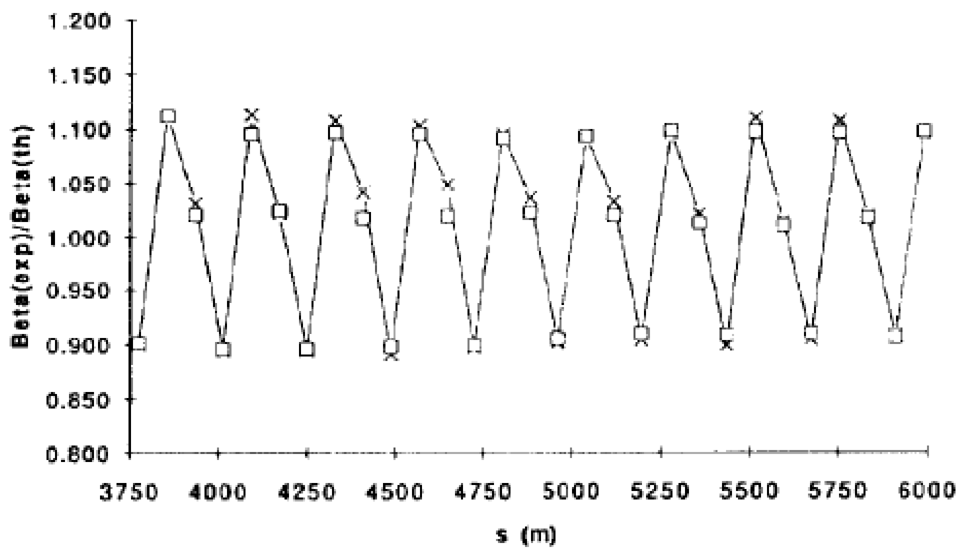
noise/error: ~ $(n_b)^{-1.5}$, half-aperture ≈ 22 mm
switch at: ~$5.3 \cdot 10^{-10}$ protons/bunch

Measurement of Response matrix

- Direct measurement of the orbit, tune, chromaticity, ... response matrix
  - perfect response matrix
  - no disentangling between beam measurement and lattice uncertainties
  - requires significant amount of time to excite/measure the response of each individual circuit: minimum of 15 s per COD circuit (1060!)
    - optics might change more often during commission

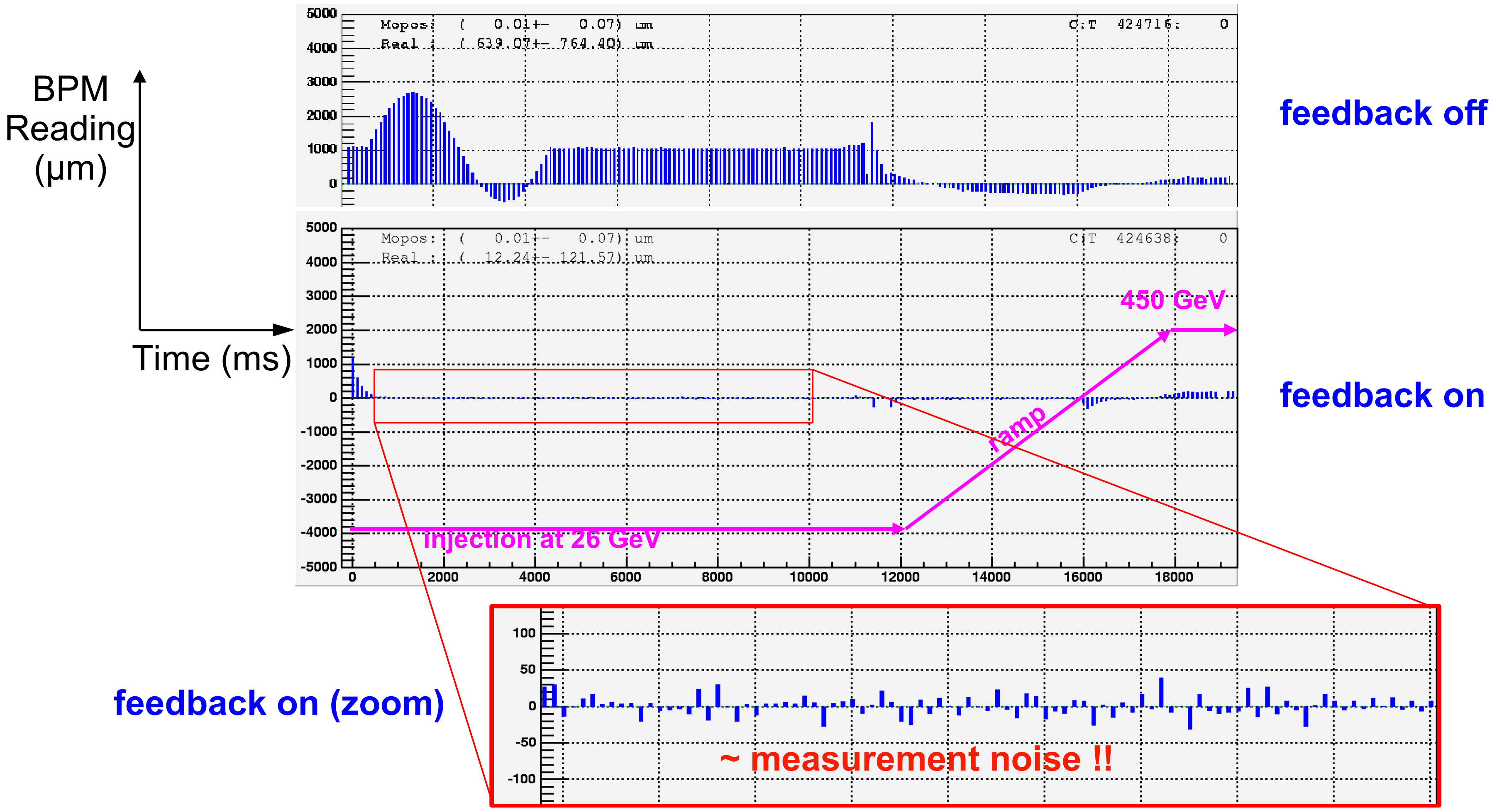
- Optics measurement through phase advance between three adjacent BPMs[1]
  – Design μ_{ij} versus measured (kick+1024 turns) ψ_{ii} phase advance:

$$\beta = \beta_0 \cdot \frac{\cot(\psi_{12}) - \cot(\psi_{13})}{\cot(\mu_{12}) - \cot(\mu_{13})} \longrightarrow$$



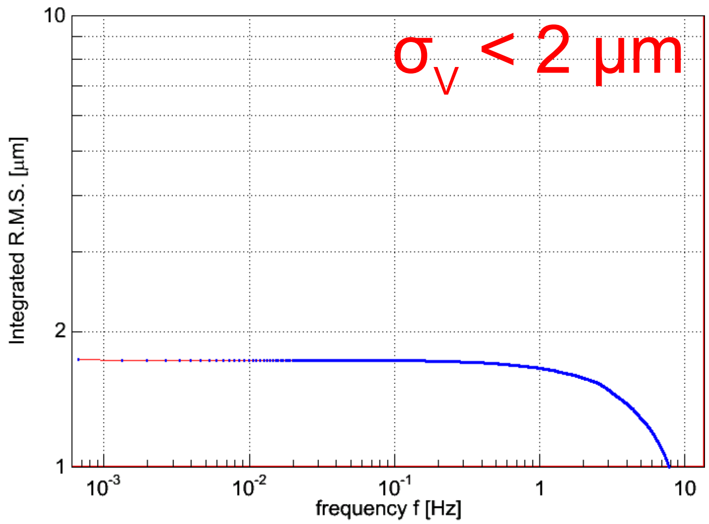
[1]P. Castro, "Betatron function measurements at LEP [..]", CERN, SL/Note 92-63-BI

