Closed Orbit and Protection

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- Combined failures: Local Orbit Bumps + Fast Failure
- Aperture Scans
- Indicator to check aperture Scans are required
  - Anticipated orbit uncertainties during operation
- Some issues concerning ramp, squeeze and physics
- Feedback a source of orbit bumps?

Closed Orbit Definition

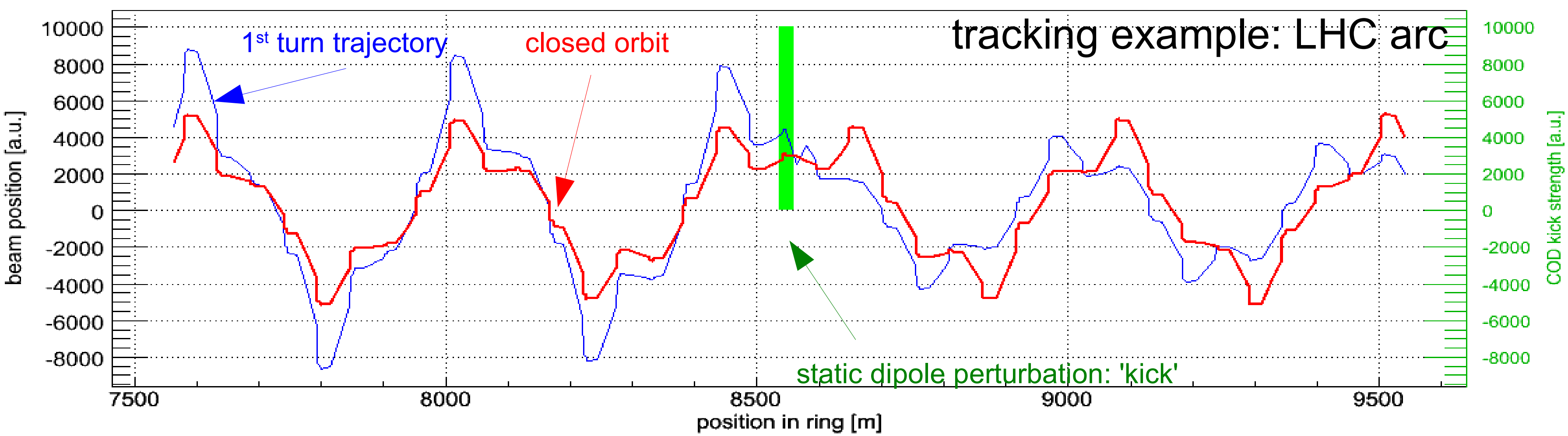
- Closed Orbit x_{co}: Single stable trajectory that maps the particle phase space coordinate on itself after each revolution in the machine.

$$\boxed{\begin{pmatrix} x_{co} \\ \dot{x}_{co} \end{pmatrix}_s = \begin{pmatrix} x_{co} \\ \dot{x}_{co} \end{pmatrix}_{s+C}}$$

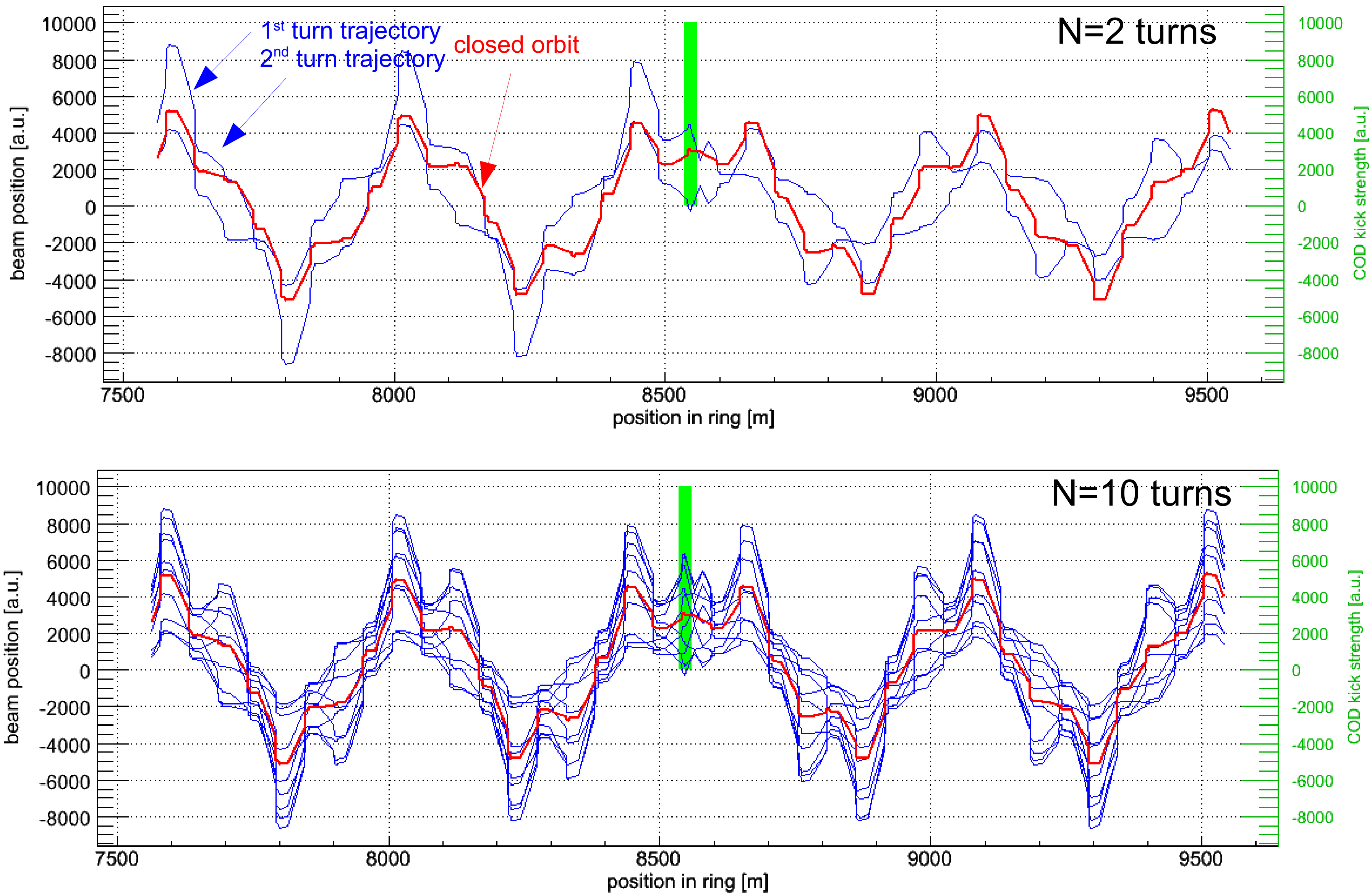
- All higher particles (/bunches) oscillate around x_{co}: $\boxed{x(s) = x_{co}(s) + x_\beta(s)}$

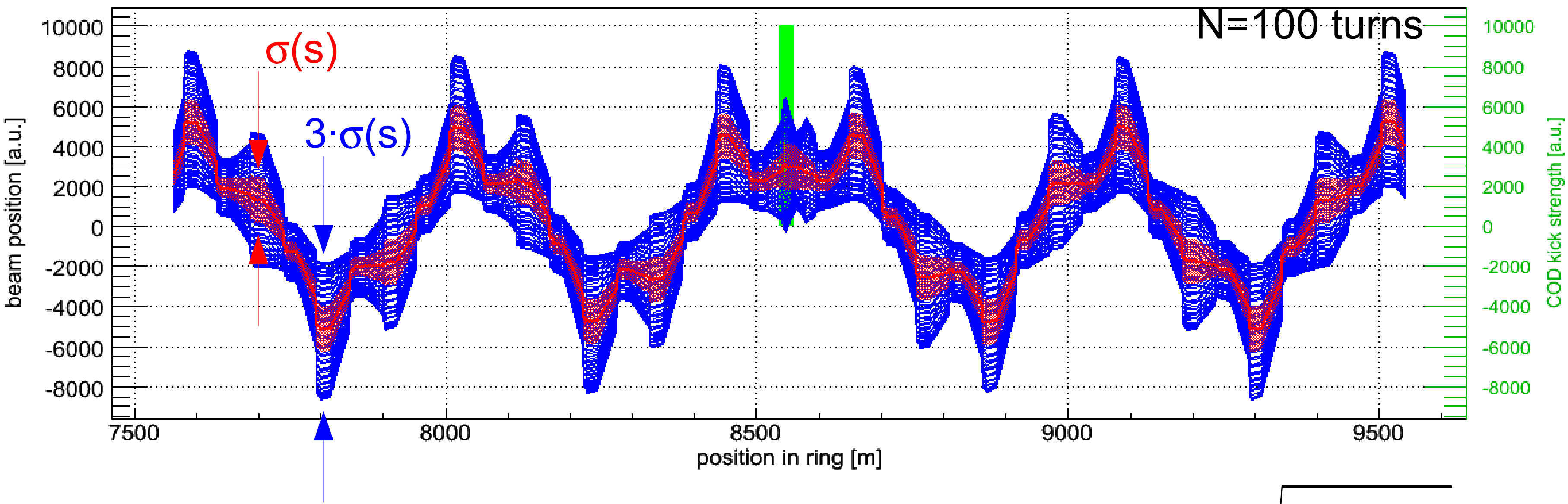
$$x_\beta(s) = \sqrt{\varepsilon\,\beta(s)} \cdot \sin\left(\mu(s) - \phi_0\right)$$

  - Twiss parameter: β: beta function, μ: phase advance, C: circumference, Initial conditions: ε: particle emittance, ϕ_0: initial particle phase



Tracking Example: LHC arc





- Collective particle oscillation → beam size $\sigma(s)=\sqrt{\varepsilon\beta(s)}=\sqrt{\frac{\varepsilon_n}{\gamma}\beta(s)}$
  - LHC: Typical max r.m.s. beam sizes in the arc:

    (norm. emittance@injection: $\varepsilon_n \approx 3.5$ µmrad, beta-function: $\beta \approx 180$ m)

- $\sigma_{450GeV} \approx 1.14$ mm resp. $3\sigma_{450GeV} \approx 3.42$ mm
- $\sigma_{7TeV} \approx 0.29$ mm resp. $3\sigma_{7TeV} \approx 0.87$ mm

Assuming Gaussian distribution:
1σ ≈ 68.3% protons/bunch
3σ ≈ 99.7% protons/bunch
nominal bunch intensity:
$3\cdot10^8$ particles are > 3σ

Closed Orbit drifts...

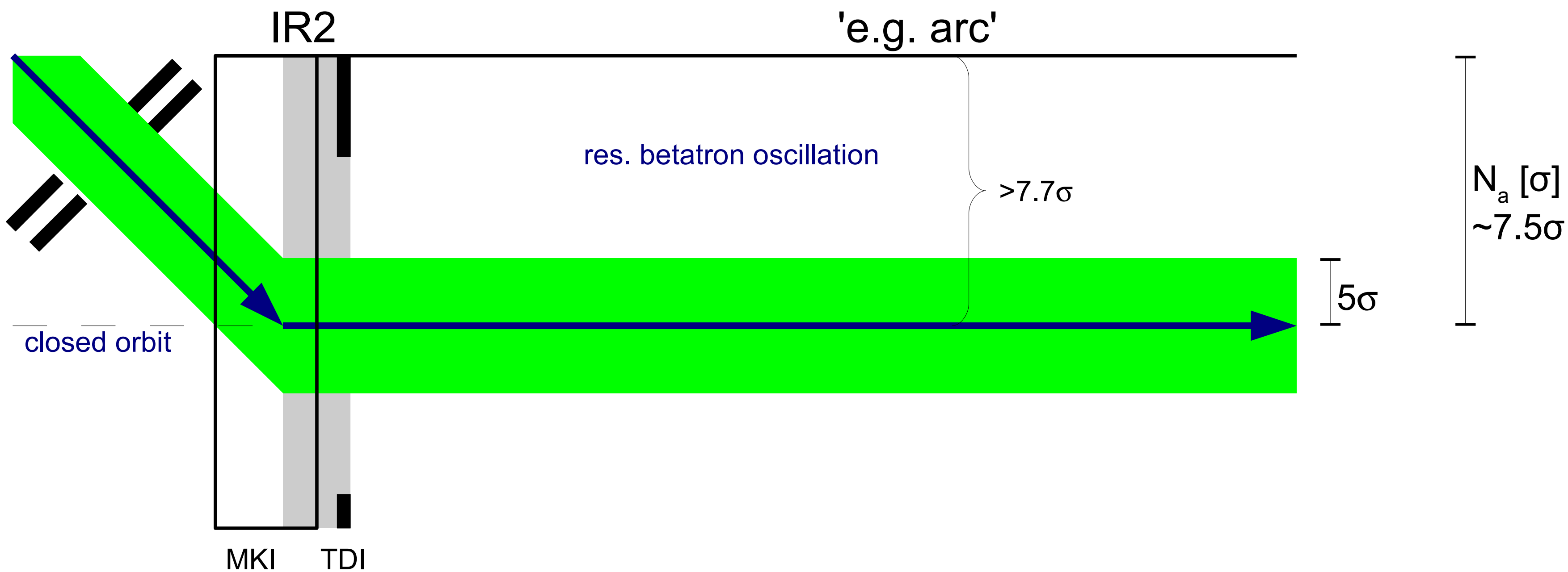
- alone are unlikely to cause damage to the machine
  - Expected drift velocities are slow: < 2 σ/s
  - Easily detectable and captured through beam loss monitors
    - independent on whether they are local or global drifts

- However, combined failures are an issue:
  - “local orbit bump” + fast other failure, e.g.:
    - Single turn failure involving injection, extraction or aperture kicker
    - fast magnet field decays
  - reduction of alignment margin at local protection devices
    - TDIs, TCDQs, Collimators etc.

- Local orbit bumps may compromise passive protection properties of absorbers and collimators for machine protection!

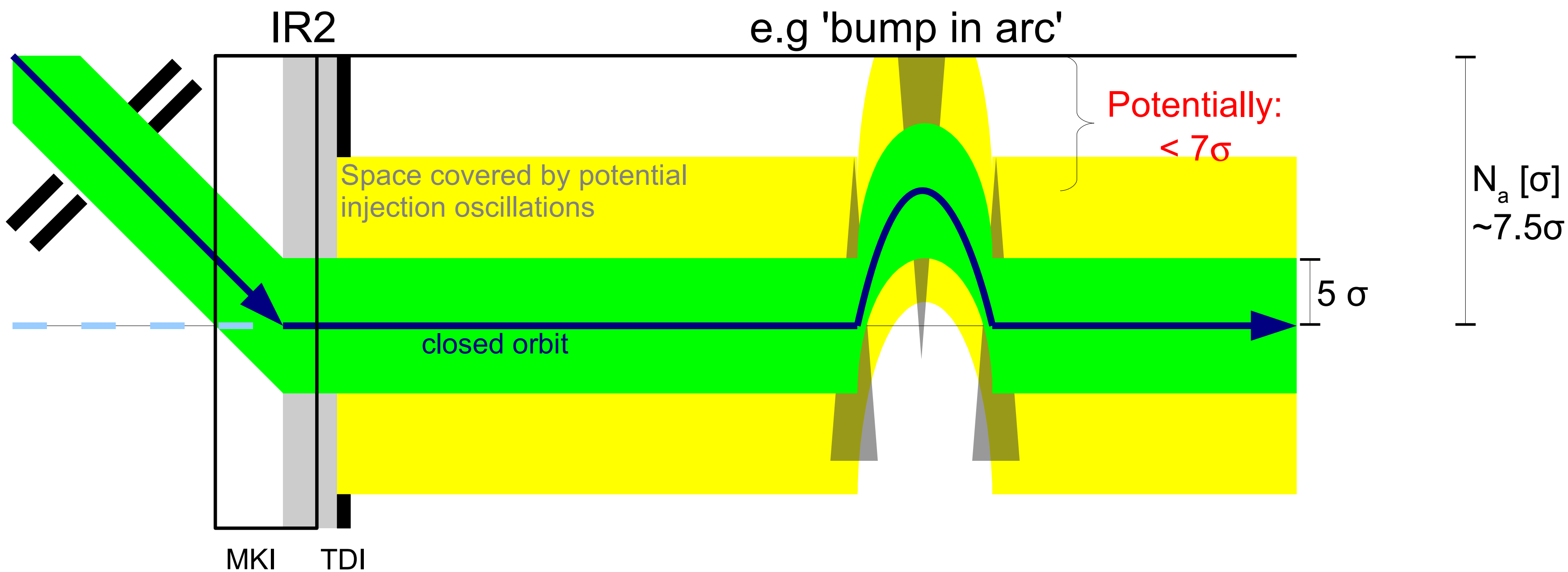
Example: Protection against Single Turn Failures

- Combined failure: (Pilot) Injection with perfect closed orbit



- TI8/TI2 collimators limits $|x_\beta(s)|_{max} < 5\ \sigma$, TDI (locally) limits $|x_\beta(s)|_{max} < 7\ \sigma$
- Perfect matching: beam circulates on closed orbit & $\varepsilon_{TI8/TI2} = \varepsilon_{ring}$
- $\Delta x, \Delta x'$/optics mismatch: → oscillation around x_{co} & filamentation $\varepsilon_{ring} > \varepsilon_{TI8/TI2}$
  - But: $\sigma_{ring} < 7\ \sigma$ globally (if proper TDI setup)
  - TDI shadows critical machine aperture
- "Ring aperture is safe", assuming only single turn (injection) failures.

- Combined failure: Local orbit bump and injection of nominal beam:

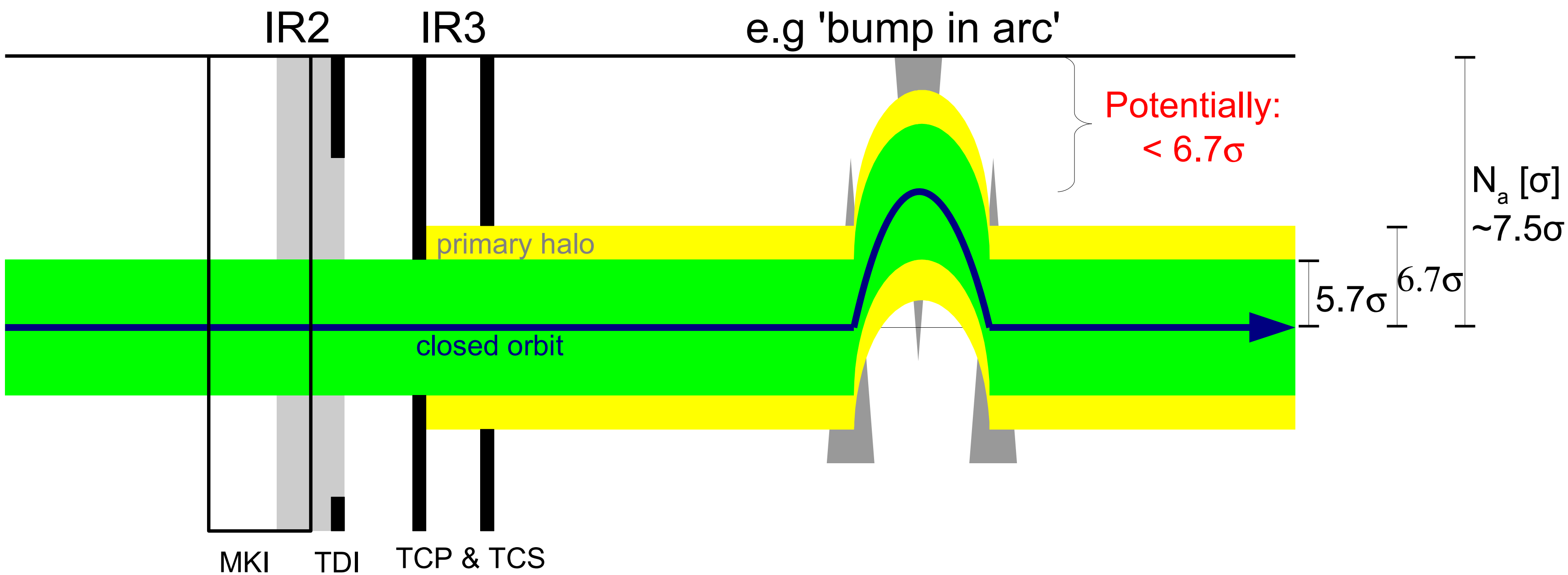


- TI8/TI2 collimators limits $|x_\beta(s)|_{max} < 5\ \sigma$, TDI (locally) limits $|x_\beta(s)|_{max} < 7\ \sigma$
  - – TDI does potentially not shadow sensitive equipment

  → Orbit bumps may compromise function of absorbers for protection if beam is closer to the aperture than to TDI

Example: Compromised Protection through Orbit Bumps

- Combined failure: Local orbit bump and collimation efficiency (/kicker failure):



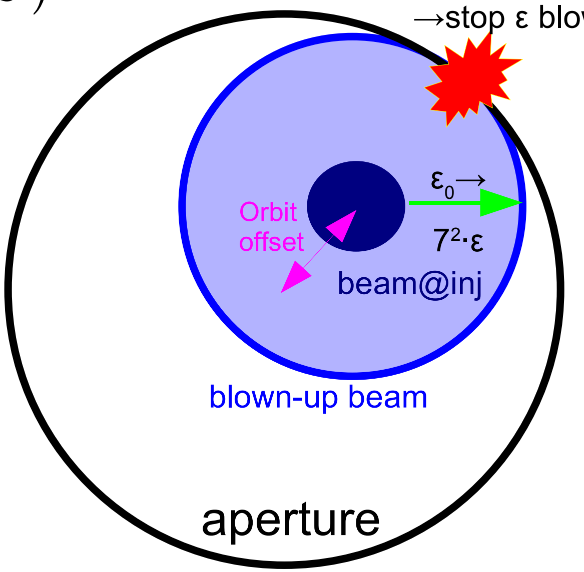
- Primary collimator (TCP) limits $|x_\beta(s)|_{max}$ locally to <5.7σ, secondary collimator (TCS) at~ 6.7σ
- To guarantee two stage cleaning efficiency/machine protection:
  - Local: TCP must be >0.7σ closer than TCS w.r.t. the beam → Orbit FB
  - Global: no other object (except TCP) closer to beam than TCS

→ Orbit bumps may compromise function of collimation if beam is closer to the aperture than to jaws!

Aperture measurement:

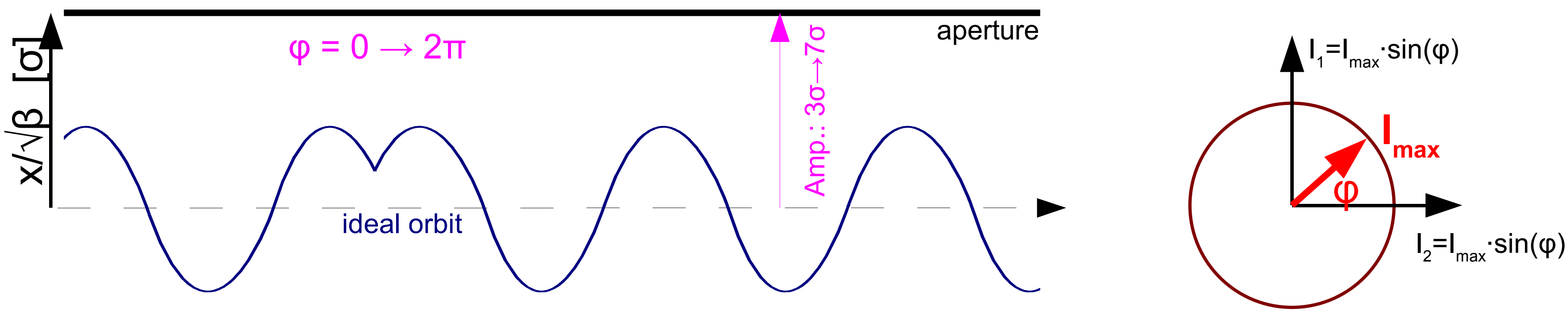
Two methods to establish whether the closed orbit is within 6.7σ of the available mechanical resp. dynamic aperture:

- Scan using emittance blow-up: $\sigma(s)=\sqrt{\varepsilon\,\beta(s)}$
  - Increase beam size in a controlled way while measuring the beam size.
    (e.g. using transverse damper and wire scanner)
  - Once particle loss above given threshold:
    → store last beam size measurement
  - "Is beam size ≥ 6.7 σ_0 ?" (σ_0: beam size at injection)
    - Yes: → mechanical aperture ≥ 6.7 σ → orbit is safe
    - No: → mechanical aperture ≤ 6.7 σ → orbit is un-safe
      - rework orbit reference (compare with old reference....)



Scan of mechanical/dynamic aperture

- Scan using two COD magnets (currents: I_1 & I_2) with π phase advance:



- Scan I_{max}/φ:
  - $\varphi = 0 \rightarrow 2\pi$ (takes ~25 second @ 7σ, due to COD power converter speed)
- Increase amplitude (COD currents) till orbit shift corresponds to 6.7σ
- Loss does not exceed predefined BLM threshold if COD settings@ 6.7σ:
  - Yes: → mechanical aperture ≥ 6.7 σ → orbit is safe
  - No: → mechanical aperture ≤ 6.7 σ → orbit is un-safe
- additional feature: compare measured with reference BPM step response (x_{co}= 0-3σ)

  → rough optics check (phase advance and beta-functions)

Mechanical Aperture Scan through...