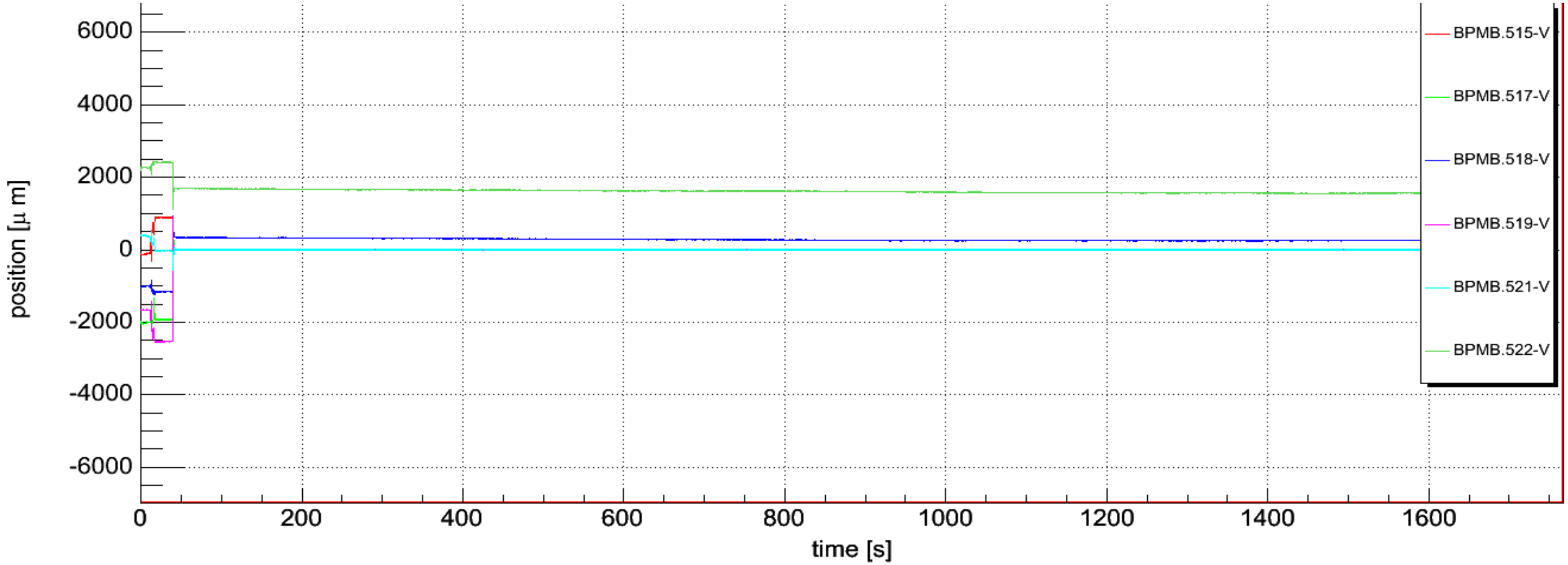
LHC Orbit Feedback Test at the SPS I/II



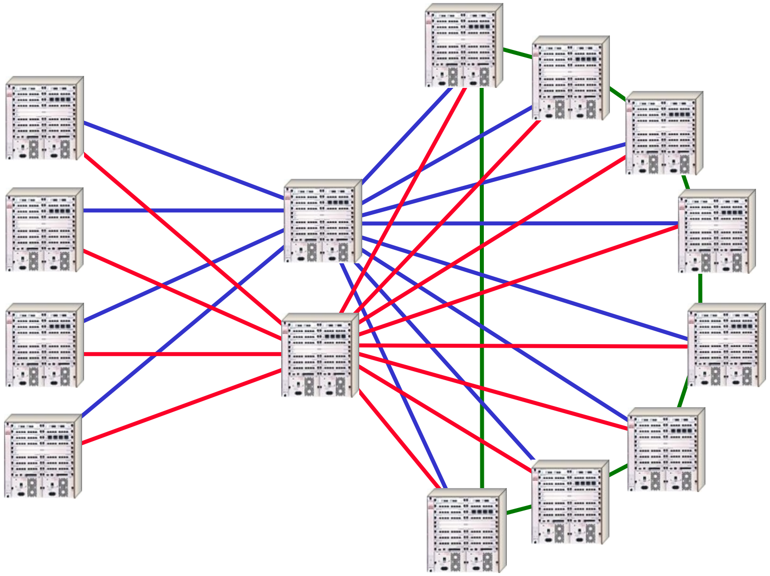
LHC Orbit Feedback Test at the SPS II/II

- Stabilisation “record” in the SPS
  - 270 GeV coasting (proton) beam, 72 nom bunches, $\beta_v \approx 100$ m
  - rivals most modern light sources
  - magnitudes better than required
- Target: maintain same longterm stability





Technical Network and Data Communication I/II

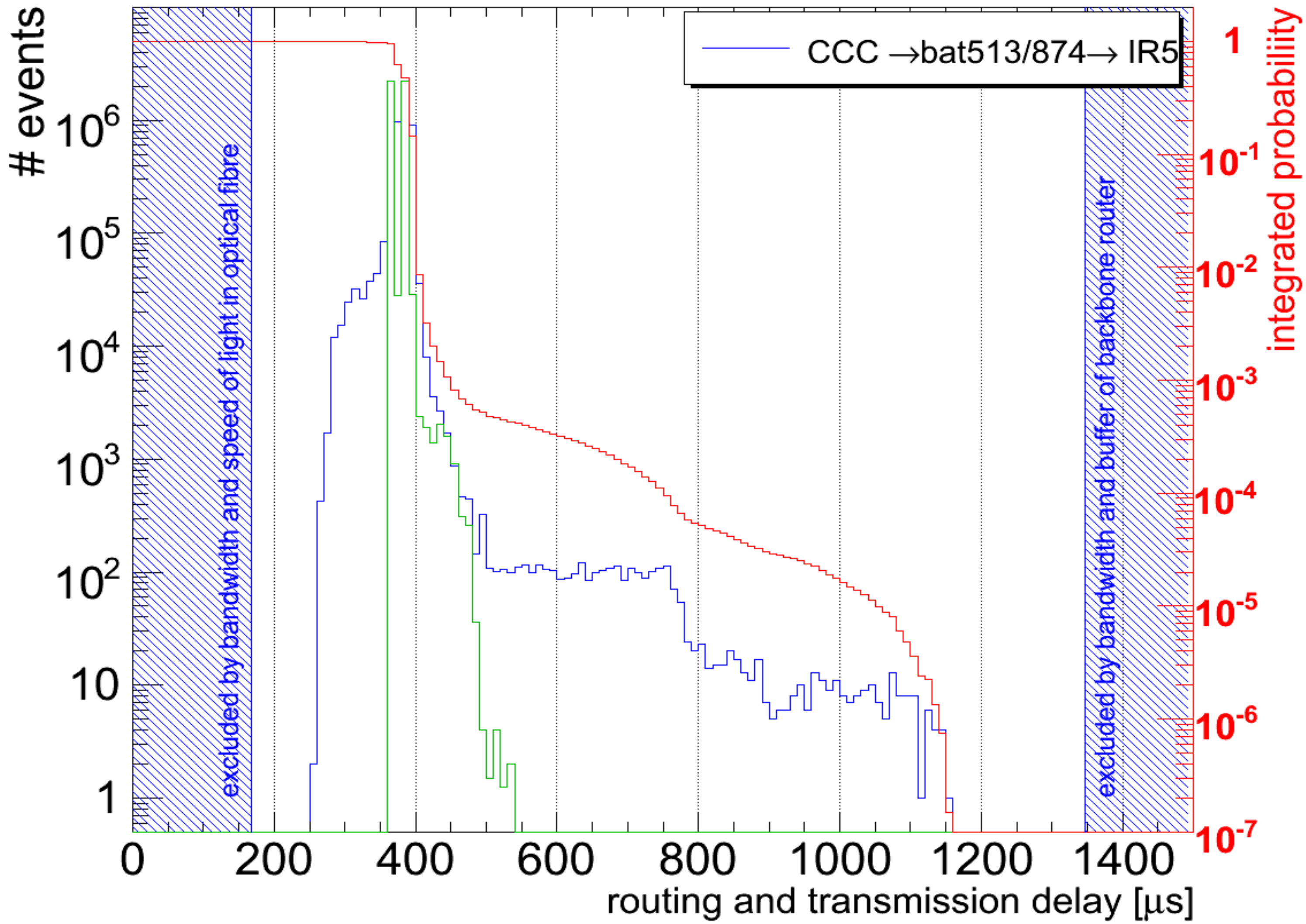


- CERN's Technical Network as backbone
  - Switched network
    - no data collisions
    - no data loss
  - double (triple) redundancy
- Core: “Enterasys X-Pedition 8600 Routers”
  - 32 Gbits/s non-blocking, $3 \cdot 10^7$ packets/s
  - 400 000 h MTBF
  - hardware QoS
    - One queue dedicated to real-time feedback
    - ~ private network for the orbit feedback
- Routing delay ~ 13 µs
- longest transmission delay (exp. verified) ~ 320 µs

  (500 bytes, IP5 -> Control room ~5 km)
  - 20% due to infrastructure (router/switches)
  - 80% due to traveling speed of light inside the optic fibre
- worst case max network jitter « targeted feedback frequency!

Technical Network and Data Communication II/II

- The maximum latency between CCC and IR5
  - tail of distribution is given by front-end computer and its operating system

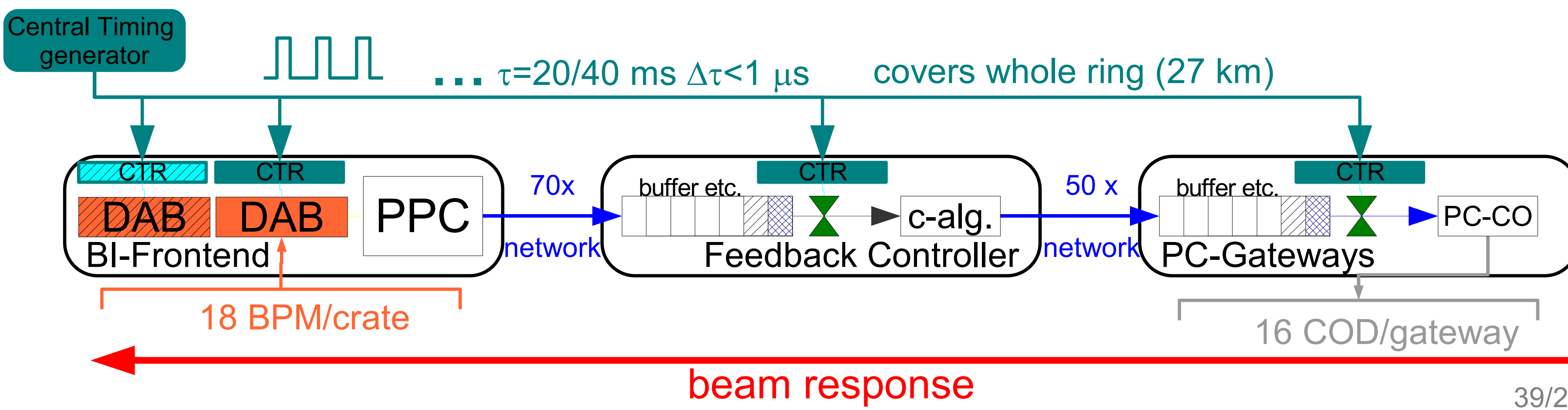


Remaining Jitter Compensation: Fix Max Loop Delay

Two main strategies:

- actual delay measurement and dynamic compensation in SP-branch:
  - high numerical complexity, due to continuously changing branch transfer function
  - only feasible for small systems

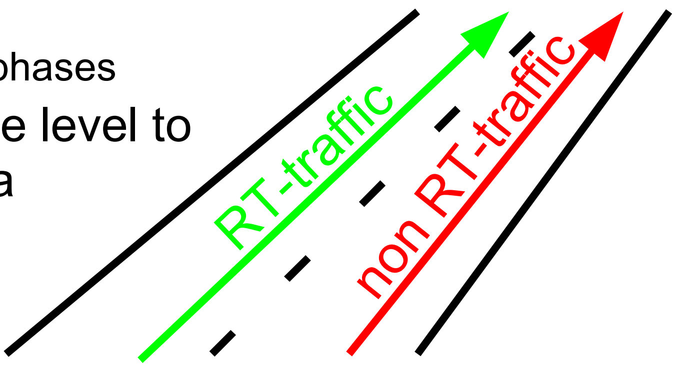
- Jitter compensation using a periodic external signal:
  - CERN wide synchronisation of events on sub ms scale that triggers:
    - Acquisition of BPM system, reading of receive buffers, processing and sending of data, time to apply in the power converter front-ends
  - The total jitter, the sum of all worst case delays, must stay within “budget”.
  - Measured and anticipated delays and their jitter are well below 20 ms.
  - feedback loop frequency of 50 Hz feasible for LHC, if required...



Bottleneck I: Network in the high-level front-ends!

- The front-end network interfaces are presently the bottleneck. e.g. feedback controller @ 50 Hz:
- lots of in-/outbound connections:
  - Two types of loads:
    - Real-Time: BPM and COD control data
      - Avg. bandwidth: ~13 Mbit/s
      - short bursts: full I/O load within few ms (100 MBit/s resp. 1GBit/s, burst duration desired to be short in order to minimise the total loop delay)
    - Non-Real-Time:
      - transfer of new settings to OFC (matrix ~30 MB)
      - PID configuration etc.
      - relay of BPM and feedback data (monitoring/logging)
      - ...