Controlled emittance blow-up:

- may check both planes at the same time
- relatively fast measurement
- reliability/robustness of beam size measurement/blow-up is an issue
- no information on injection optics
- Tests rather dynamic than mechanical aperture if $a_{dyn} < a_{mech}$
- Destructive measurement
  - beam has to be dumped after scan
  - cannot be used for collimator setup
  - increased beam loss during extraction

Betatron oscillation scan:

- non-destructive measurement
  (could be done to check during each injection)
- rough information on injection optic
- Independent information on planes
- checks only one plane at a time
- What to do if on COD is down?
  - spares: longer measurement
- requires ~30 s for a scan at 7σ
- Required:
  - inhibit injection during scan
  - COD setting reset after scan

- Both methods:
  - Determine the available aperture
  - should be performed with low-intensity beams
  - need time and exclusive control of the machine
- in order to minimise the need for too frequent aperture scans:

  → perform above checks only when exceed given window

Beam Position Monitors:

- Procedure:

A: Initial check whether Orbit is safe:

- aperture scan (ε blow-up, betatron-oscillation)
  - Potential bump scans to determine location of aperture
- save "safe BPM reference" current settings → x_{ref} = "SAFE SETTING"

B: Check: if ($|x_{meas.} - x_{ref}| < \Delta x_{tol}$) {...}

- FALSE: potential orbit bump detected
- TRUE: Orbit is safe

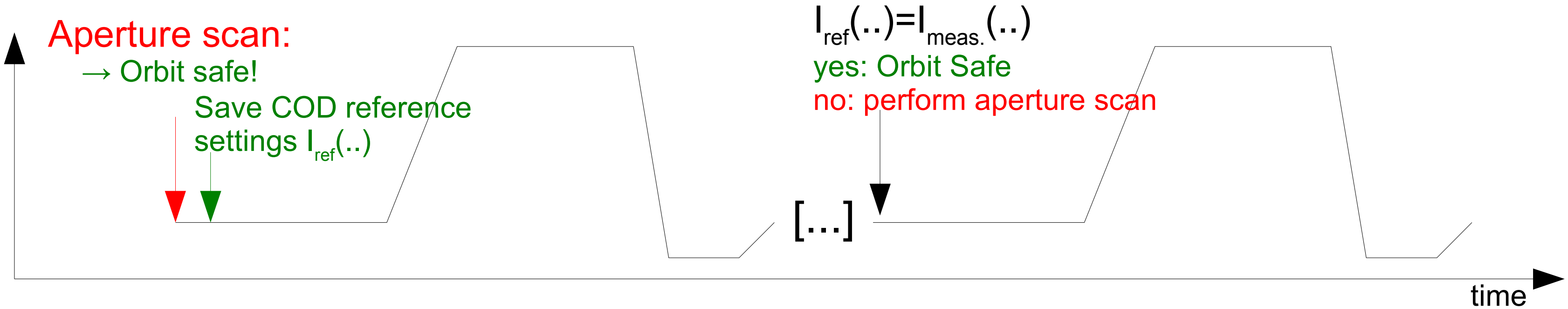
no

yes

– Pro's:
  - Easy to check with circulating beam
  - Less dependent on machine optics
  - Sensitive to most orbit manipulations

– Con's:
  - erroneous BPMs
  - No information before injection
  - Bunch intensity systematics (gain settings) and change of BPM calibration
  - Potential cross-talk with orbit feedback

Magnet Current Surveillance I/II



- Proposed Procedure:

A: Initial check whether Orbit is safe:

- aperture scan (ε blow-up, betatron-oscillation)
  - Potential bump scans to determine location of aperture
- Save "safe COD reference" current settings → $I_{ref}(...)$ = "SAFE SETTING"

B: Each cycle:

- Compare with actual current reference $I_{meas}(..)$:

if $(|I_{meas}(..) - I_{ref}(...)| < \Delta I_{tolerances})$ {…}

- FALSE: Orbit may contain potential bumps → State A
- TRUE: Orbit can be considered to be safe → State B

no

yes

- Current Surveillance:
  - Pro's
    - Can be used to check even before first injection
    - Can run continuously with orbit feedback in operation
  - Con's
    - Less sensitive to complicated orbit bumps
    - No precise&simple '$\Delta I \rightarrow \Delta x$' transfer function available
      - depends on machine optic, energy
      - CODs create not only bumps but compensate
        - » ground motion,
        - » decay & snap-back,
        - » multipole field errors, ..

→ Current tolerance level $\Delta I_{tolerances}$ (“SAFE SETTINGS”) should include margin for
- orbit feedback operation
- expected compensation of closed orbit uncertainties = “natural effects”

Closed Orbit Uncertainties at injection

- LPR501 specification[1]:	
– nom.: $(\Delta p/p)_{max} \approx 10^{-4}$	$0.25\ \sigma$ (MD: max ≈ 3.7 σ)
– $b_2+b_3\cdot\Delta x$ decay: $(\Delta\beta/\beta)_{3\sigma} \approx 2.5\%$	$0.03\ \sigma$
- Moon/sun tides[2] ($\Delta p/p \leq 5.0\cdot 10^{-5}$)	$0.14\ \sigma$
- Main Bends, random $b_1 \approx 0.75$ units[34] (dipole kick)	$0.11\ \sigma$
- Random ground motion[5] (10 hours)	$\sim 0.3 – 0.5\ \sigma$
- Systematic ground motion drifts:	~?? σ
- MCB hysteresis[6]	$0.01\ \sigma$
- MCB ±8V/±60A PC stability[7] (16bit ADC)	$0.10\ \sigma$
- Total (abs):	~ 0.9 - $1.1\ \sigma$ (max: 4.6 σ)

1: M. Giovannozzi: FQWG Meeting on 8th of March 2005
2: J. Wenninger: "Observation of Radial Ring Deformation using Closed Orbits at LEP"
3: M. Haverkamp, "Decay and Snapback in Superconducting Accelerator Magnets", CERN-THESIS-2003-030
4: FQWG-Homepage: http://fqwg.web.cern.ch/fqwg/
5: RST: "Analysis of Ground Motion at SPS and LEP, implications for the LHC", AB note to be published
6: W. Venturini: "Hysteresis measurements of a twin aperture MCB orbit corrector", 19th October 2005
7: Q. King, L. Ceccone: private communications

Example: Tolerance and COD Constraints

- Mechanical aperture: $N_a = n\,\sigma$ (e.g. n=7.5)
- Deductions:
  - Collimation: 6.7 σ
  - Momentum correction
  - Known uncertainties: 1.1 σ
  - Unknown: ~?? σ
- safe window for dynamic closed orbit modifications: ~ "- 0.3 σ"???
  - Evident: aperture check required!

- Possible MCB tolerance levels:
  - ... 1 σ orbit excursion using CODs one needs e.g.:
    - All CODs with a r.m.s. kick of ~ 1.4 μrad ↔ ≈ 0.07 A@450 GeV
    - 3COD bump: 2x ~12 (-0.1) μrad ↔ ≈ 0.5 (0.05)A@450 GeV

→ Vicious bump: smaller strengths and larger local displacement possible!

  - ... 1 σ orbit excursion through dispersion one needs ($\Delta p/p \approx 4\cdot10^{-4}$):
    - Coherent shift of all MCBH CODs ≈ 0.5 A@450 GeV

→ MCB current change of 0.5 A is likely to cause a orbit bump/shift of 1 σ.

Continuation of Interlock Scheme through Ramp

- Scheme may be extended through the ramp till squeeze:
  - Similar effects as in injection that perturb the orbit dynamically:
    - Snapback (= inverse of Decay), ground motion,...
  - But: effect of each dipole (deflections) depends on energy:
    - Interlock window and its centre has to be scaled with energy:
      - $0.5\ A/\sigma_{orbit}$ @450 Gev $\rightarrow 7.8\ A/\sigma_{orbit}$ @7 TeV
- Continuation through β*-Squeeze seems to be tricky:
  - CODs do not compensate only ground motion/decay
  - Squeeze induced orbit shifts due to systematic (mis-)alignment of the orbit inside the insertion quadrupoles. If not corrected:
    - Squeeze induced orbit drift up to 30 mm $\leftrightarrow$ 100 σ!

→ No simple window to subtract squeeze induced COD changes from those creating bumps.

Preliminary Conclusions

- Closed Orbit drifts alone are unlikely to cause fast particle losses.
- But may become an issue through combined failures:

(all fast failures occur around the closed orbit)

  - local orbit bumps + fast kicker failure
  - local orbit bumps + collimation efficiency

- Aperture scan to check whether orbit containing potential bumps is safe:
  - Controlled emittance blow-up
  - Betatron-oscillation using closed orbit correction dipoles

- Both methods can only be performed with low-intensity beam at 450 GeV
  - Indicators available to check whether aperture scan is required:
    - Using BPMs (direct) & Surveillance of COD currents (indirect)
    - Continuation of scheme through ramp needs refinement (β^*-squeeze not obvious!)

- Orbit manipulations within 0.8 σ seem to be safe (arc: $\Delta x < 1$ mm @ β=180 m).
  - Very tight: Should be checked during early LHC operation.

LHC Orbit feedback: Among Others, “A Suspect for Creating Bumps?”

- LHC Orbit feedback:
  - Does not find optimal orbit!
  - Minimises orbit perturbation around predefined reference
  - Runs continuously from ~ 100 ms after first injection till beam dump

- However:
  - Feedback may potentially create bumps in case of false BPM readings or wrong orbit reference settings.

→ To be robust, the feedback is designed to be insensitive to COD/BPM failures:

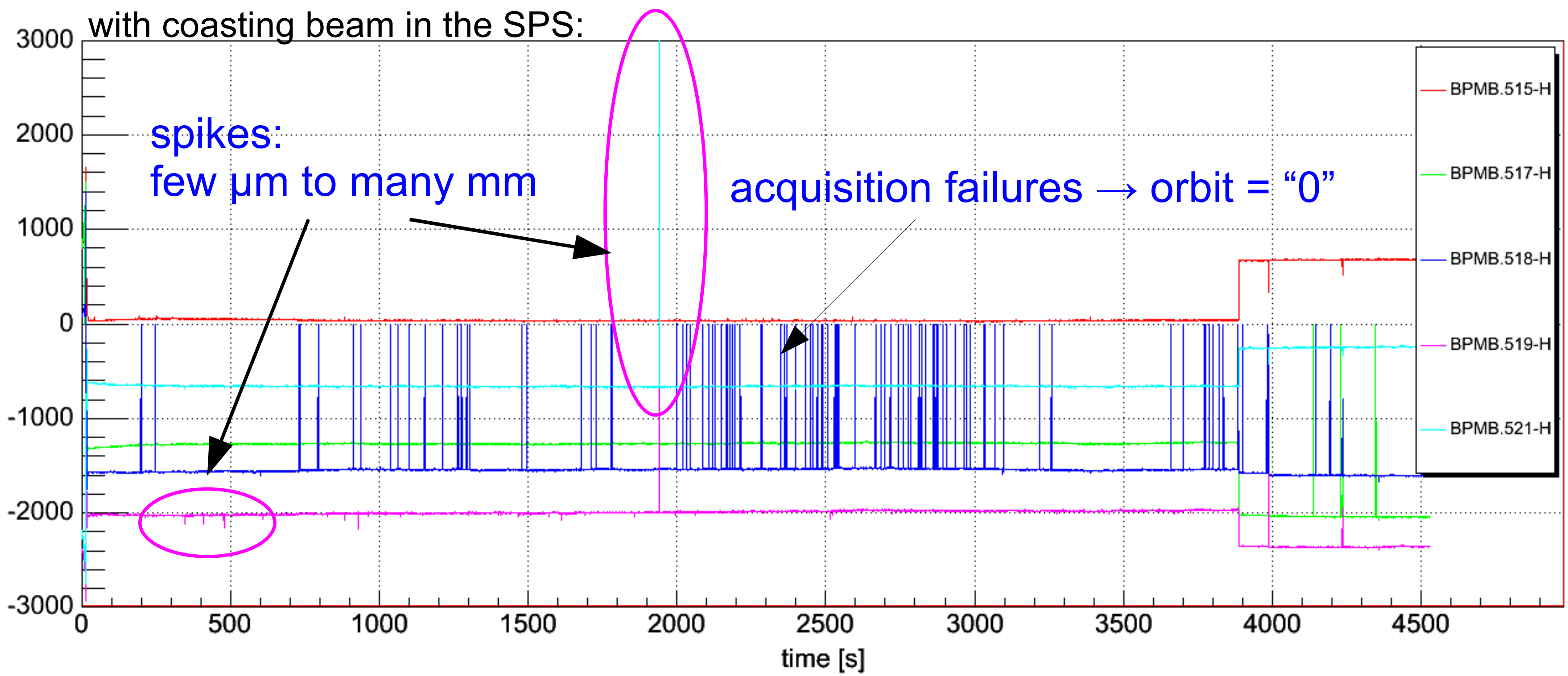
- Closed Orbit Dipole (COD) failures → see MPWG meeting #46
- BPM:
  - Double sampling w.r.t. betatron oscillations
  - Detect BPM failures at an early stage
  - Eliminate per construction corrections that potentially may create bumps

Anticipated BPM Failures/Noise/Systematics

LHC BPM Prototype in the SPS:

- Most common: acquisition failure = no orbit info available and spikes
  - Short term (few ms-s): Zero Order Holder (ZOH)
  - Long term: Disable BPM in feedback and recalculate SVD pseudo-inverse matrix
- Only a few drifts observed: systematic on bunch length & bunch intensity
  - within 1% of BPM half aperture ↔ 250 μm (complies with specification)

LHC BPM test in 2004
with coasting beam in the SPS:



Orbit Feedback Robustness against BPM Failures/Errors

1. BPM phase advance of ~π/4:
   - Twice the sampling than minimum required to detect β-oscillation
   - Distribution of consecutive BPMs on different front-ends (minimise impact of front-end drop outs)
2. Detection of erroneous BPM failures (SPS: mostly spikes)
   ($x_i(n)$=position at i[th] monitor, n: sampling index; σ_{orbit}= residual orbit r.m.s.)
   - Reject BPM if the following applies:
     - Cuts in Space Domain:
       - $x_i(n)$ > machine aperture
       - $x_i(n) - x_{i,ref} > 3 \cdot \sigma_{orbit}$
       - possible: interpolate position from neighbouring BPMs (implemented in APS)
       - (BPMs marked by the front-end itself)
     - Cuts in Time Domain:
       - $\Delta x_i(n)=x_i(n)-x_i(n-1)$ > 500 µm/s Spike detection!(BPM noise < 20 µm and ofb@25Hz assumed)
       - filters to reduce noise (e.g. low integrator gain)
       - ...
   - Difficult to detect coherent, very slow or systematic drifts

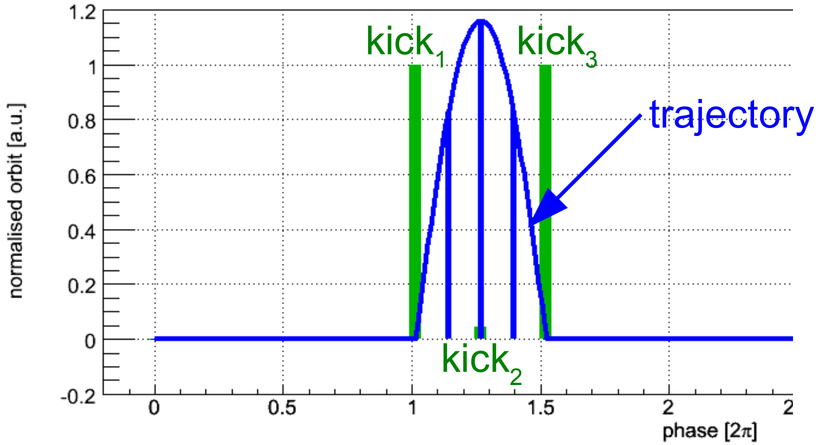
     (e.g drift of BPM electronics vs. systematic ground motion, temperature drifts, tides... etc.)
3. Use SVD based correction → less sensitive to BPM errors

SVD Robustness: Properties

- Global orbit feedback with local constraints
  - Based on SVD algorithm → see attachment for details
  - Expands orbit using orthogonal "eigen-orbits"

- Important mathematical properties:
    - SVD minimises orbit & deflection strengths
    - Uses rather many CODs with small than few with large kicks
    - Solutions are sorted by their 'effectiveness': large eigenvalues λ_i (solutions) first
    - Local 'bump-like' solutions corresponds to small eigenvalues
  - "number of used eigenvalues" $\#\lambda_{svd}$ controls OFB robustness vs. precision
    - more #eigenvalues → more precise correction (collimation requirement)
    - less #eigenvalues → more robustness against BPM & optic failures
  - discard deliberately solutions with small eigenvalues (=local bumps)
    → SVD cannot generate (= correct) those bumps
  - However: Will use all (local SVD) eigenvalues regions like collimation.
    (due to precision requirement)

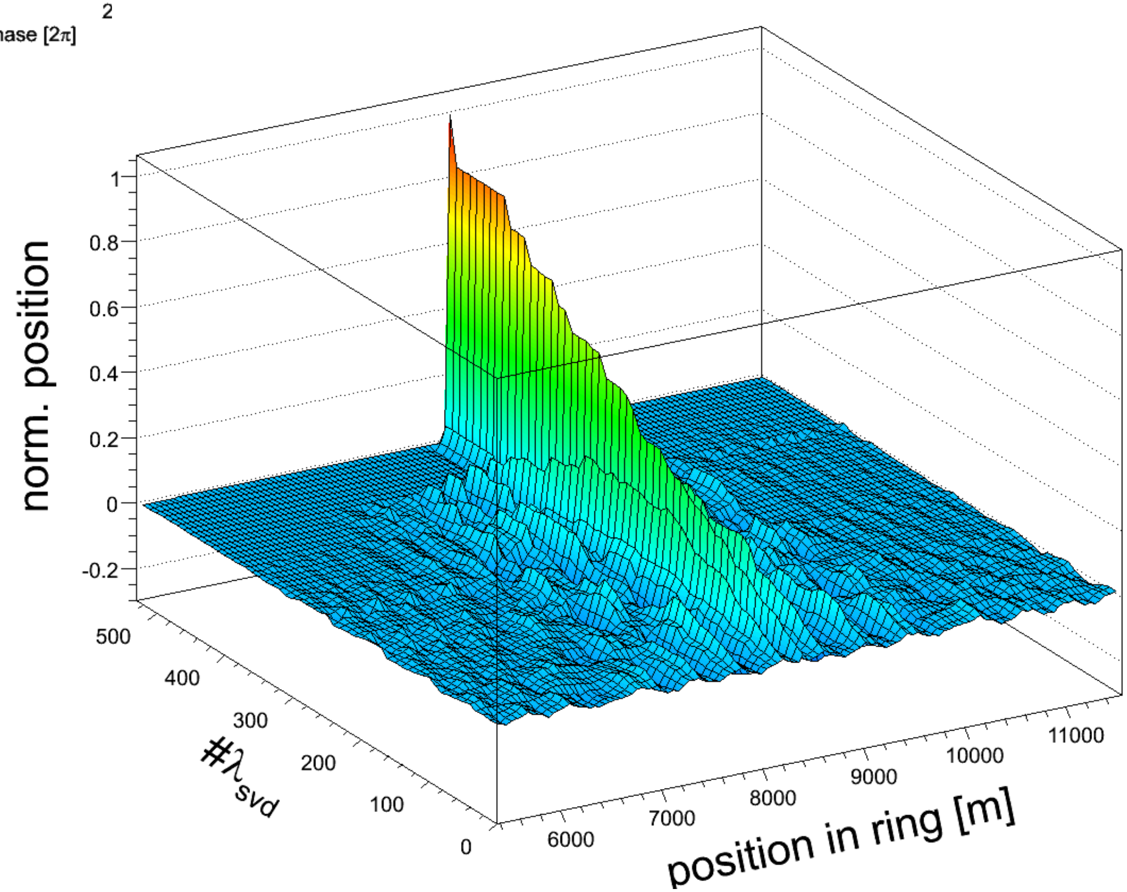
Robustness Examples



- E.g. simplest Three-Corrector-Bump is sampled with at least three BPMs
  - erroneous or noisy BPM has less effect on total correction
-

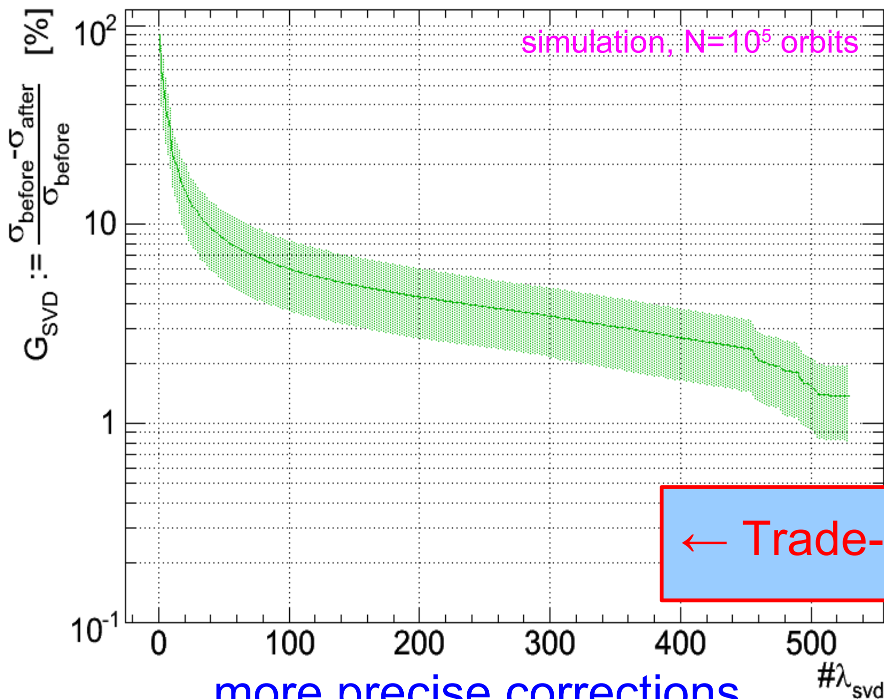
Example: Single BPM spike

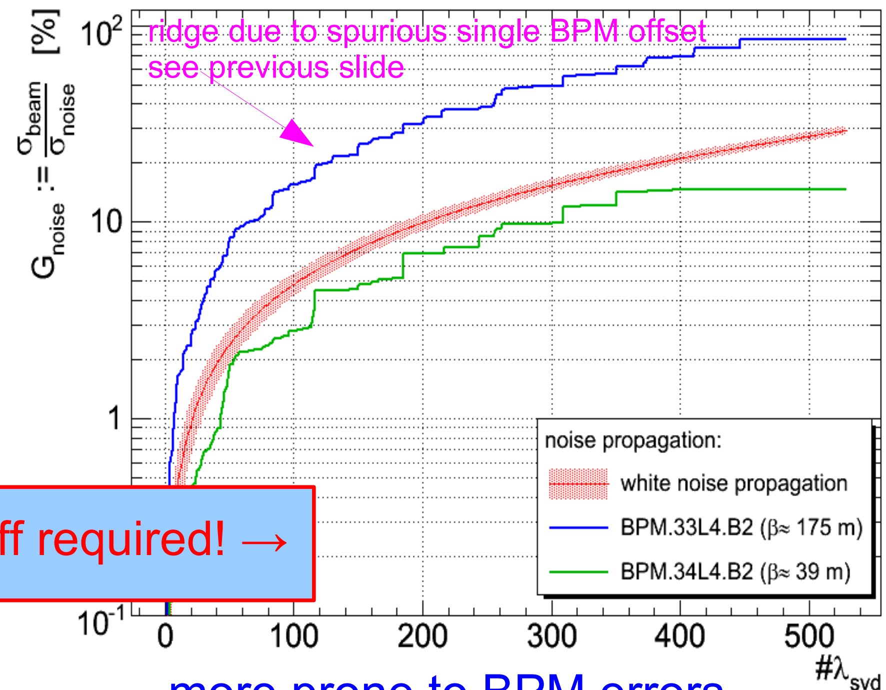
- perfect orbit (=0)
- BPM.33L4.B2 with spurious offset
- SVD corrects the spurious offset (ridge in surface plot), if a large number of eigenvalues $\#\lambda_{svd}$ is used for the orbit correction
- e.g. $\#\lambda_{svd} = 100 \rightarrow$ spurious offset propagates to 16 % to the orbit



Feedback Sensitivity to BPM Failure

- Propagation of single (arc) BPM failure with $x_i(n) < 3\cdot\sigma_{orbit} < \sigma_{beam}$
  - #λ≈250: < 40% (β ≈ 175m) resp. < 10% (β ≈ 39 m)
- Propagation of random (white) noise on all BPMs
  - 30% (worst case #λ=529) resp. 10% (OFB operation with #λ≈250)
- BPM induced noise on orbit (single bunch):
  - Single BPM failure: < 0.01 - 0.4 σ
  - White BPM noise: < 0.001 σ (inj) resp. 0.02 σ (coll)





← Trade-off required! →

more precise corrections

more prone to BPM errors

Conclusions

- Closed Orbit drifts alone are unlikely to cause fast particle losses.
- But may become an issue through combined failures:

  (all fast failures occur around the closed orbit)
  - local orbit bumps + fast kicker failure
  - local orbit bumps + collimation efficiency

- Aperture scan to check whether orbit containing potential bumps is safe:
  - Controlled emittance blow-up
  - Betatron-oscillation using closed orbit correction dipoles

- Both methods can only be performed with low-intensity beam at 450 GeV
  - Indicators available to check whether aperture scan is required:
    - Using BPMs (direct) & Surveillance of COD currents (indirect)
    - Continuation of scheme through ramp needs refinement (β^*-squeeze not obvious!)

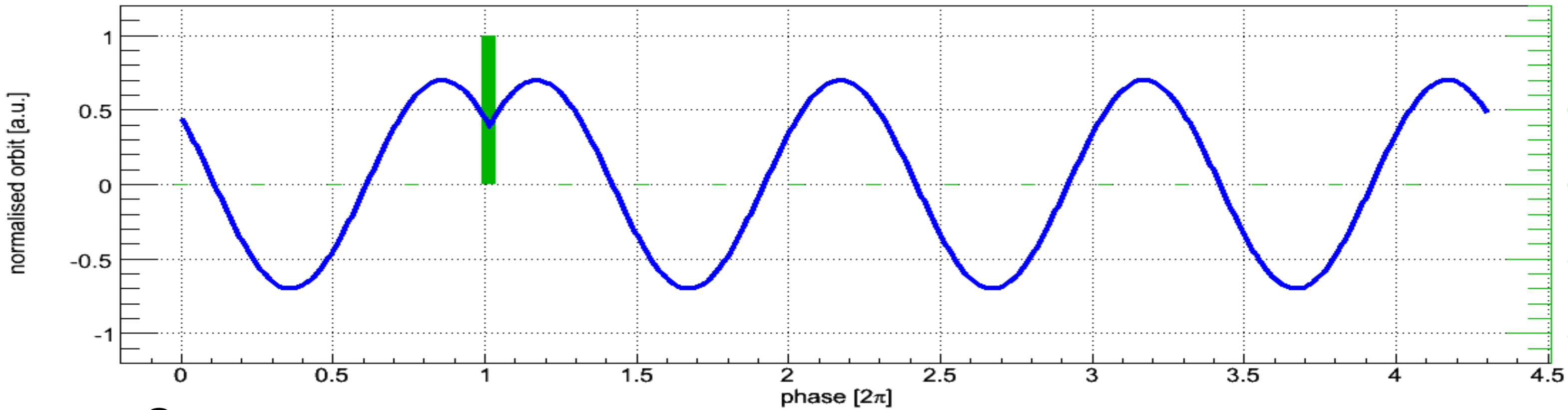
- Orbit manipulations within 0.8 σ seem to be safe (arc: $\Delta x < 1$ mm @ $\beta = 180$ m).
  - Very tight: Should be checked during early LHC operation.