  - (Peak) load similar to high-end network servers
    - Nearly constant full load during certain operational phases
- network interface should be scheduled on the device level to provide a Quality of Service (QoS) for real-time data
  - One reserved FIFO queue for feedback data
  - General purpose queue for other data



Bottleneck I: Network in Front-Ends: Data Rates

Hardware:

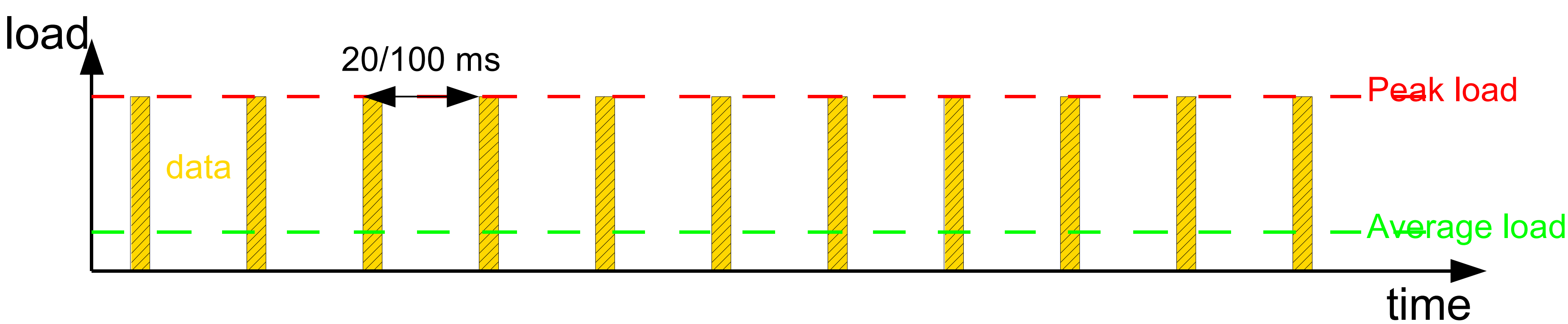
- both rings covered by 1056 BPMs
- Measure both planes (2112 readings)
- Organised in front-end crates (PowerPC/VME) in surface buildings
  - 18 BPMs (hor & vert) ⇔ 36 positions / VME crate
  - 68 crates in total, 6-8 crates /IR

Data streams:

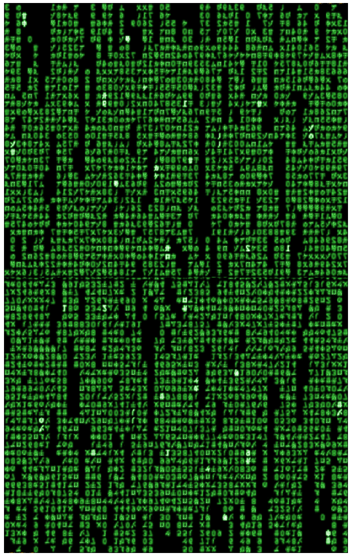
- Average data rates per IR:

– 18 BPMs x 20 bytes+overhead	~1500	bytes	/ sample / crate
– 1056 BPMs x 20 byte	~ 94	kbytes	/ sample
@ 10 Hz:	~ 7.7	Mbit/s	
@ 50 Hz:	~ 38.4	Mbit/s	

- Peak data rates (bursts): 100Mbit/s resp. 1Gbit/s (depending on Ethernet interface)



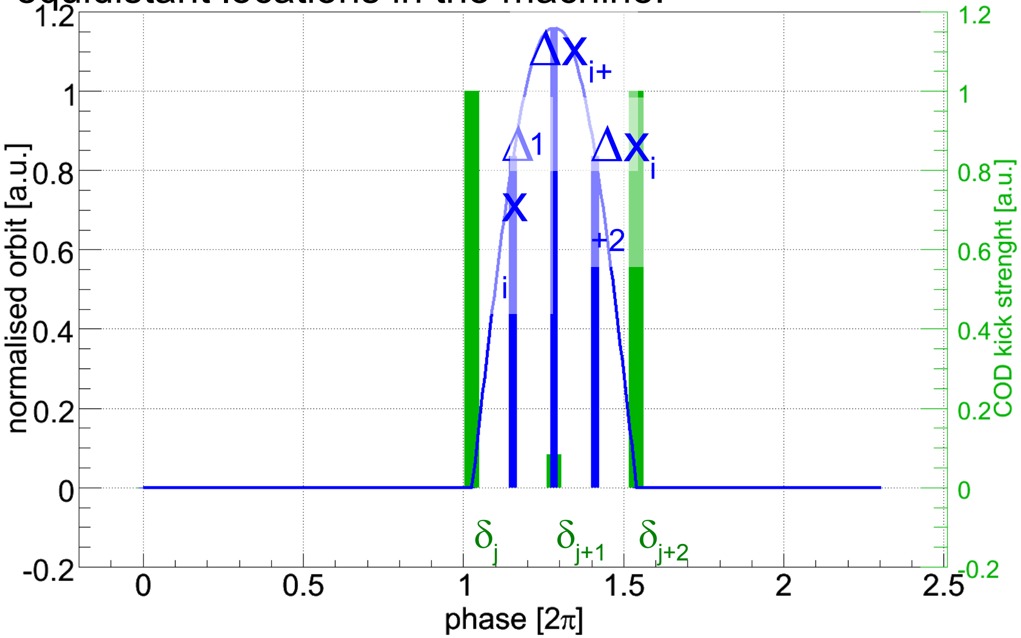
Bottleneck II: Orbit Feedback Controller!



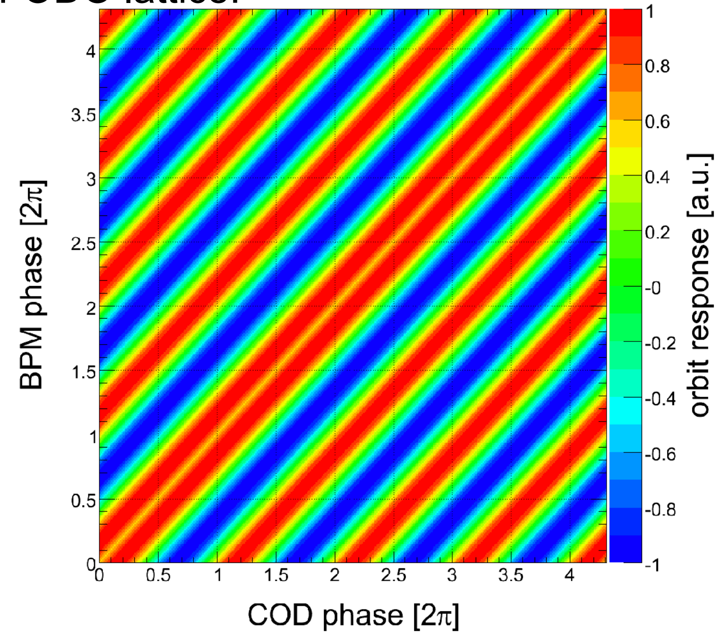
- Controller: must handle large matrices (~30 MB)
  - core of orbit correction:
    - multiplication of inverse orbit response matrix with input position vector: $\sim 4 \cdot 10^6$ *double* multiplications per sample @50Hz: ~ 400 MFLOPS
    - 1.5 GByte/s local memory data transfer
    - several ms processing time on a high-end SMP system
  - Requirements as for high-end web, file or database servers:
    - high performance & high reliability, but:
    - hard real-time constraints:
      total execution time has to be deterministic and less than 20/40 ms to fit the 25/50 Hz feedback frequency requirement
- present test solution:
  - x86 based SMP server: (HP Proliant 380 DL, 2.8GHz Xeon SMP, 3 GByte RAM)
  - 2 x Gigabit Ethernet connection (one dedicated card to service unit)
  - hardware redundancy (2 power supplies, 2 disks, hw monitor, watchdog, remote ...)
  - Processing duration per feedback cycle: ~12 ms

Automated Orbit Correction using Singular Value Decomposition

The orbit is sampled at *m* discrete not necessarily equidistant locations in the machine:



orbit response matrix example of a regular FODO lattice:



The superimposed beam position shift at the i[th] monitor due to single dipole kicks is described through the orbit response matrix R. It can be written as