- Orbit Feedback designed to be less sensitive to COD/BPM faults and errors
  - Expected single BPM failure propagation: < 0.01 - 0.4 σ ($\sigma \approx 1.1$ mm @450 GeV & β = 180 m)

reserve slides

Closed Orbit Perturbations

- ...due to dipole kick δ_0: $$\Delta x_{co}(s)=\frac{\sqrt{\beta_0\beta(s)}}{2\sin(\pi Q)}\cdot\cos(\mu(s)-\pi Q)\cdot\delta_0$$



– Sources:
  - closed orbit dipole magnets
  - Misalignments ($\Delta x \approx$ 0.3...0.5 mm r.m.s) and ground motion:
    – Main quadrupole magnets: $\delta_0 = k\cdot\Delta x$
    – (strong multipole magnets: $\delta_0 = m_n\cdot(\Delta x)^n$)

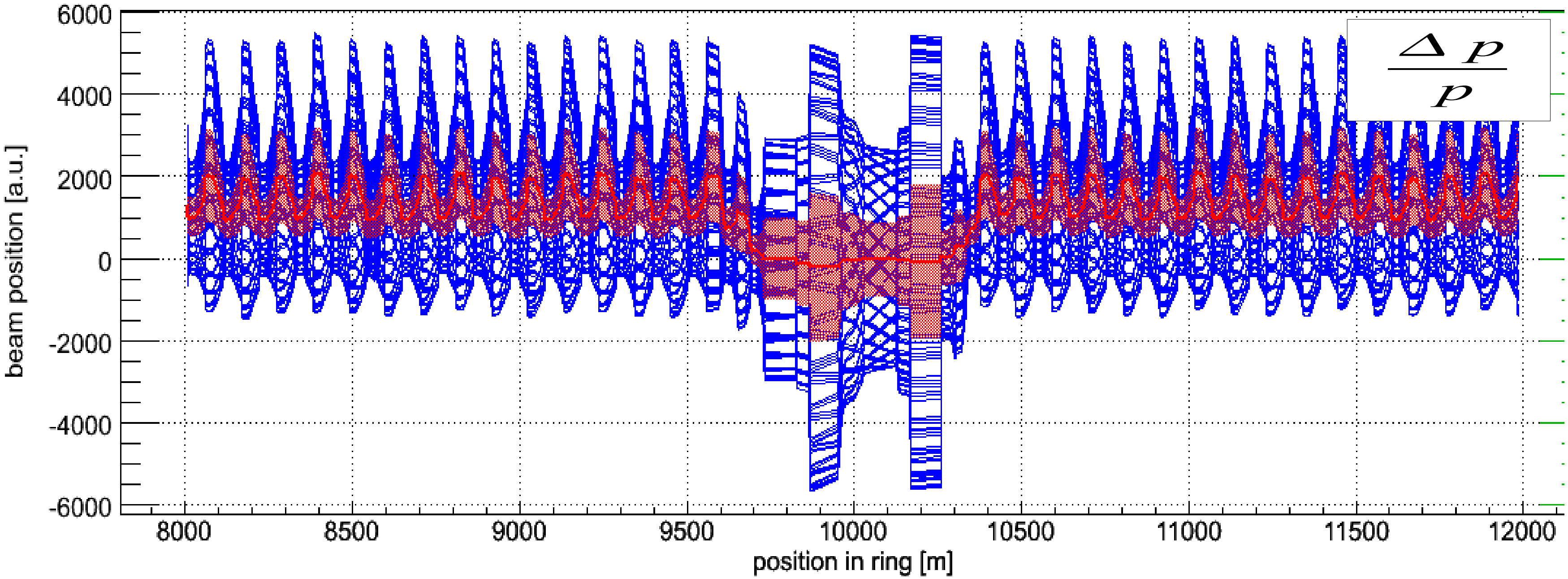
– Typical COD orbit response in LHC arcs ($\beta\approx$170 m, $Q_x\approx$64.28, $Q_y\approx$64.31):

$$\Delta x_{co}[\mu m]\leqslant 110\cdot\delta_x[\mu rad]$$
$$\Delta y_{co}[\mu m]\leqslant 100\cdot\delta_y[\mu rad]$$

or:

$$\boxed{\begin{aligned}\Delta x_{co}[\sigma]&\leqslant 0.10\cdot\delta_x[\mu rad]\\ \Delta y_{co}[\sigma]&\leqslant 0.09\cdot\delta_y[\mu rad]\end{aligned}}$$

- ...due to particle momentum shift: $\Delta x_{co}(s) = D(s) \cdot \frac{\Delta p}{p}$



– Dispersion D(s):
  - Insertions: $D_{x/y}(s) \approx 0$
  - LHC arcs: $D_y(s) \approx 0...0.1$ m resp. $D_x(s) \approx 1...2$ m
    – At dispersion suppressors: $D_x(s)|_{max} \approx 2.8$ m

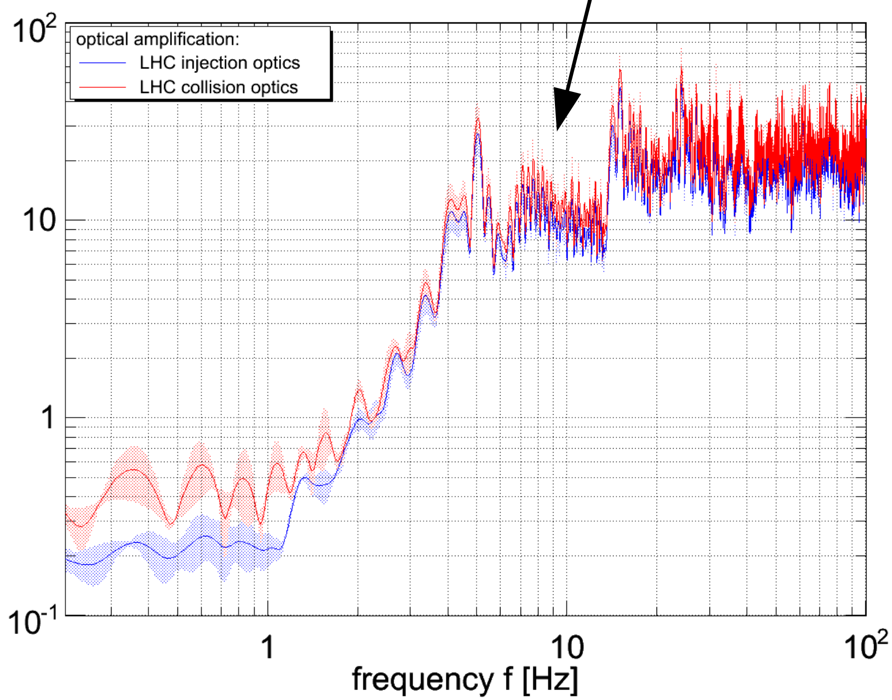
– $\frac{\Delta p}{p}$ typically: 10^{-4} ... 10^{-3}

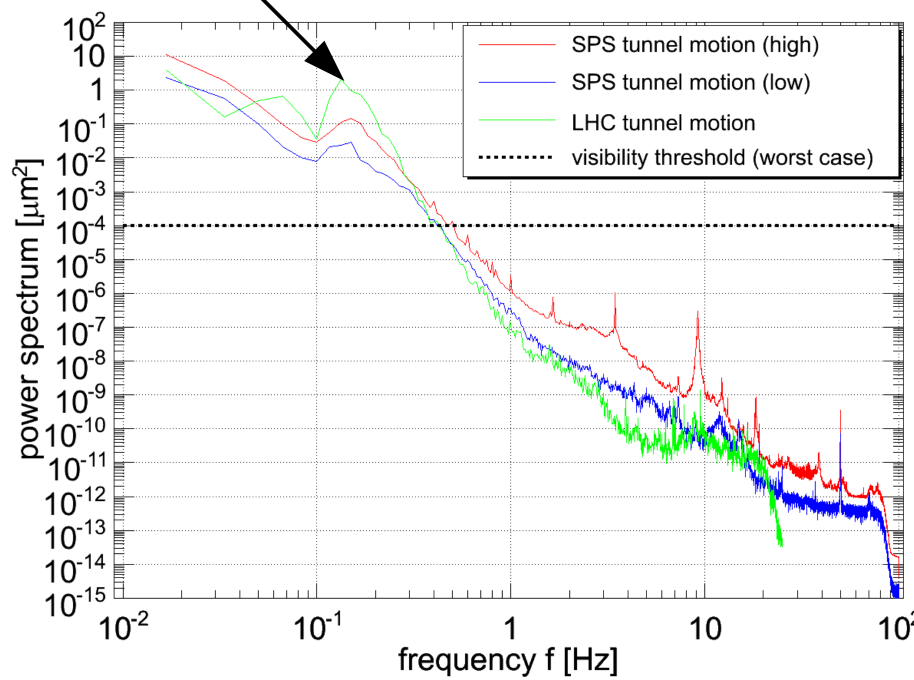
Correlated vs. Random Ground Motion I/II

Two classes of ground motion:

- Correlated ground motion waves: 'Cultural Noise', ocean swelling, tidal waves, ...
  - Assuming visibility threshold of 1 μm and κ≈1000
    → coherent ground motion negligible above 1 Hz

$$\boxed{\sigma_{beam}[\mu m] = \kappa(f) \cdot \sigma_{ground}[\mu m]}$$





Correlated vs. Random Ground Motion II/II

- Random ground motion (Brownian motion):
  - amplitudes increases with ~√t
    - LEP and SPS based measurements:

$$\sigma_{ground}[\mu m] \approx 5-6\cdot 10^{-2}\left[\frac{\mu m}{\sqrt{s}}\right]\cdot\sqrt{t}$$

  - Propagation of random ground motion onto orbit r.m.s. σ_{beam}:

$$\boxed{\sigma_{beam}[\mu m] = \kappa\cdot\sigma_{ground}[\mu m]}$$

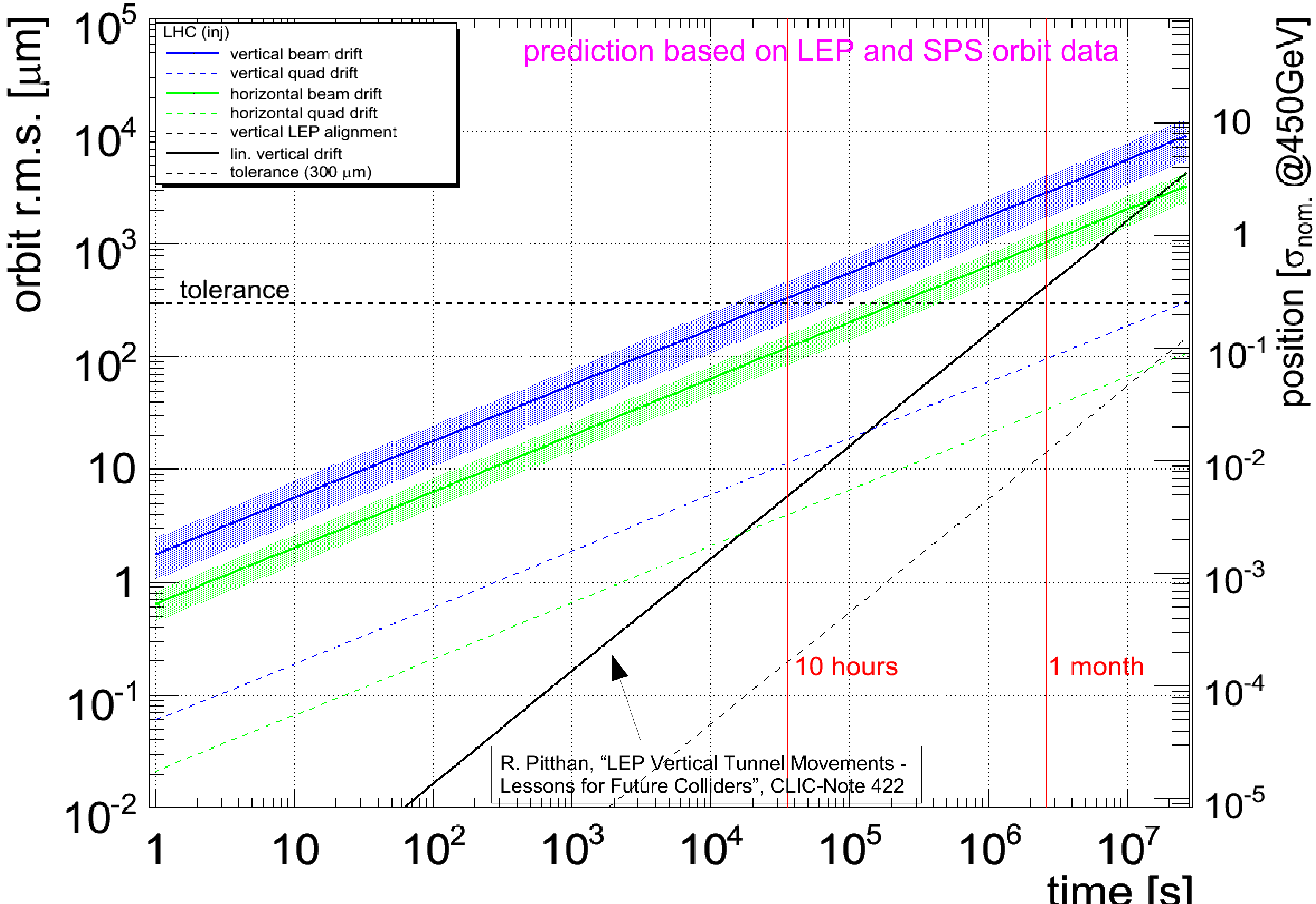
    - LHC injection optics:

κ_H=30.5±11.5 and κ_V=29.6±9.0

    - LHC collision optics:

κ_H=63.3±32.5 and κ_V=62.1±25.5

"Analysis of Ground Motion at SPS and LEP, Implications for the LHC", AB Report to be published



- → closed Orbit drifts after 10 hours ≈ 0.3 σ

Tides and Main Bend Field Uncertainties

- Relative energy shift Δp/p depends on the main dipole field stability:

$$\frac{\Delta p}{p} = \frac{\Delta B}{B} = 10^{-4}\, b_1 \quad [units]$$

  - Decay&Snapback
    - Systematic component: $b_1 \approx 2.6$ units (corrected by feed-forward)
      → $\Delta x = D_{max} \cdot \Delta p/p \cdot = 728\ \mu m \approx 0.64\ \sigma$
    - Random component: $b_1 \approx 0.75$ units
      → random dipole kicks: $\Delta x \approx 126\ \mu m \approx 0.11\ \sigma$

- Moon/sun tides change the geometric circumference of the machine:

$$\frac{\Delta p}{p} = -\frac{1}{\alpha_p} \cdot \frac{\Delta C}{C}$$

  - LHC: $\Delta C \approx \pm 0.5$ mm, momentum compaction factor $\alpha_p = 3.2 \cdot 10^{-4}$
    - $\Delta p/p \approx 5.8 \cdot 10^{-5}$
      → $2\Delta x = 2 \cdot D_{max} \cdot \Delta p/p \cdot = 326\ \mu m \approx 0.29\ \sigma$

- Orbit Correction will consist of two steps (which may alternate repetitively):

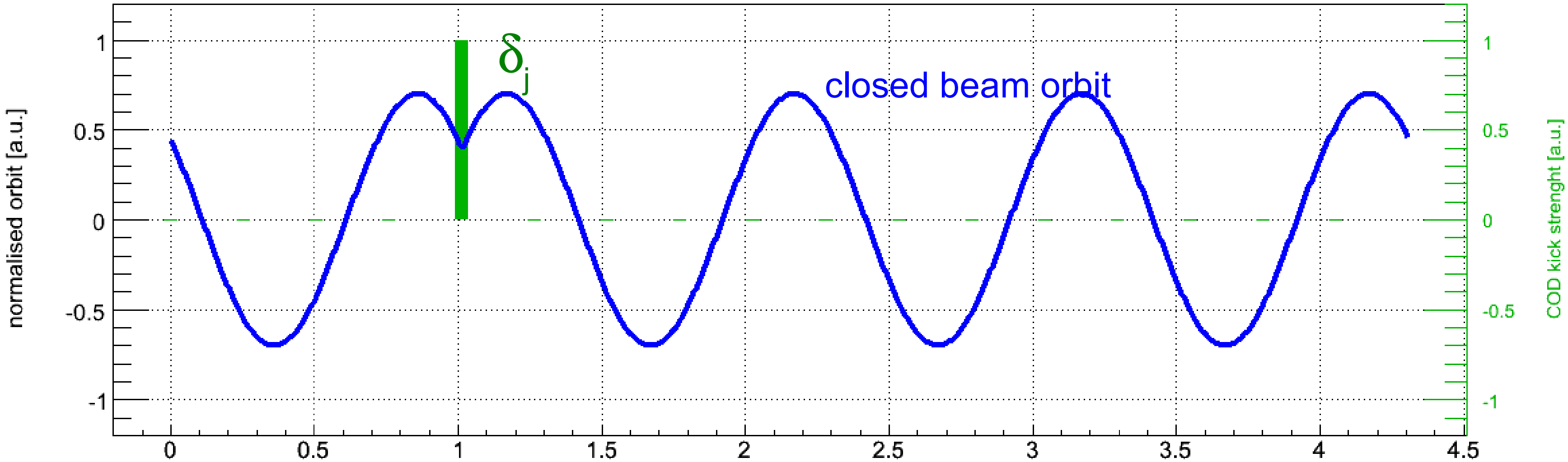
  – Initial setup: "Find a good orbit" (mostly feedback "off")
    - establish circulating beam
    - compensate for each fill recurring large perturbations:
      – static quadrupole misalignments
      – static magnetic dipole field imperfections
      – partially: decay & snapback, ramp, ...
      – ...
    - tune for optimal orbit
      – keep aperture limitation
      – rough jaw-orbit alignment in cleaning insertions
      – Squeeze: minimise the effect of the squeeze due to non-centred beam in IR1 & IR5 quadrupoles (esp. final-focus triplets)

    → reference orbit

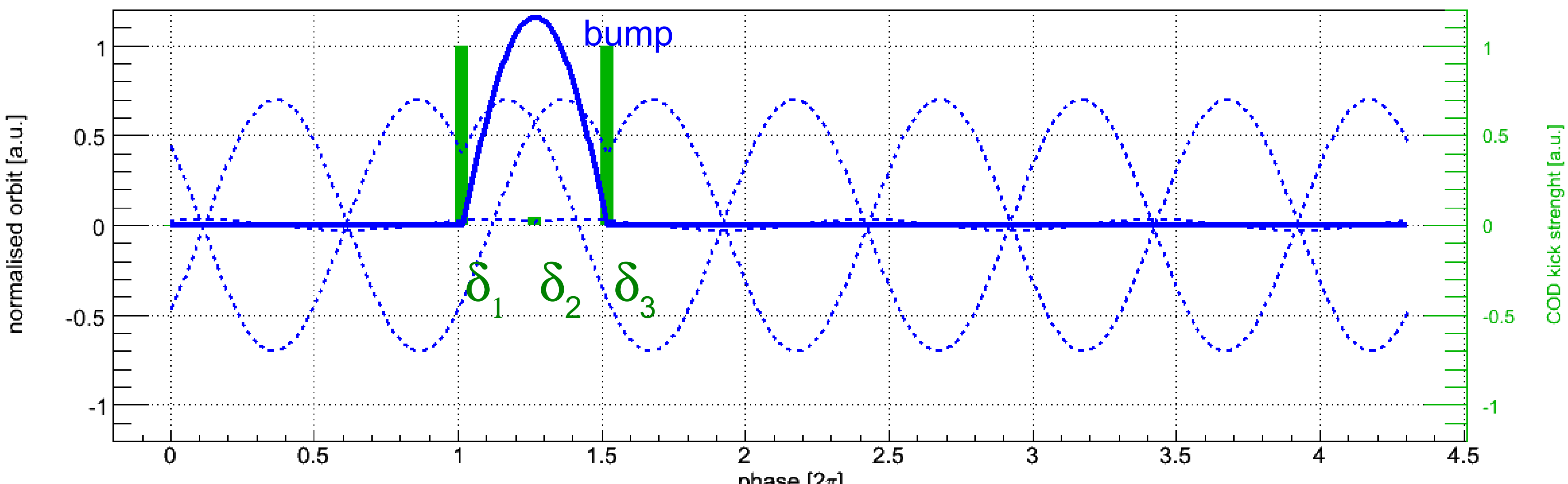
  – During fill: "Stabilise around the reference orbit" (feedback "on"):
    - correct for small and random perturbations Δx
      – environmental effects (ground-motion, girder expansion, ...)
      – compensate for residual decay & snapback, ramp, squeeze
    - optimise orbit stability at collimator jaws/roman pots.

Space Domain: Orbit Response

Each dipole kick δ_j (or shifted quad: $\delta_j = k\,\Delta x$) creates a characteristic betatron oscillation in the accelerator:

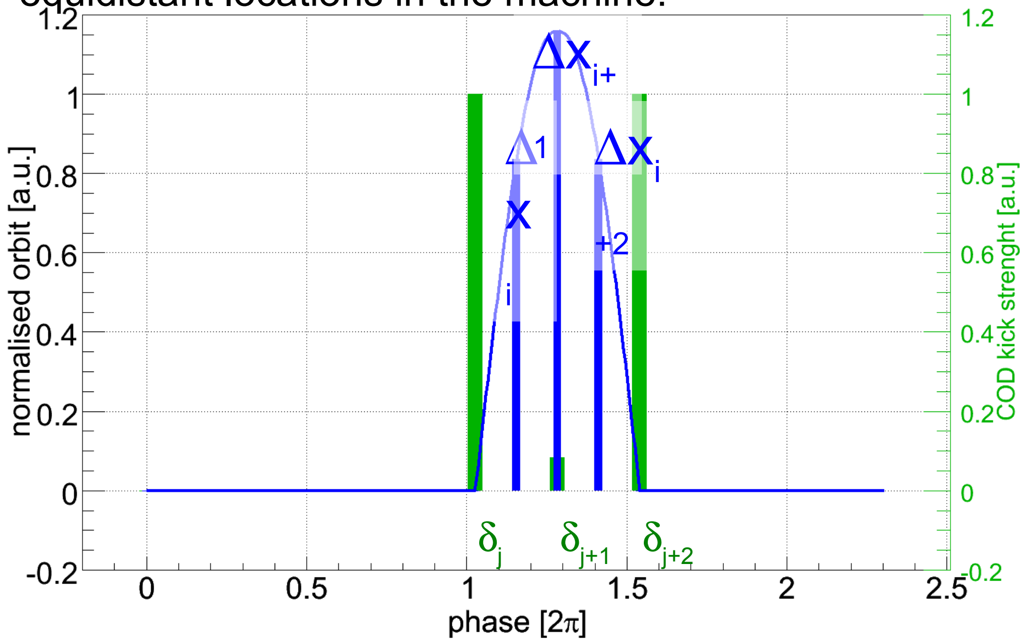


Multiple betatron oscillations superimpose linearly to a resulting closed orbit (e.g. bump):

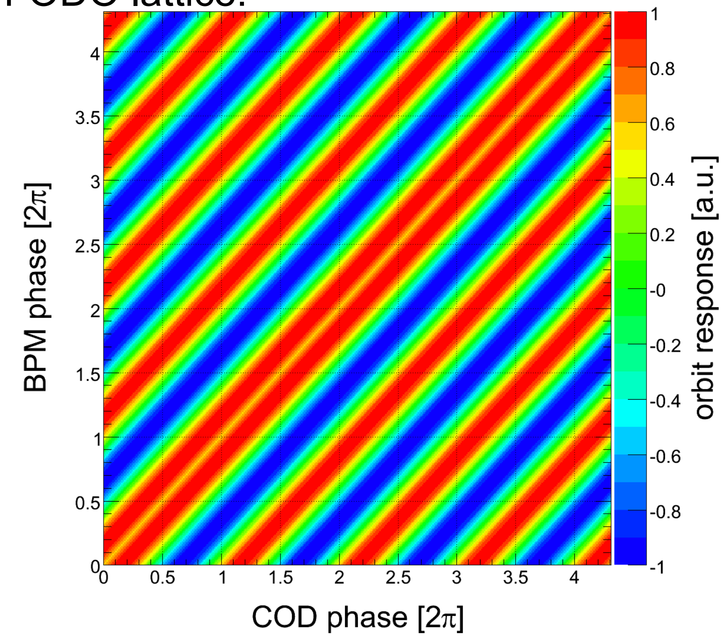


Space Domain: Orbit Response Matrix

The orbit is sampled at *m* discrete not necessarily equidistant locations in the machine:



orbit response matrix example of a regular FODO lattice:



The superimposed beam position shift at the i[th] monitor due to single dipole kicks is described through the orbit response matrix R. It can be written as

$$\Delta x_i = \sum^{n} R_{ij} \cdot \delta_j \quad with \quad R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi Q)} \cdot \cos(\Delta \mu_{ij} - \pi Q)$$

$$\Leftrightarrow \quad \Delta \vec{x} = \sum_{j=0}^{n} \delta_j \vec{u}_j \quad with \quad \vec{u}_j = (R_{1j}, \ldots, R_{mj})^T \Leftrightarrow \boxed{\Delta \vec{x}(t) = \underline{R} \cdot \vec{\delta}_{ss}}$$

where (β,μ,Q) depends on the machine optic (example: Q=4.31).

Space Domain:

Task in space domain:

Solve linear equation system and/or find (pseudo-) inverse matrix R^{-1}

$$\left\|\vec{x}_{ref}-\vec{x}_{actual}\right\|_2=\left\|\underline{R}\cdot\vec{\delta}_{ss}\right\|_2<\epsilon \;\rightarrow\; \vec{\delta}_{ss}=\tilde{R}^{-1}\Delta\vec{x}$$

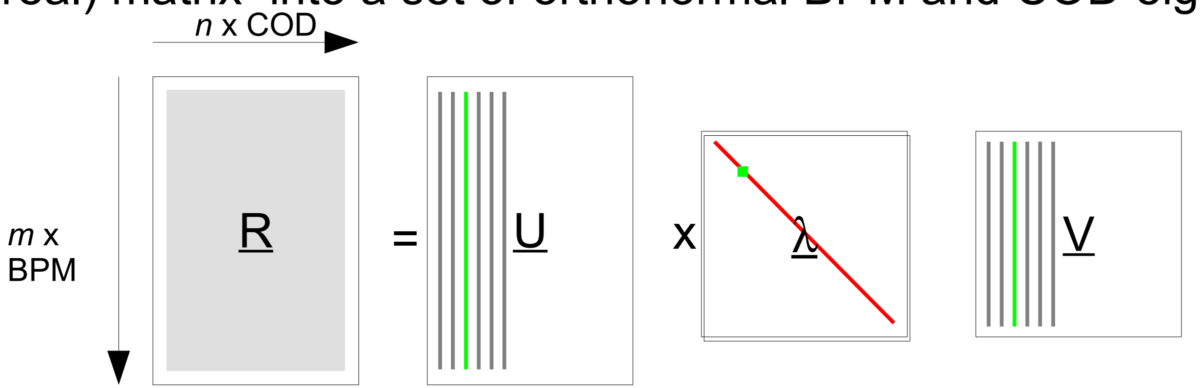
- Singular Value Decomposition (SVD) is the preferred orbit feedback workhorse:
  - standard and proven eigenvalue approach
  - insensitive to COD/BPM faults and their configuration (e.g. spacing)
  - minimises orbit deviations <u>and</u> COD strengths
  - numerical robust:
    - guaranteed solution even if orbit response matrix is (nearly) singular

      (e.g. two CODs have similar orbit response ↔ two rows are (nearly) the same)
    - easy to identify and eliminate singular solutions
  - high complexity:
    - Gauss(MICADO): $O= \frac{1}{2} mn^2 + 1/6\, n^3$
    - SVD: $O= 2mn^2+4n^3$

  m=n: SVD is 9 times more expensive, even on high-end CPUs full initial decomposition may take several seconds (LHC: ~15 s/plan), but once decomposed and inverted: simple matrix multiplication ($O(n^2)$) complexity, LHC: ~15ms!)