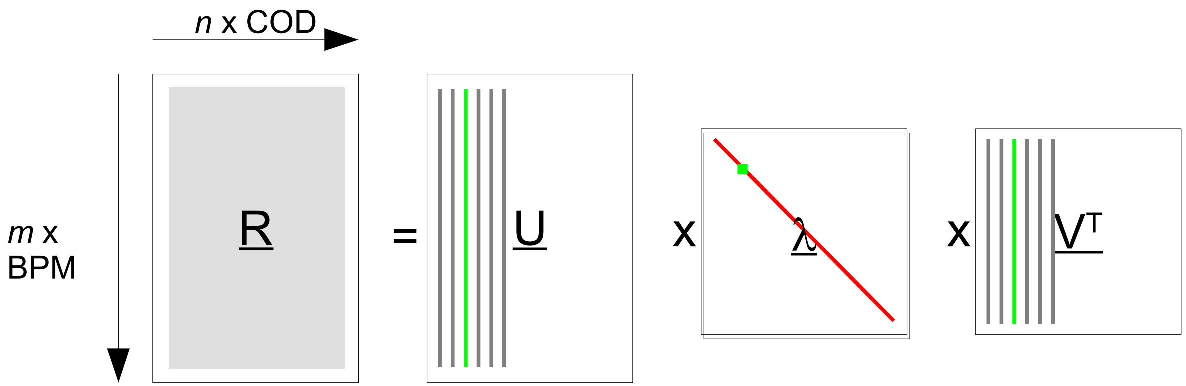
$$\Delta x_i = \sum^{n} R_{ij} \cdot \delta_j \quad with \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cdot \cos(\Delta \mu_{ij} - \pi Q)$$

$$\Leftrightarrow \quad \Delta \vec{x} = \sum_{j=0}^{n} \delta_j \vec{u}_j \quad with \quad \vec{u}_j = (R_{1j}, \ldots, R_{mj})^T \Leftrightarrow \boxed{\Delta \vec{x}(t) = \underline{R} \cdot \vec{\delta}_{ss}}$$

where (β,μ,Q) depends on the machine optic (example: Q=4.31).

Singular Value Decomposition (SVD) on a slide

Theorem from linear algebra*:



eigen-vector relation:

$$\lambda_i \vec{u}_i = \underline{R} \cdot \vec{v}_i$$

$$\lambda_i \vec{v}_i = \underline{R}^T \cdot \vec{u}_i$$

- final correction is a simple matrix multiplication
- large eigenvalues ↔ bumps with small COD strengths but large effect on orbit

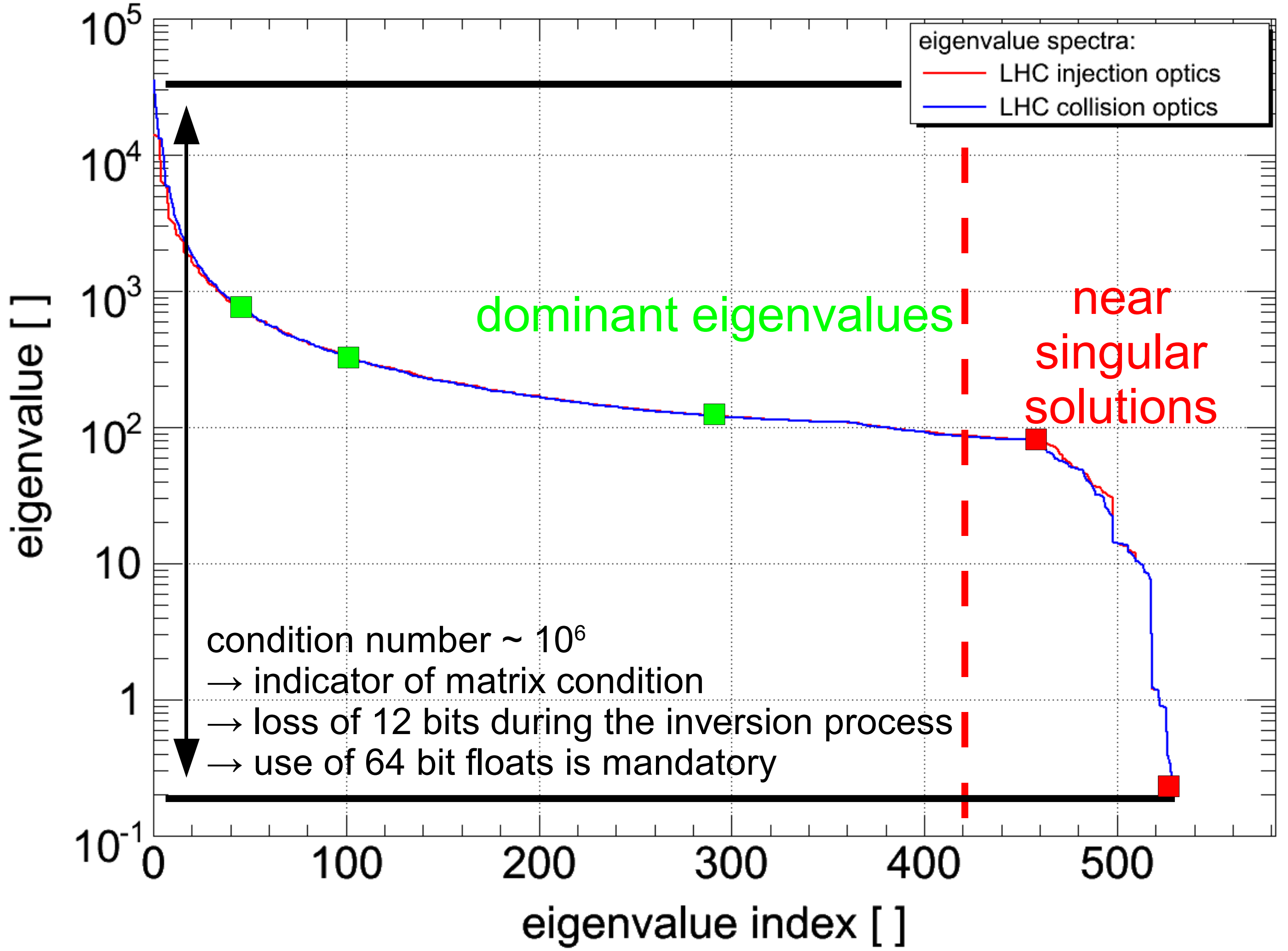
$$\boxed{\vec{\delta}_{ss} = \tilde{R}^{-1} \cdot \Delta \vec{x} \quad \text{with} \quad \tilde{R}^{-1} = \underline{V} \cdot \underline{\lambda}^{-1} \cdot \underline{U}^T} \quad \Leftrightarrow \quad \vec{\delta}_{ss} = \sum_{i=0}^{n} \frac{a_i}{\lambda_i} \vec{v}_i \quad \text{with} \quad a_i = \vec{u}_i^T \Delta \vec{x}$$

- Easy removal of singular (=undesired, large corrector strengths) eigen-values/solutions:
  - near singular eigen-solutions have $\lambda_i \sim 0$ or $\lambda_i = 0$
  - to remove those solution: $\lim_{\lambda_i \to \infty} 1/\lambda_i = 0$
- discarded eigenvalues corresponds to bumps that won't be corrected by the fb