Singular Value Decomposition (SVD) on a slide

Theorem from linear algebra*: "It is always possible to decompose a orbit response (real) matrix into a set of orthonormal BPM and COD eigenvectors"



eigen-vector relation:

$$\lambda_i \vec{u}_i = \underline{R} \cdot \vec{v}_i$$

$$\lambda_i \vec{v}_i = \underline{R}^T \cdot \vec{u}_i$$

- final correction is a simple matrix multiplication
- large eigenvalues ↔ bumps with small COD strengths but large effect on orbit

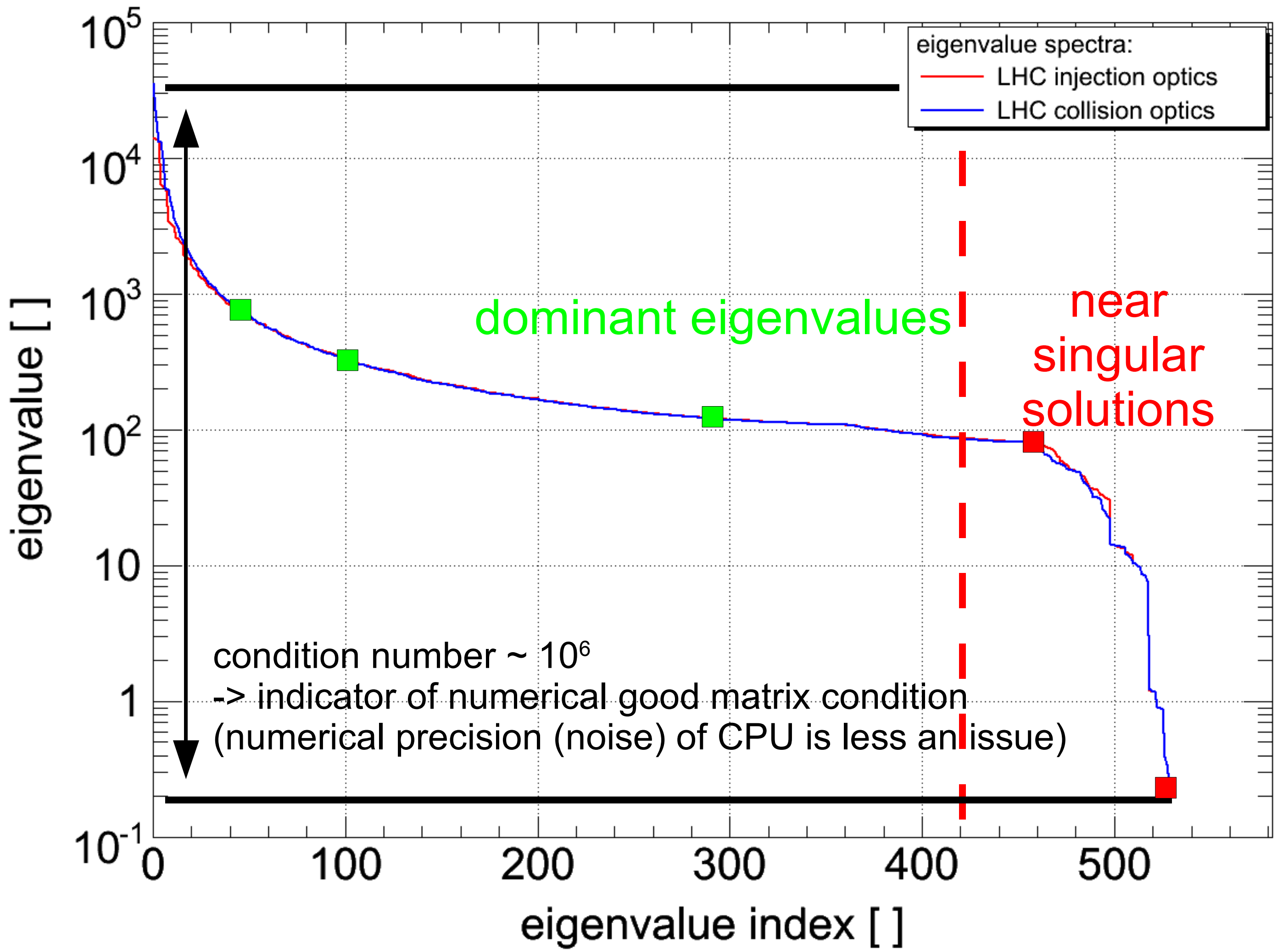
$$\boxed{\vec{\delta}_{ss} = \tilde{R}^{-1} \cdot \Delta \vec{x} \quad with \quad \tilde{R}^{-1} = \underline{V} \cdot \lambda^{-1} \cdot \underline{U}^T} \quad \Leftrightarrow \quad \vec{\delta}_{ss} = \sum_{i=0}^{n} \frac{a_i}{\lambda_i} \vec{v}_i \quad with \quad a_i = \vec{u}_i^T \Delta \vec{x}$$

- Easy removal of singular (=undesired, large corrector strengths) eigen-values/solutions:
  - near singular eigen-solutions have $\lambda_i \sim 0$ or $\lambda_i = 0$
  - to remove those solution: lim $\lambda_i \to \infty$ $1/\lambda_i = 0$
- discarded eigenvalues corresponds to bumps that won't be corrected by the fb

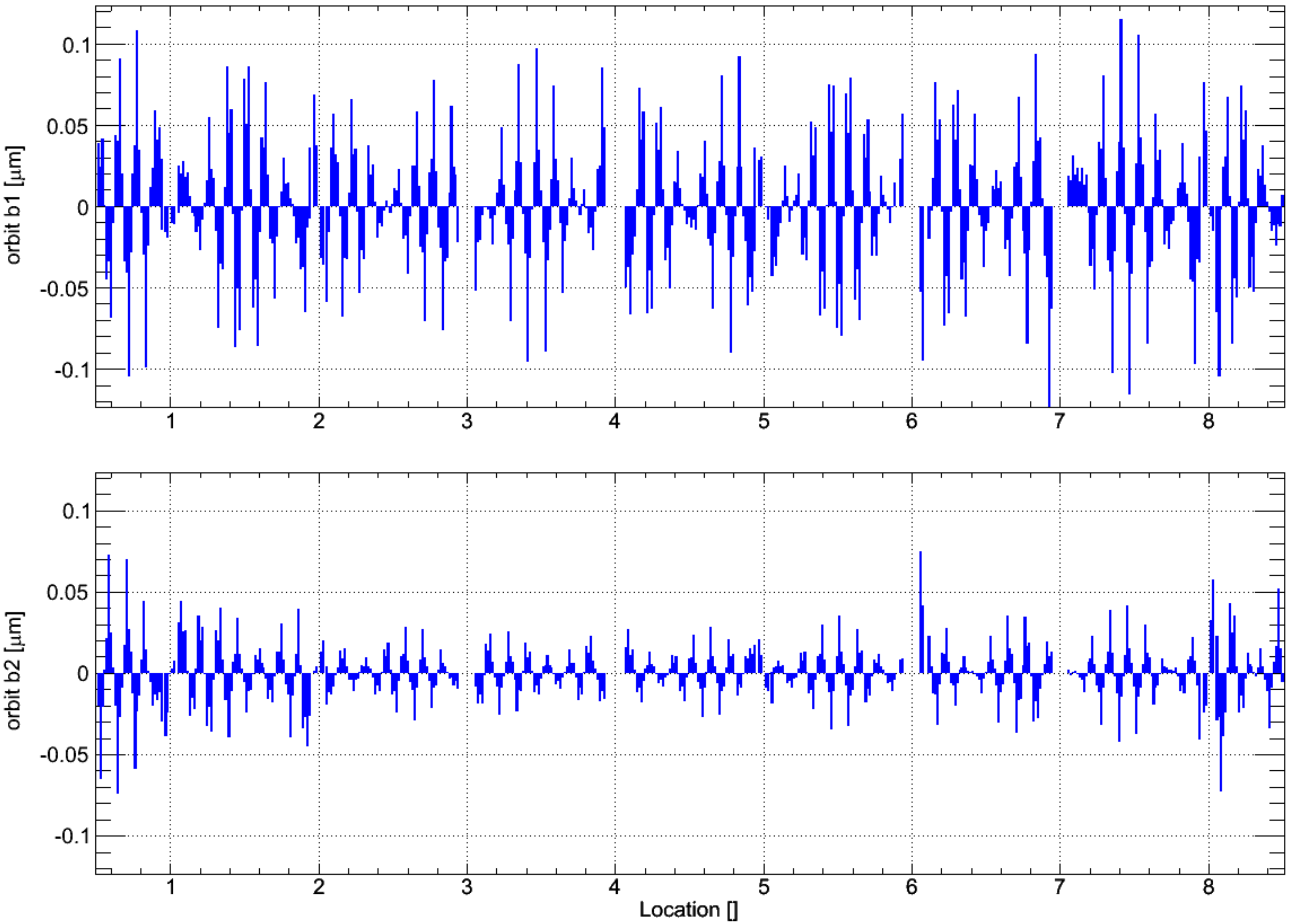
*G. Golub and C. Reinsch, "*Handbook for automatic computation II, Linear Algebra*", Springer, NY, 1971

SVD example: LHC eigenvalue spectrum

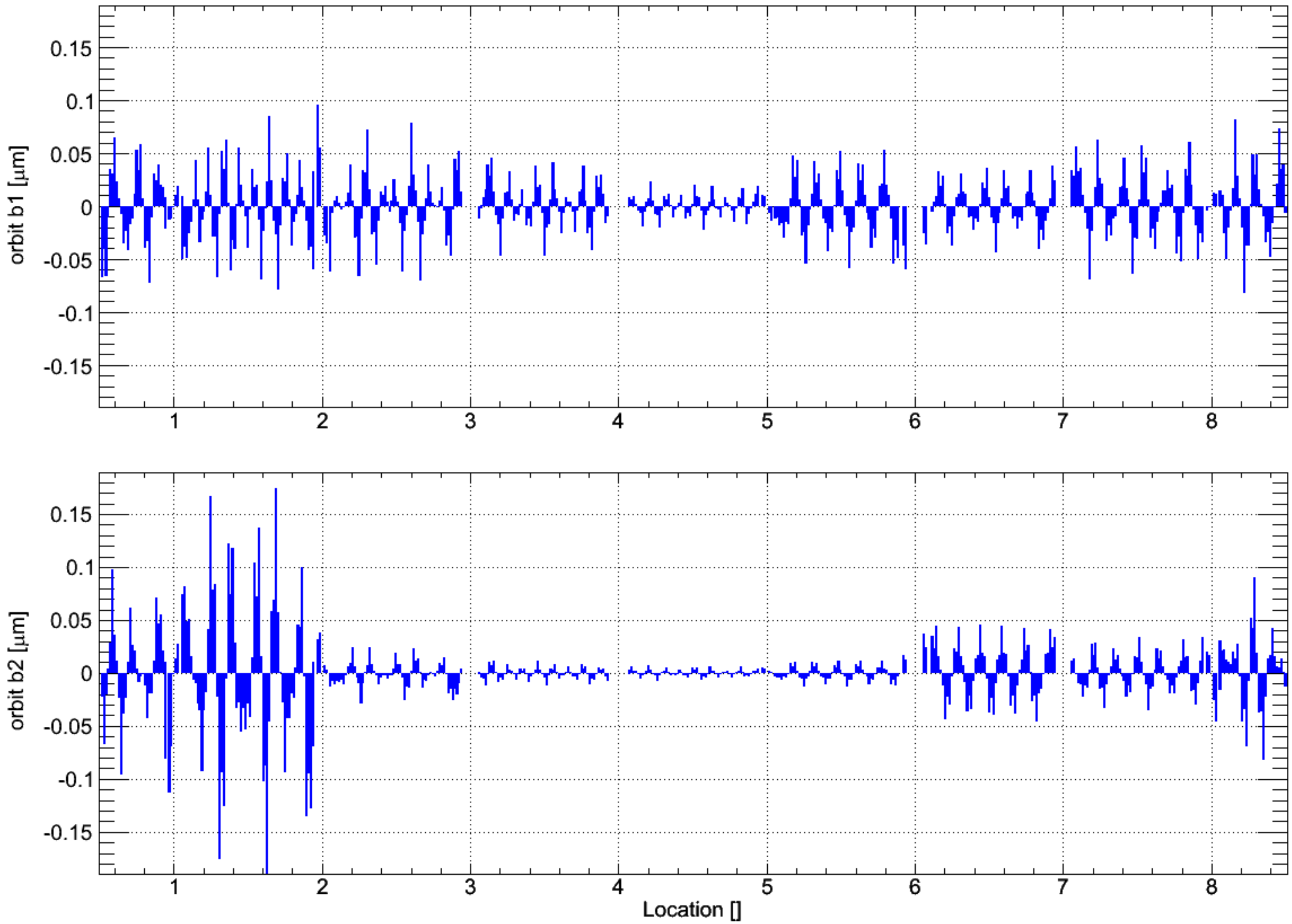
Eigenvalue spectra for vertical LHC response matrix using all BPM and COD:



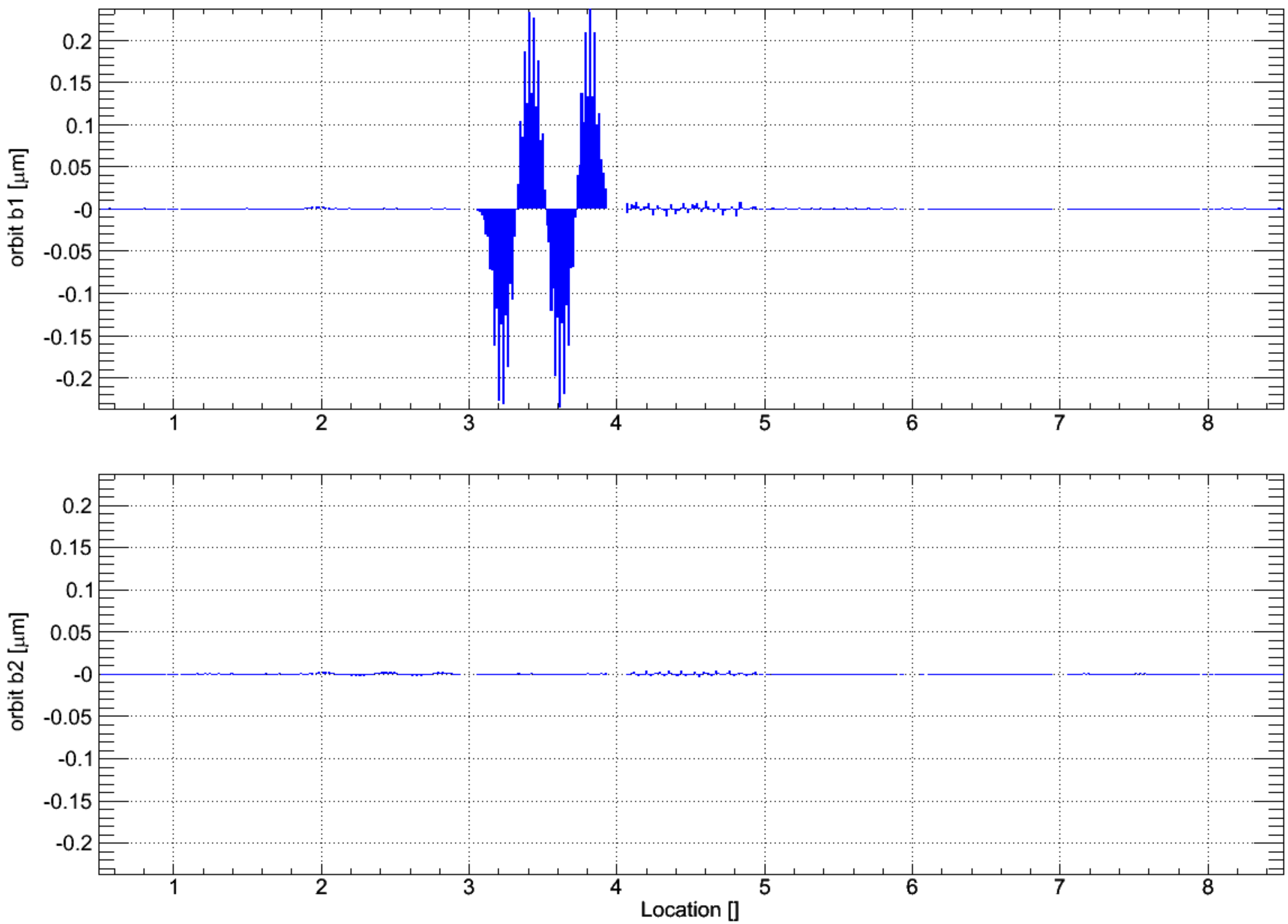
LHC example: BPM eigenvector #50 $\lambda_{50}= 6.69 \cdot 10^2$



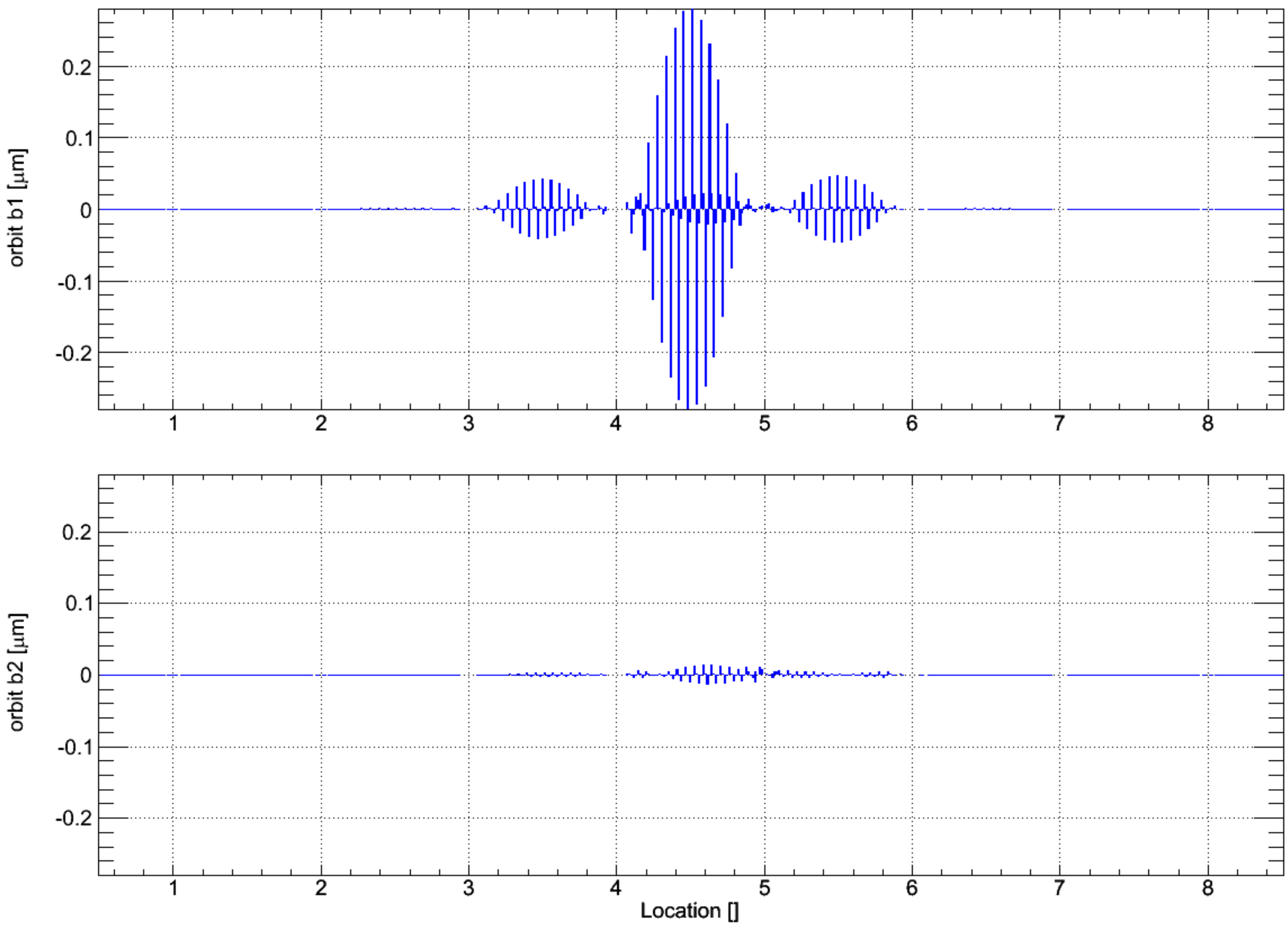
LHC example: BPM eigenvector #100 $\lambda_{100}= 3.38\cdot10^2$



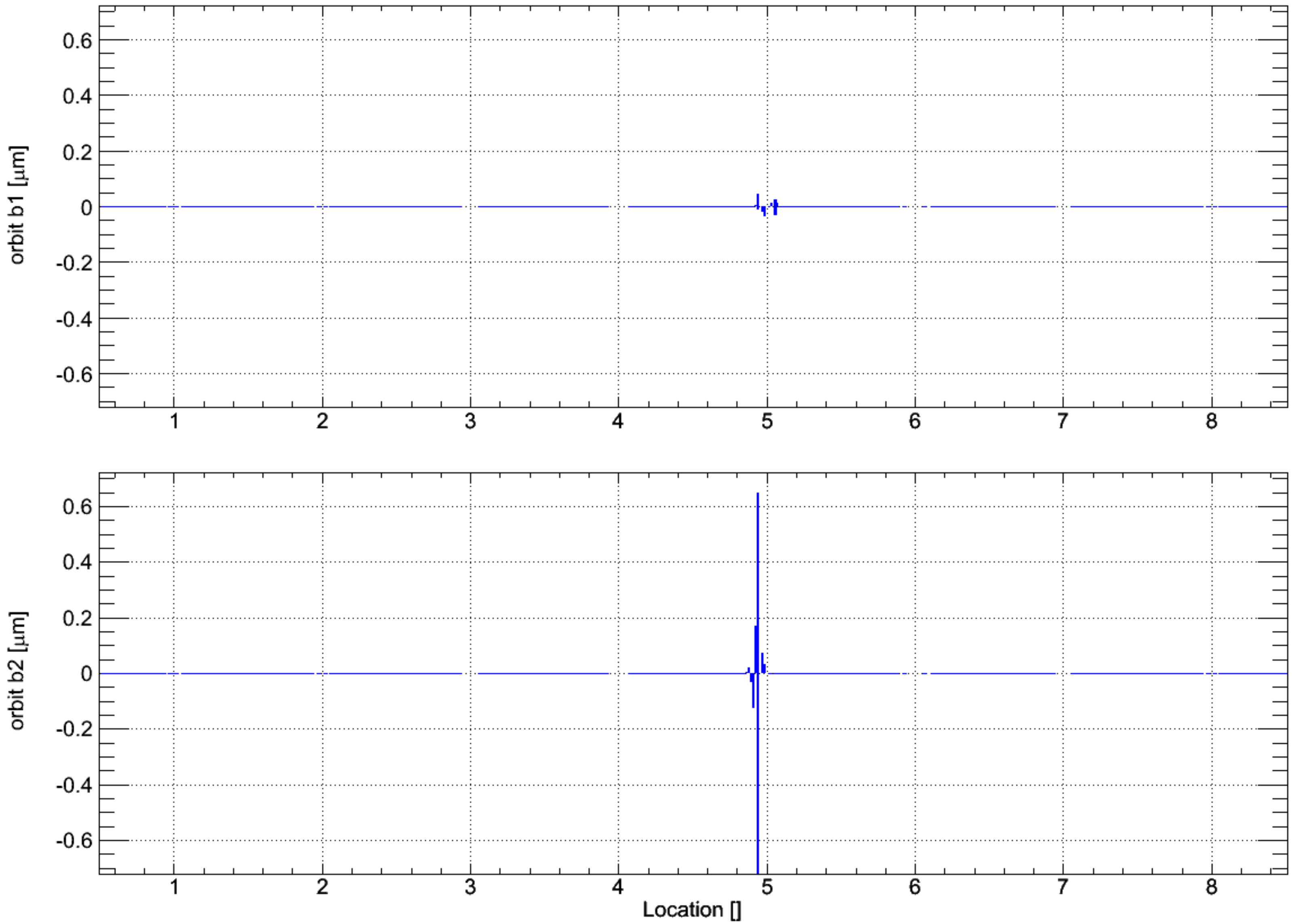
LHC example: BPM eigenvector #291 λ_{291}= 2.13•10^2



LHC example: BPM eigenvector #449 $\lambda_{449} = 8.17 \cdot 10^{1}$



LHC example: BPM eigenvector #521 $\lambda_{521}= 1.18\cdot10^{0}$



Gretchen Frage: “How many eigenvalues should one use?”

low number of eigenvalues:

(e.g. ~20% of total # e-values)

- more global type of correction:
  - use arc BPM/COD to steer in crossing IRs
  - less sensitive to BPM noise
  - less sensitive to single BPM faults/errors
  - less sensitive to single COD/BPM faults/errors
- robust wrt. machine imperfections:
  - beta-beat
  - calibration errors
- easy to set up
- ...
- poor correction convergence
- leakage of local perturbations/errors
  - not fully closed bump affects all IRs
  - squeeze in IR1&IR5 affects cleaning IRs
- ...

high number of eigenvalues:

(still without using singular solutions)

- more local type of correction
  - more precise
  - less leakage of local sources onto the ring
  - perturbations may be compensated at their location
- good correction convergence
- ...
- more prone to imperfections
  - calibration errors more dominant
  - instable for beta-beat > 70%
- more prone to false BPM reading
  - Errors & faults
- ...

orbit stability requirement
feedback stability requirement

Space Domain: local within global correction

- The orbit and feedback stability requirements vary with respect to the location in the two LHC rings. In order to meet both requirements:
  - Implement robust global correction (low number of eigenvalues)
  - fine local correction where required (high number of eigenvalues or simple bumps):
    - Cleaning System in IR3 & IR7
    - Protection devices in IR6
    - TOTEM

coarse global SVD with fine local “SVD patches” (no leakage due to closed boundaries)

minor disadvantage: longer initial computation
(global + local SVD + merge vs one local SVD)

coarse global SVD with weighted monitors where required (ω = 1 ... 10)

disadvantage:
•total number of to be used eigenvalues less obvious
•Matrix inversion may become instable

free orbit manipulation (within limits) while still globally correcting the orbit