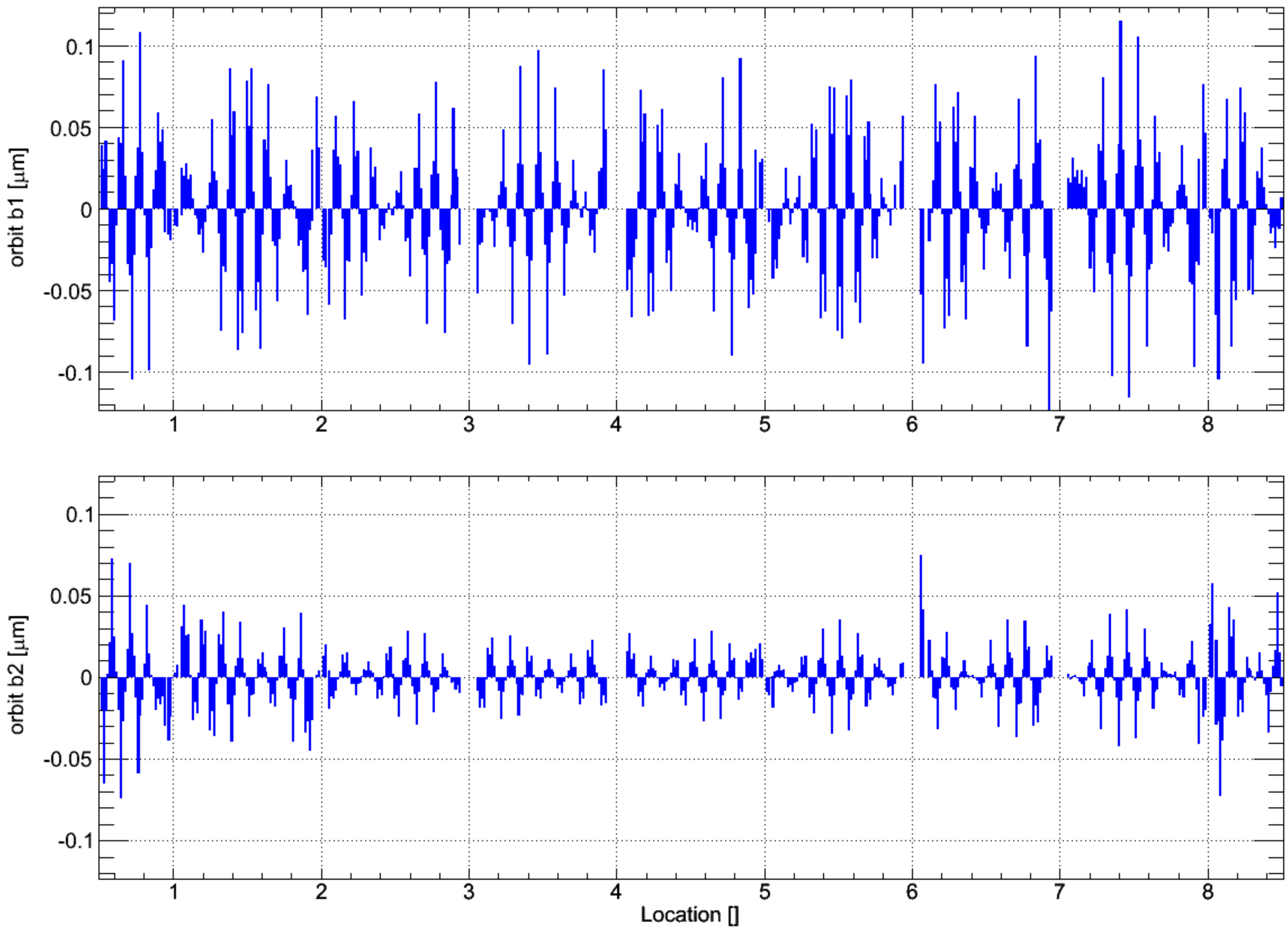
*G. Golub and C. Reinsch, “*Handbook for automatic computation II, Linear Algebra*”, Springer, NY, 1971

SVD example: LHC eigenvalue spectrum

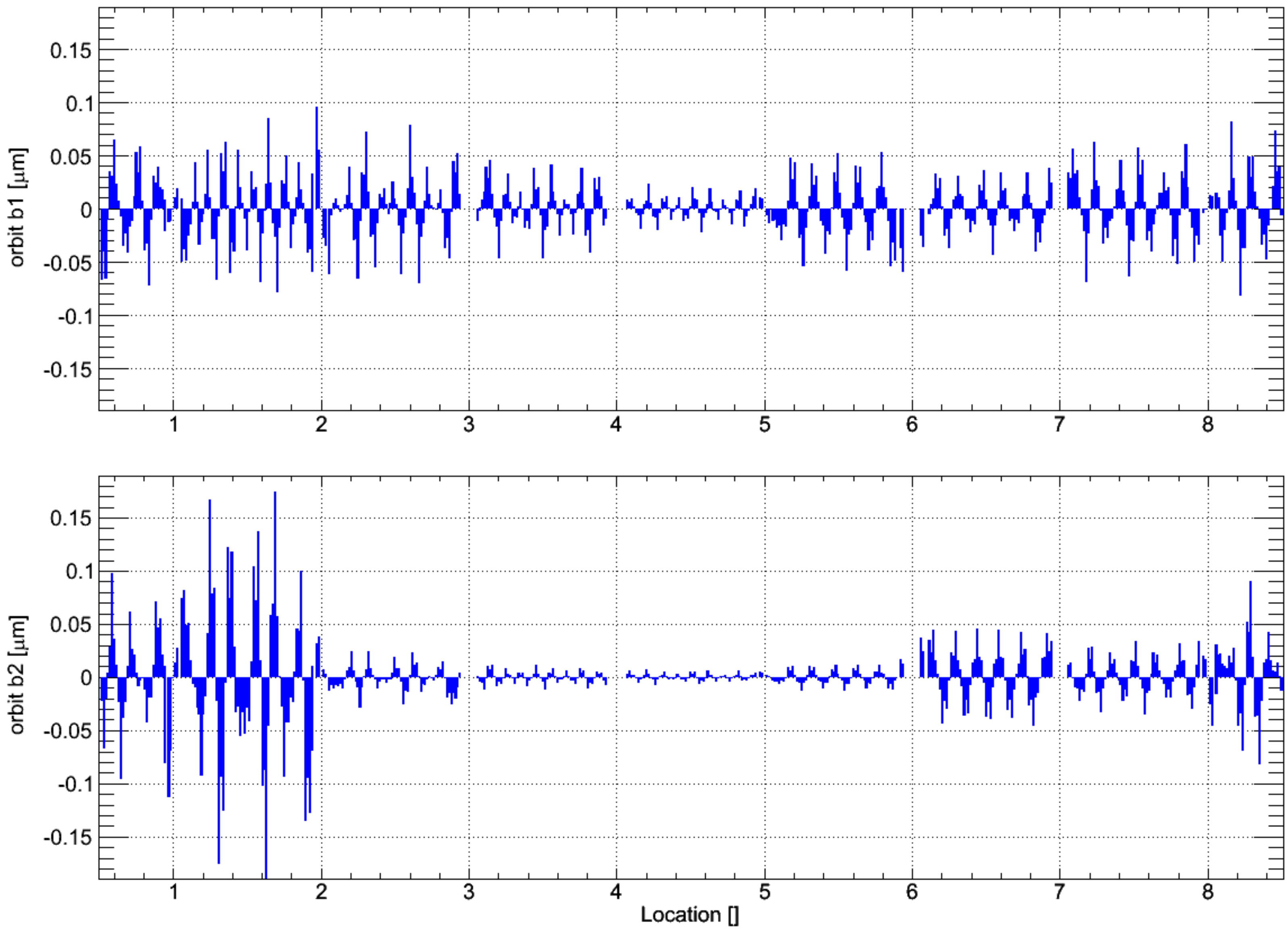
Eigenvalue spectra for vertical LHC response matrix using all BPMs and CODs:



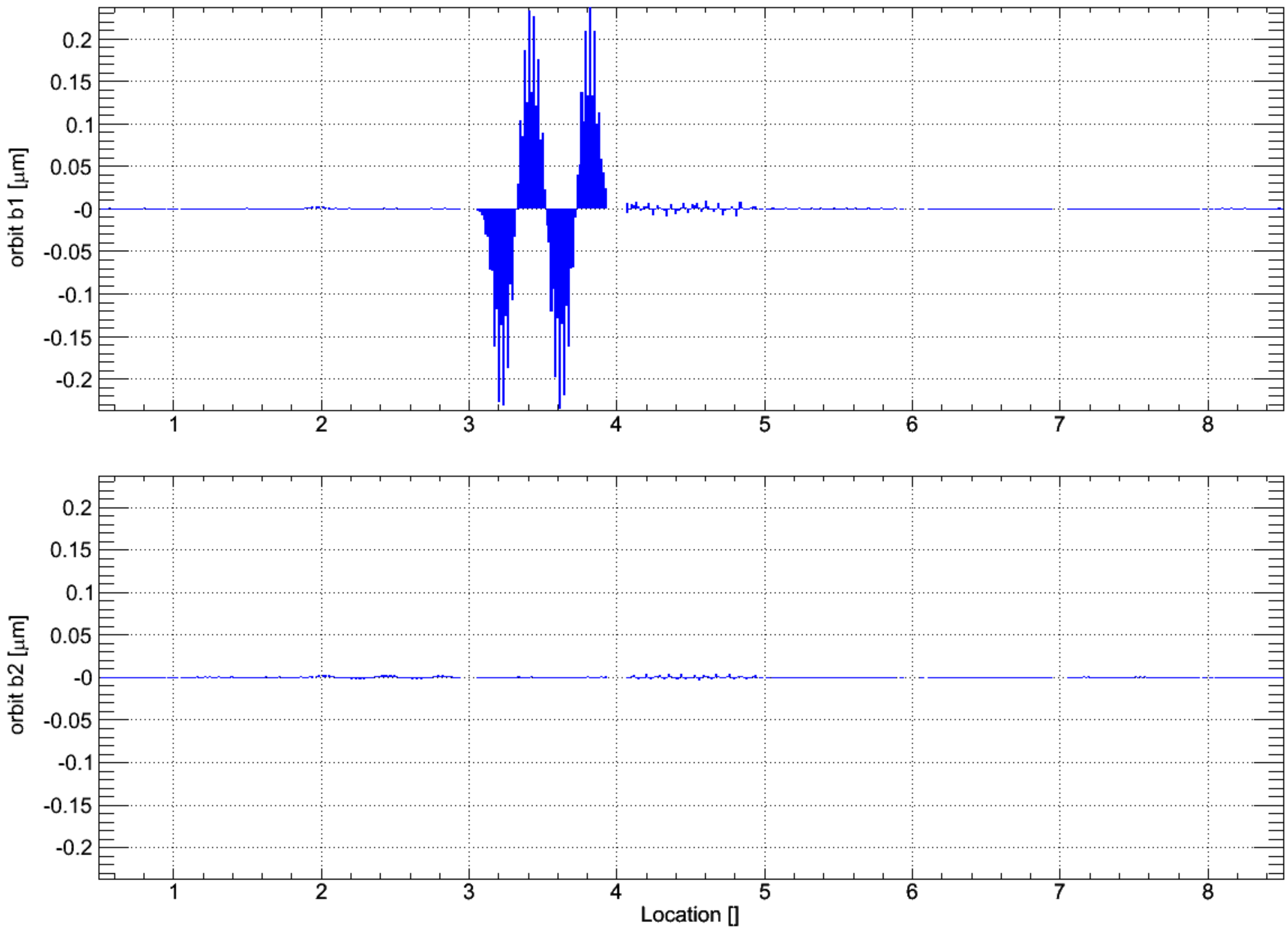
LHC example: BPM eigenvector #50 $\lambda_{50} = 6.69 \cdot 10^2$



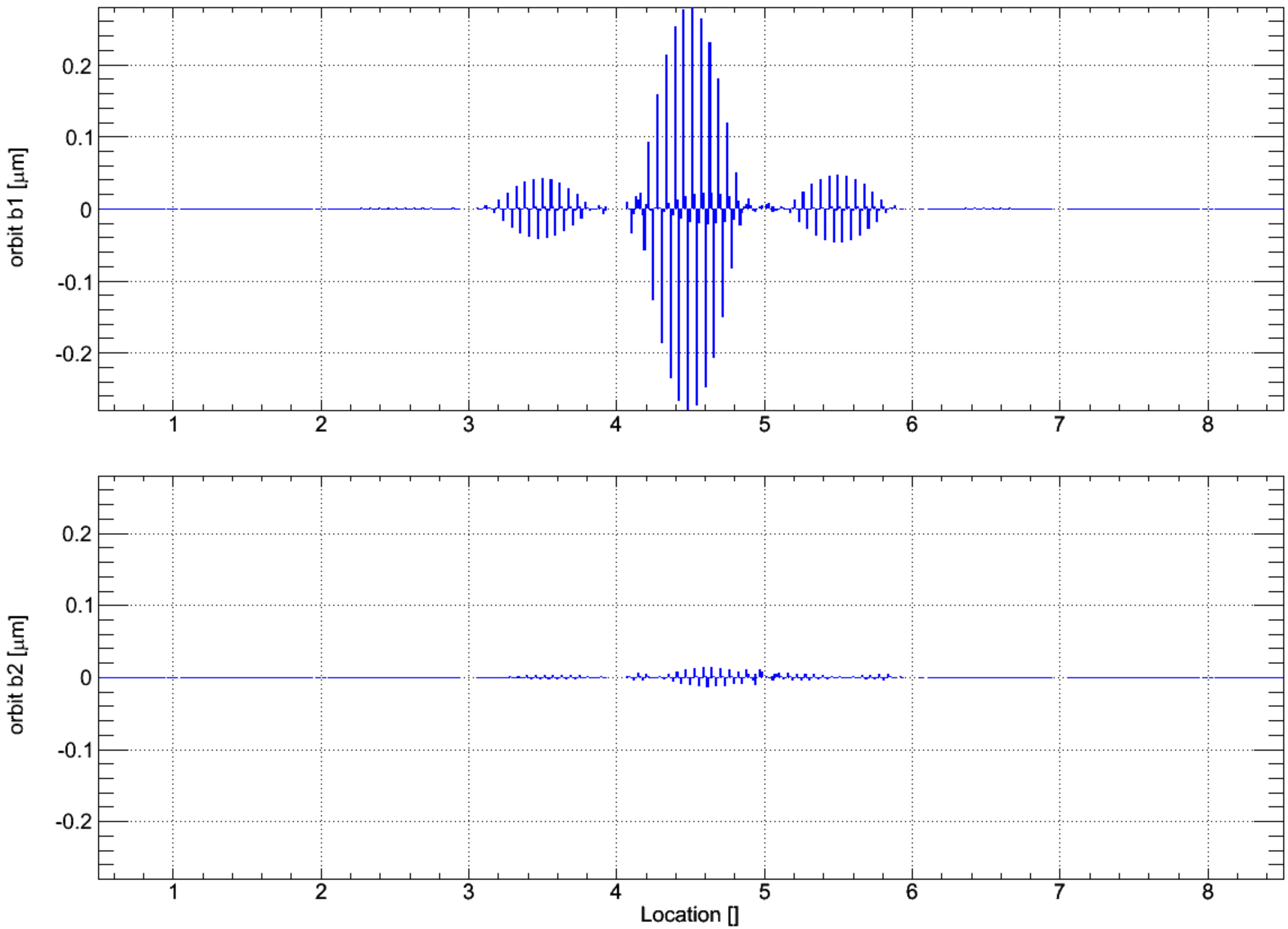
LHC example: BPM eigenvector #100 $\lambda_{100} = 3.38 \cdot 10^2$



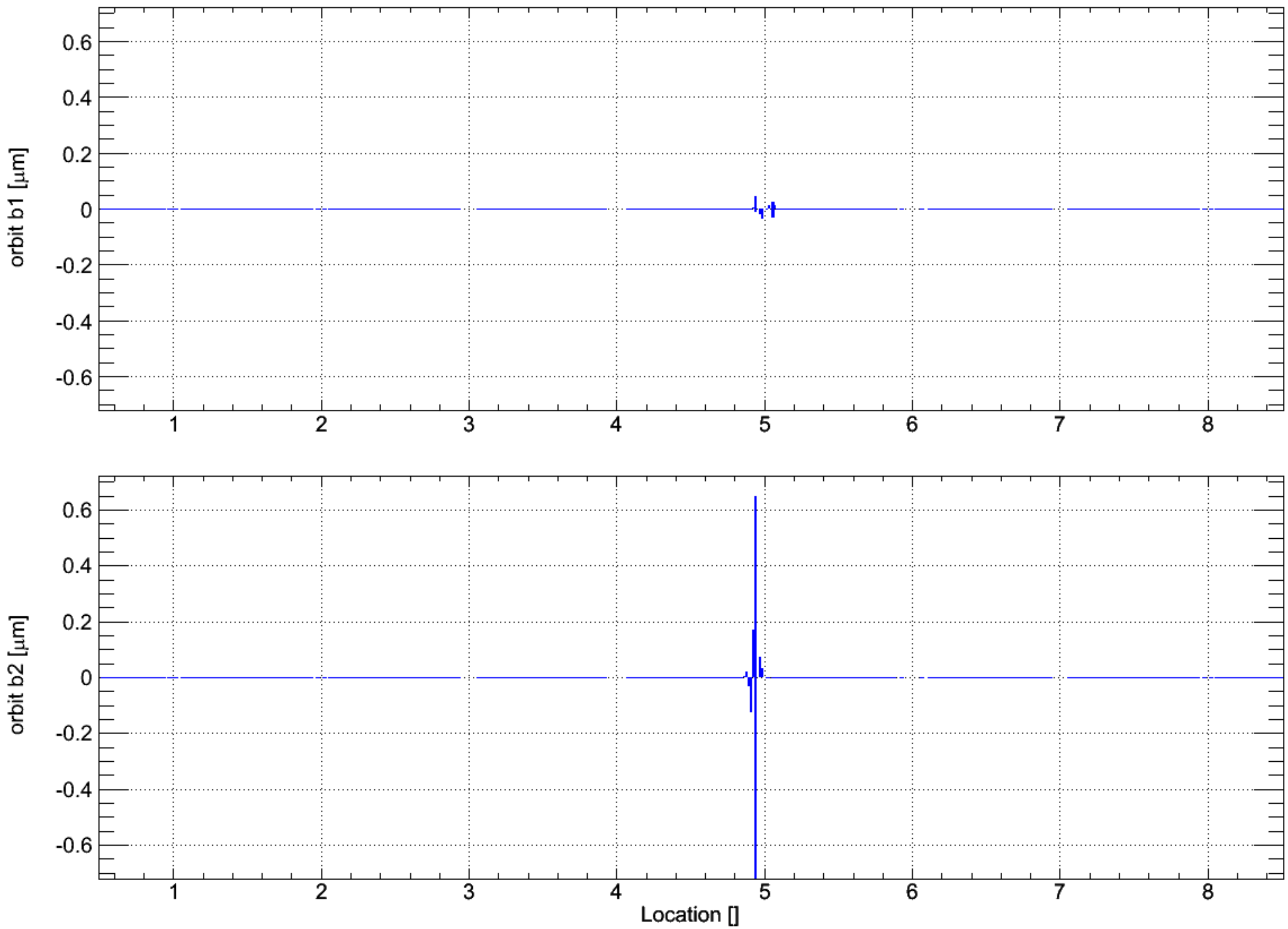
LHC example: BPM eigenvector #291 $\lambda_{291} = 2.13 \cdot 10^2$



LHC example: BPM eigenvector #449 $\lambda_{449} = 8.17 \cdot 10^1$



LHC example: BPM eigenvector #521 $\lambda_{521} = 1.18 \cdot 10^0$



Space Domain: Number of used eigenvalues?

Gretchen Frage: “How many eigenvalues should one use?”

small number of eigenvalues:

- more coarse type of correction:
  - use arc BPM/COD to steer in crossing IRs
  - less sensitive to BPM noise
  - less sensitive to single BPM faults/errors
  - less sensitive to single COD/BPM faults/errors
- robust wrt. machine imperfections:
  - beta-beat
  - calibration errors
- easy to set up
- ...
- poor correction convergence
- leakage of local perturbations/errors
  - not fully closed bump affects all IRs
  - squeeze in IR1&IR5 affects cleaning IRs
- ...

large number of eigenvalues:

- more local type of correction
  - more precise
  - less leakage of local sources onto the ring
  - perturbations may be compensated at their location
- good correction convergence
- ...
- more prone to imperfections
  - calibration errors more dominant
  - instable for beta-beat > 70%
- more prone to false BPM reading
  - Errors & faults
- ...

parameter stability requirement

feedback stability requirement

Choice for Q, Q', C^- is much simpler: only two out of *n* non-vanishing eigenvalues!

Space Domain: local within global correction

- The orbit and feedback stability requirements vary with respect to the location in the two LHC rings. In order to meet both requirements:
  - Implement robust global correction (low number of eigenvalues)
  - fine local correction where required (high number of eigenvalues or simple bumps):
    - Cleaning System in IR3 & IR7
    - Protection devices in IR6
    - TOTEM

coarse global SVD with fine local "SVD patches" (no leakage due to closed boundaries)

minor disadvantage: longer initial computation (global + local SVD + merge vs one local SVD)

coarse global SVD with weighted monitors where required (ω = 1 ... 10)

disadvantage:
- total number of to be used eigenvalues less obvious
- Matrix inversion may become instable

free orbit manipulation (within limits) while still globally correcting the orbit

- Youla's affine parameterisation for stable plants[1] - showed that all stable closed loop controllers D(s) can be written as:

$$D(s)=\frac{Q(s)}{1-Q(s)G(s)} \quad (1)$$

- Simplifies the form of the system transfer $T_0(s)$ and sensitivity function $S_0(s)$:

$$T_0(s) = Q(s)G(s) \quad (2)$$
$$S_0(s) = 1-Q(s)G(s) = 1-T_0(s) \quad (3)$$

- Use following common *ansatz* for solving (1): $Q(s)=F_Q(s)G^i(s)$ (4)
- In case of a "perfect" inverse response function (no unstable poles) (2) (3) yield simply:

$$T'_0(s) = F_Q(s)$$
$$S'_0(s) = 1-F_Q(s)$$

- → effective closed loop response can be deduced by construction of $F_Q(s)$

[1]D. C. Youla et al., *"Modern Wiener-Hopf Design of Optimal Controllers"*, IEEE Trans. on Automatic Control,1976, vol. 21-1,pp. 3-13 & 319-338